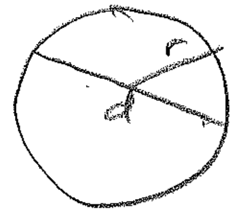
Geometry, 10.1: Circles

Definitions

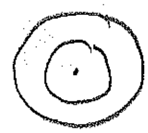
radius – line segment, center to edge of circle

diameter – line segment dividing circle in half

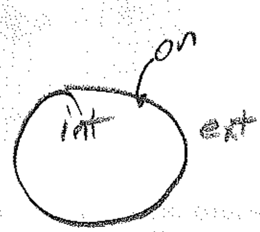


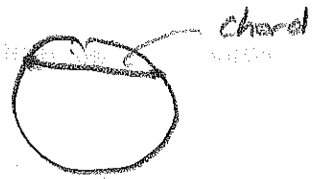
congruent circles – same radius

concentric circles – same center, different radii



interior / exterior of circle



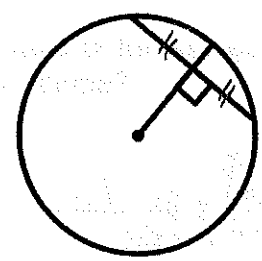


chord – line segment with endpoints on circle

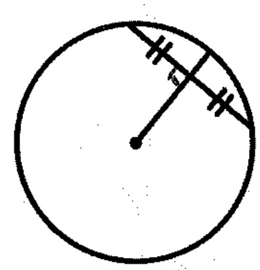
Circumference and Area: $C = 2\pi r$ $A = \pi r^2$

3 related theorems about circles:

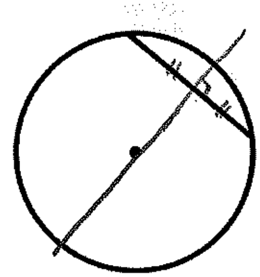
- If a radius is perpendicular to a chord, then it bisects the chord:



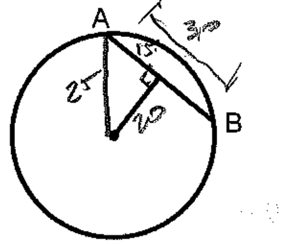
- If a radius bisects a chord, it is perpendicular to the chord:

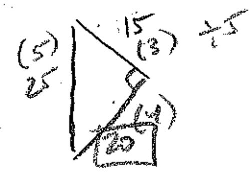


- The perpendicular bisector of a chord passes through the center of a circle:



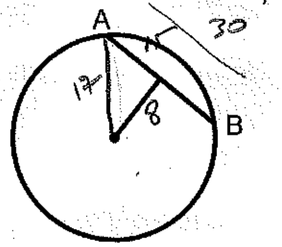
Ex: If AB=~~15~~ 30 and radius=~~10~~ 25, what is the distance from center to the chord AB?





$\boxed{20}$

Ex: If diameter=34, and distance from center to chord is 8, what is the length of the chord?



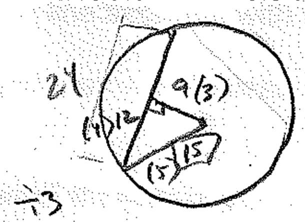
$d = 34$

$r = \frac{1}{2}d = 17$

$\boxed{30}$

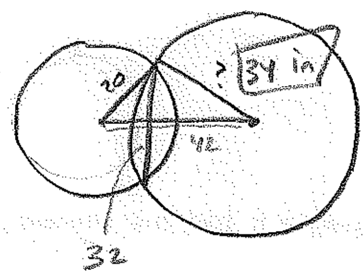
Practice:

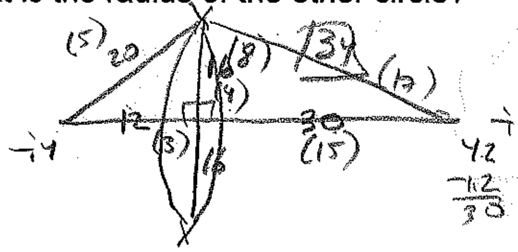
#1. If length of a chord on circle P is 24, and distance from center of circle to the chord is 9, what is the radius of circle P?



$r = \boxed{15}$

#2. Two circles intersect and have a common chord 32 in long. The centers of the circles are 42 in apart. The radius of one circle is 20. What is the radius of the other circle?





$\boxed{34 \text{ in}}$

#3. $\odot P$ just touches the x-axis. P= (20,13) and Q=(24,16).

a) Find the radius of $\odot P$ $= \boxed{13}$

b) Find PQ $= \boxed{5}$

c) Find the length of $\overline{AB}$ $= \boxed{24}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

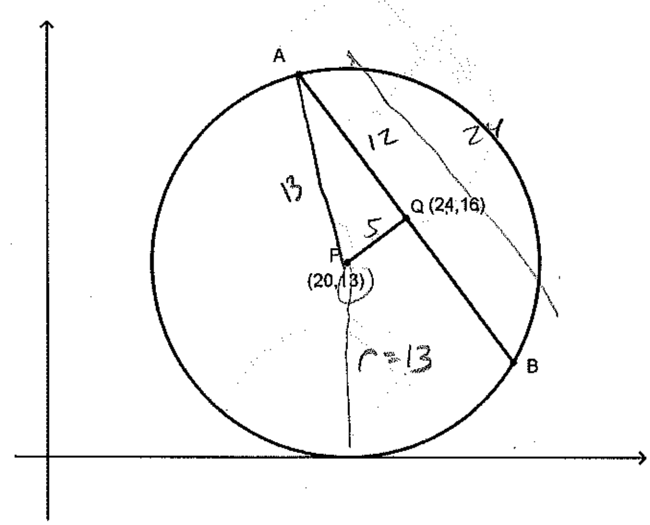
$d = \sqrt{(24-20)^2 + (16-13)^2}$

$d = \sqrt{(4)^2 + (3)^2}$

$d = \sqrt{16 + 9}$

$d = \sqrt{25}$

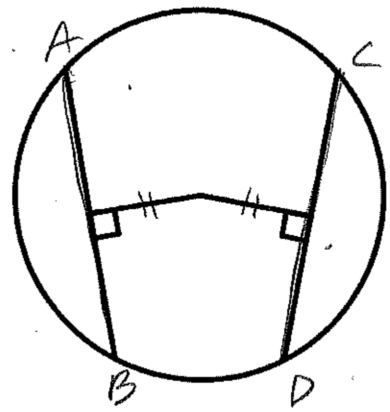
$d = 5$



Geometry, 10.2: Chords and Circles

2 related theorems about chords and circles

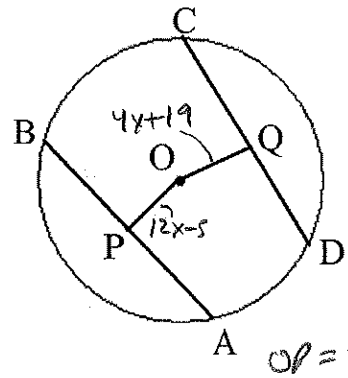
$\overline{AB} \cong \overline{CD}$



- If two chords of a circle are equidistant from the center, then they are congruent.
- If two chords of a circle are congruent, then they are equidistant from the center of the circle.

Examples:

Given: $\odot 0$, $\overline{AB} \cong \overline{CD}$
OP = 12x - 5, OQ = 4x + 19
Find: OP



$\overline{OP} \cong \overline{OQ}$

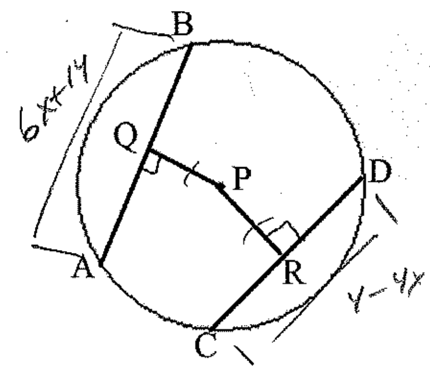
$12x-5=4x+19$
$-4x \quad -4x$
$8x-5=19$
$+5 \quad +5$
$\frac{8x}{8}=\frac{24}{8}$

$x=\frac{24}{8}=3$

$OP = 31$

$OP = 12x-5$
$=12(3)-5$
$=36-5$
$=\boxed{31}$

Given: $\odot 0$, $\overline{PQ} \cong \overline{PR}$, AB = 6x + 14, CD = 4 - 4x
Find: AB



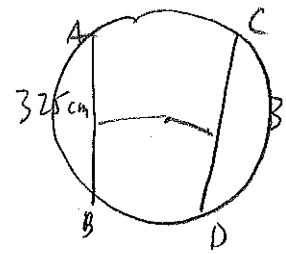
$\overline{AB} \cong \overline{CD}$
$6x+14=4-4x$
$+4x \quad +4x$
$10x+14=4$
$-14 \quad -14$
$\frac{10x}{10}=\frac{-10}{10}$
$x=-1$

$AB=6x+14$
$AB=6(-1)+14$
$AB=-6+14$
$AB=\boxed{8}$

In a circle, chord AB is 325 cm long and chord CD is 3 1/4 m long. Which is closer to the center?

$3\frac{1}{4}m = 3.25\,m \cdot \frac{100\,cm}{1\,m} = 325\,cm$

$4\overline{)1.00}$ → $.25$



$3\frac{1}{4}m = 325\,cm$

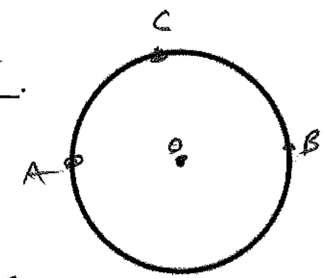
$\overline{AB} \cong \overline{CD}$
so same distance

Geometry, 10.3: Arcs of a Circle

Arcs:

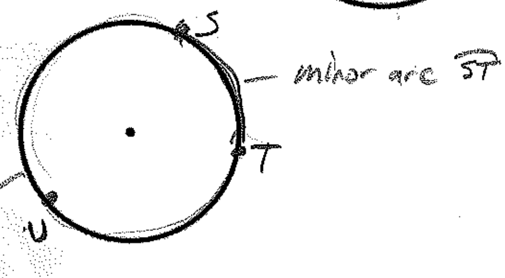
arc = a portion of a circle, consists of 2 endpoints and all the points on the circle between these endpoints:

semicircle: an arc whose endpoints are the endpoints of a diameter.
(Named using 3 points) $\overset{\frown}{ACB}$



minor arc: an arc that is less than a semicircle:
(Named using only the 2 endpoints) $\overset{\frown}{ST}$

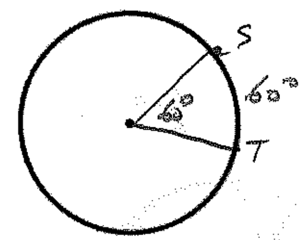
major arc: an arc that is more than a semicircle:
(Named using 3 points)



Measure of an arc:

Minor arc or semicircle: measure of arc is same as the measure of the central angle that intercepts that arc.

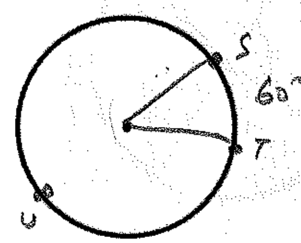
$m\overset{\frown}{ST} = 60^\circ$



Major arc: measure of a major arc is 360° minus the measure of the minor arc with same endpoints.

$m\overset{\frown}{SUT} = 360^\circ - 60^\circ$

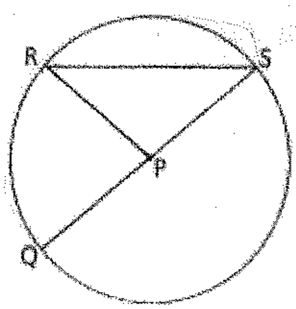
$m\overset{\frown}{SUT} = 300^\circ$



Examples:

1 Match each item in the left column with the correct term in the right column.

a $\overset{\frown}{QRS}$	**1** Radius
b $\overline{QS}$	**2** Diameter
c $\overset{\frown}{RQS}$	**3** Chord
d $\overset{\frown}{RS}$	**4** Minor arc
e $\overline{RS}$	**5** Major arc
f $\angle RPQ$	**6** Semicircle
g $\overline{PS}$	**7** Central angle

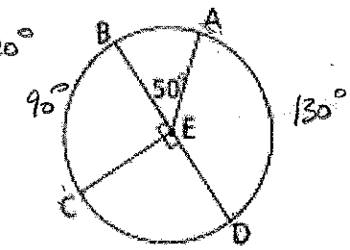


3 In circle E, find each of the following.

a $m\widehat{BC}$ 90° **c** $m\widehat{ACD}$ 230° **e** $m\widehat{ADC}$ 220°

b $m\widehat{AD}$ 130° **d** $m\widehat{BAD}$ 180°

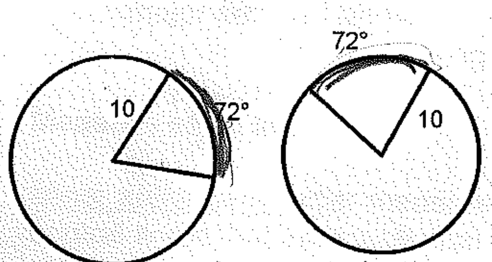
180 − 50 = 130   360 − 130 = 230   360 − 140 = 220



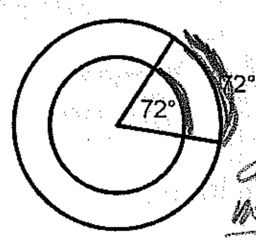
Congruent arcs:

Two arcs are congruent whenever they have the same measure

and are part of the same circle or congruent circles.

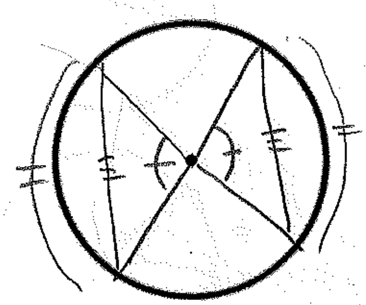


Congruent arcs



arcs not congruent

Congruent chords - arcs - central angles:



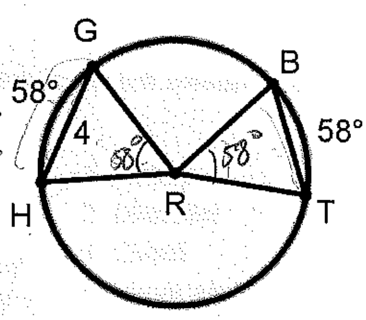
The textbook has 6 theorems on p. 453. They can be summarized as:

In the same circle or in congruent circles:

congruent chords ⟺ congruent arcs ⟺ congruent central angles

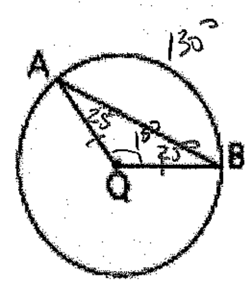
Examples:

Find:
a) BT = 4
b) $m\angle GRH$ = 58°
c) $m\angle BRT$ = 58°



4 Given: ⊙Q, ∠A = 25°
Find: $m\widehat{AB}$ = 130°

$m\angle AQB = 180 - 50$
$= 130°$

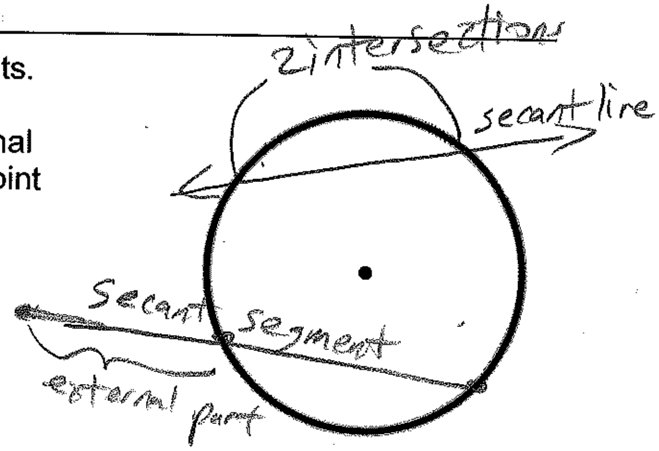


Geometry, 10.4 day 1: Secants and Tangents

Secant – a line that intersects a circle at exactly **two** points.

Secant segment – part of a secant line from an external point, to the farthest intersection point of secant and circle.

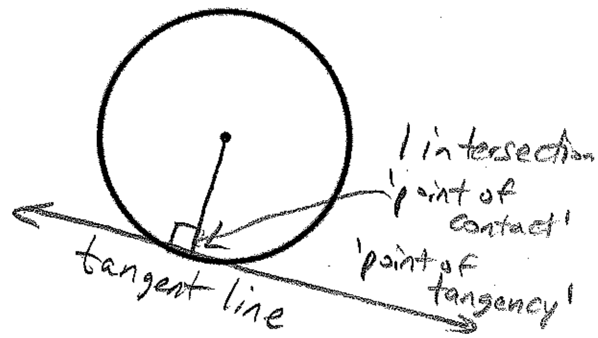
External part – of a secant segment is the part outside the circle.



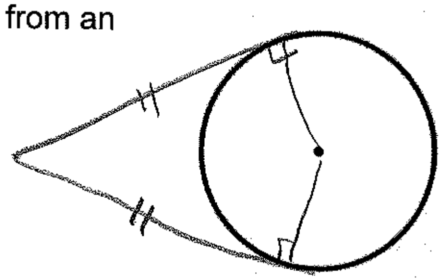
Tangent – a line that intersects a circle at exactly **one** point.

A tangent line is perpendicular to the radius drawn to the point of contact.

If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.



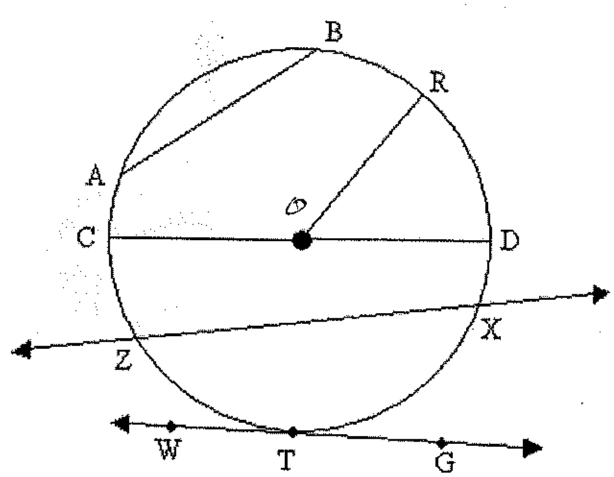
Two-tangent Theorem: If two tangents are drawn to a circle from an external point, then those segments are congruent.



Practice:

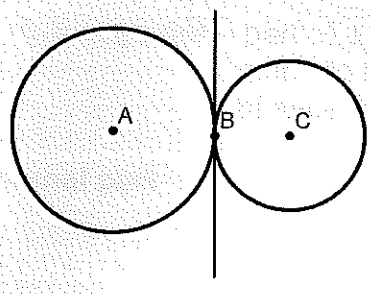
Use the circle to identify the parts of the circle:

Center	O
Radius	$\overline{OR}$
Chord	$\overline{AB}$
Diameter	$\overline{CD}$
Point of tangency	T
Tangent	$\overleftrightarrow{WG}$
Secant	$\overleftrightarrow{ZX}$
Minor arc	$\overset{\frown}{RD}$
Major arc	$\overset{\frown}{RZD}$
Semi-circle	$\overset{\frown}{CTD}$

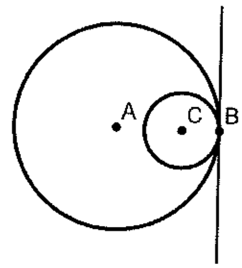


Tangent Circles – circles that intersect each other at exactly one point.

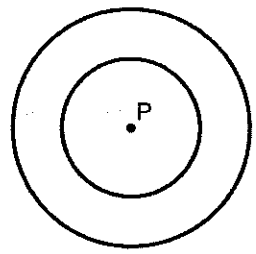
Externally tangent circles



Internally tangent circles

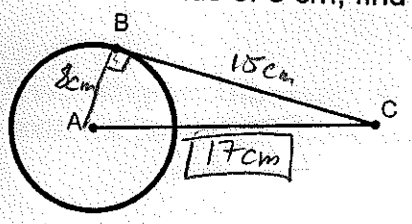


Concentric Circles – have the same center

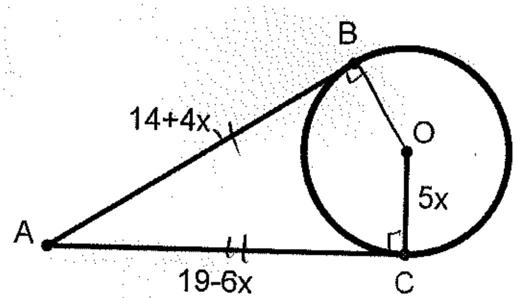


Examples:

If tangent segment BC is 15 cm long, and circle has radius of 8 cm, find length AC.



Find OC:



$14+4x=19-6x$

$+6x \qquad +6x$

$14+10x=19$

$-14 \qquad -14$

$10x=5$

$x=\frac{5}{10}=\frac{1}{2}$

$OC=5x$

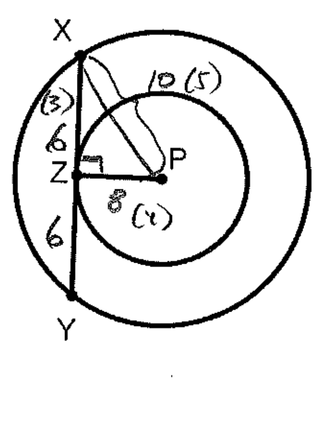
$OC=5\left(\frac{1}{2}\right)$

$OC=\boxed{\frac{5}{2}}$

HW problem #2:

Concentric circles with radii 8 and 10.

Find length XY

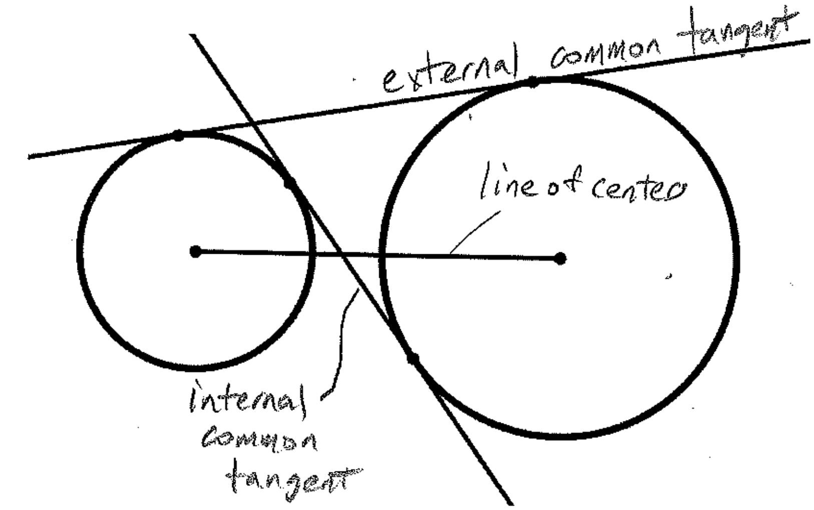


$XY=\boxed{12}$

Line of Centers and Common Tangents:

Line of Centers – line connecting centers of circles.

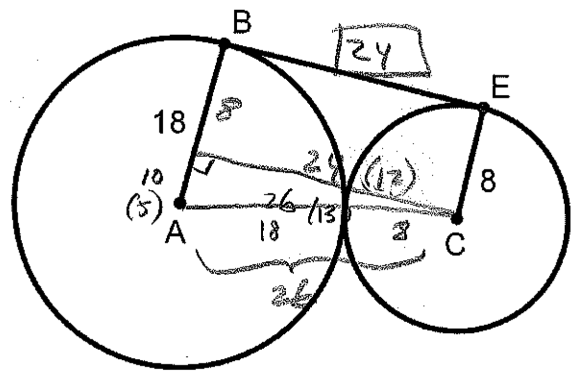
Common Tangent - line tangent to two circles.



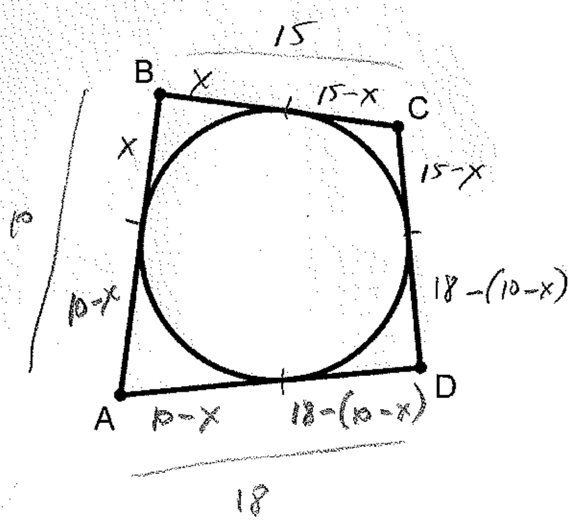
The Common-Tangent Procedure:

1) Draw the segment joining the centers.
2) Draw the radii to the points of contact.
3) Through the center of the smaller circle, draw a line parallel to the common tangent.
4) This line will intersect the radius of the larger circle to form a rectangle and right triangle.
5) Use Pythagorean Theorem and rectangle properties to solve.

Example: Find length BE



Example: 'walk-around' problem:
Each side of ABCD is tangent to the circle.
If AB=10, BC=15, AD=18, find CD.



$CD = 15 - x + 18 - (10 - x)$

$CD = 15 - \cancel{x} + 18 - 10 + \cancel{x}$

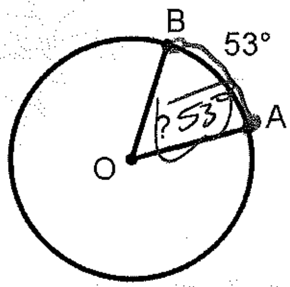
$CD = \boxed{23}$

Geometry, 10.5: Angles Related to a Circle

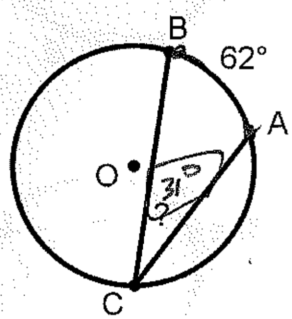
Names of different angles in circles and how angle measure relates to arc measure:

Central angles: **Vertex in center --> angle = arc**

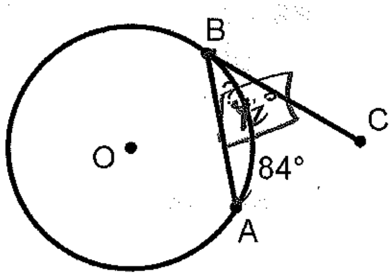
arc = where the sides of angle 'cut' the circle



Inscribed and Tangent-chord angles: **Vertex on circle -> angle = $\frac{1}{2}$ arc**



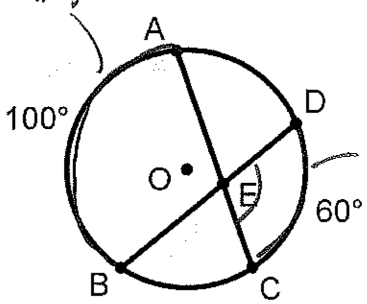
$\frac{62}{2} = 31°$



$\frac{84}{2} = 42°$

Chord-chord angles: **Vertex in circle, not at center -> angle = $\frac{1}{2}$(big arc + little arc)**

big arc = 100°



little arc = 60°

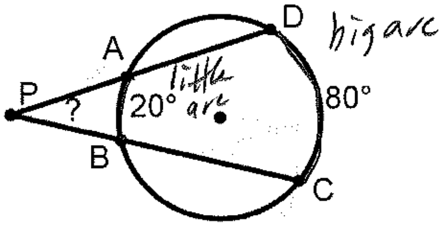
$m\angle E = \frac{1}{2}(100+60)$

$= \frac{1}{2}(160)$

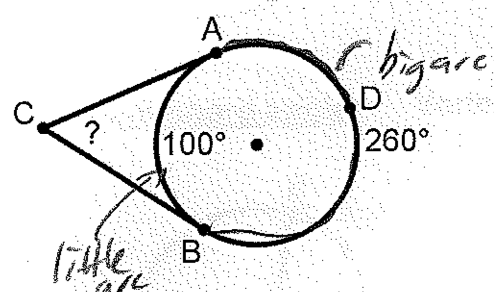
$= \boxed{80°}$

Secant-secant, Tangent-tangent, and Secant-tangent angles:

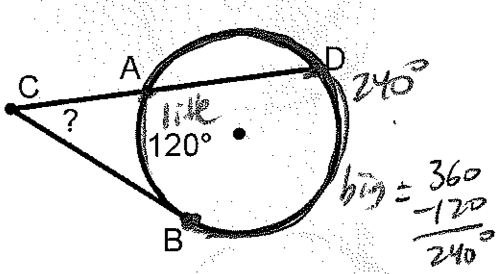
Vertex outside circle -> angle = $\frac{1}{2}$ (big arc - little arc)



$m\angle P = \frac{1}{2}(80 - 20)$
$= \frac{1}{2}(60)$
$= \boxed{30°}$



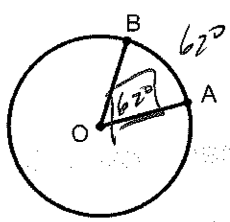
$m\angle C = \frac{1}{2}(260 - 100)$
$= \frac{1}{2}(160)$
$= \boxed{80°}$



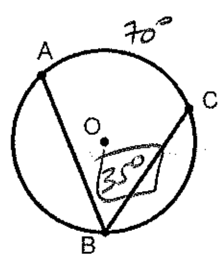
$m\angle C = \frac{1}{2}(240 - 120)$
$= \frac{1}{2}(120)$
$= \boxed{60°}$

Practice:

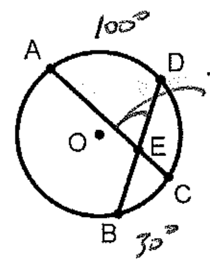
#1. $m\widehat{AB} = 62°$, $m\angle O = ?$



#2. $m\widehat{AC} = 70°$, $m\angle B = ?$

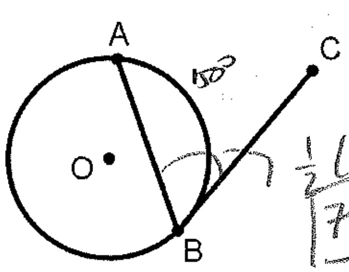


#3. $m\widehat{AD} = 100°$, $m\widehat{BC} = 30°$,
$m\angle AED = ?$



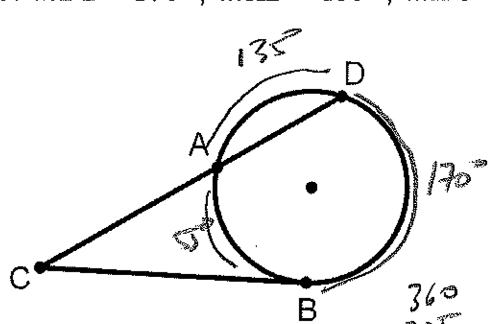
$\frac{1}{2}(100+30)$
$\frac{1}{2}(135)$
$\boxed{65°}$

#4. CB is tangent at B
$m\widehat{AB} = 150°$, $m\angle CBA = ?$



$\frac{1}{2}(150)$
$\boxed{75°}$

#5. $m\widehat{DB} = 170°$, $m\widehat{AD} = 135°$, $m\angle C = ?$



$m\angle C = \frac{1}{2}(170 - 55)$
$= \frac{1}{2}(115)$
$= \boxed{\frac{115}{2}}$

135
+170
305

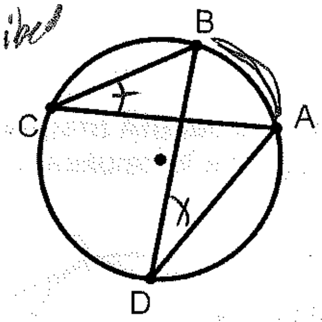
360
−305
55

Geometry, 10.6: Arc-Angle Theorems

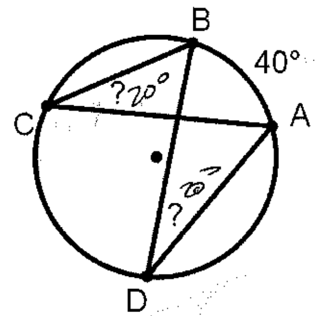
Same arc, Inscribed and Tangent-chord angles

If 2 inscribed angles or 2 tangent-chord angles intercept the same arc, they are congruent.

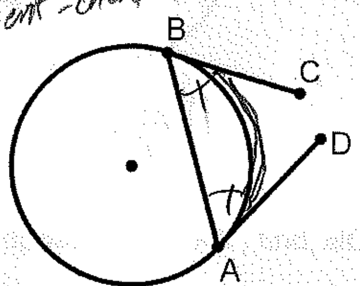
inscribed



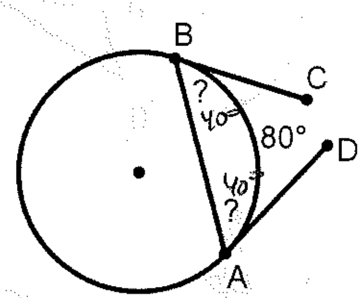
Example:



Tangent-chord



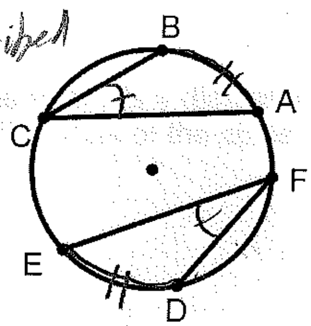
Example:



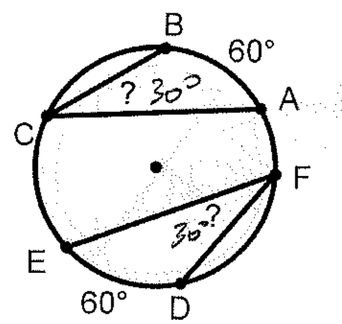
Congruent arcs, Inscribed and Tangent-chord angles

If 2 inscribed angles or 2 tangent-chord angles intercept 2 congruent arcs, they are congruent.

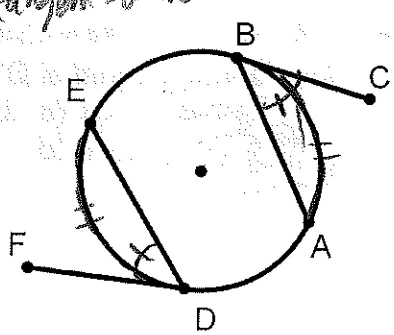
inscribed



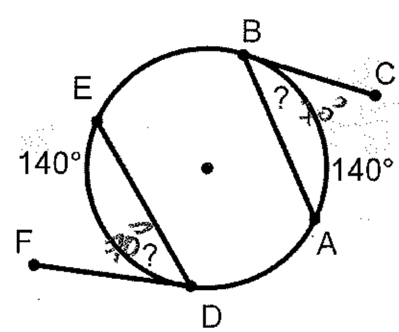
Example:



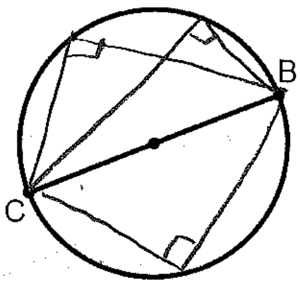
tangent-chord



Example:

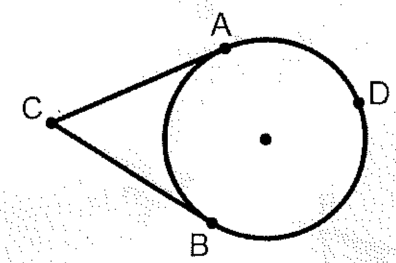


Angles Inscribed in Semicircles – are right angles

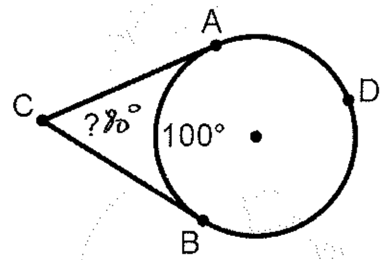


Tangent-Tangent Angles and Minor Arcs

The sum of measures of a tangent-tangent angle and its minor arc is 180.



$m\angle C + m\overparen{AB} = 180°$



Practice:

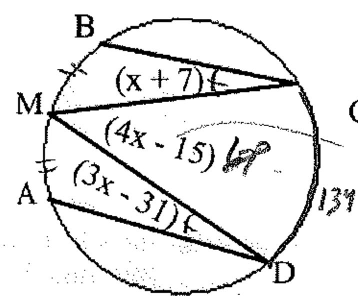
#1. M is midpoint of arc $\overparen{AB}$, find $m\overparen{CD}$ $\boxed{=134°}$

$x+7 = 3x-31$

$7 = 2x-31$

$38 = 2x$

$19 = x$



4(19)−15

76

−15

61

3

19

×4

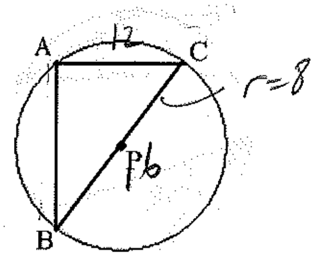
76

67

×2

134

#2. In circle P, BC is a diameter, AC=12 and BC=16.
Find the radius of the circle.



#3. $m\angle S = 88°$, $\overparen{QT} = 104°$, $\overparen{ST} = 94°$

PQ is tangent to the circle.

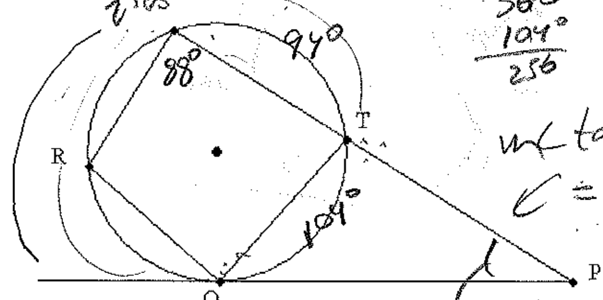
Find: a) $m\angle P$ $\boxed{=76°}$

b) $m\angle STQ$ $\boxed{=83}$

856

−94

168



366

104°

256

m∠ tan-sec ∠

∠ = ½ diff.

256

104

152

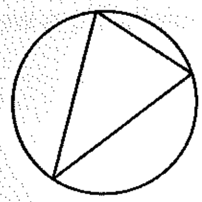
½ (256−104)

½ 152

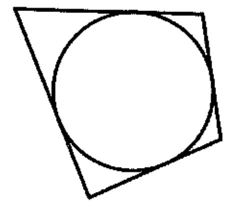
76

Geometry, 10.7: Inscribed and Circumscribed Polygons

Inscribed = drawn inside

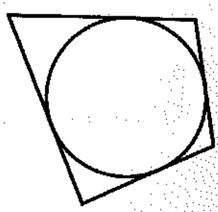


a triangle inscribed in a circle

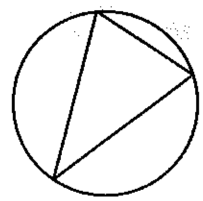


a circle inscribed in a quadrilateral

Circumscribed = drawn around



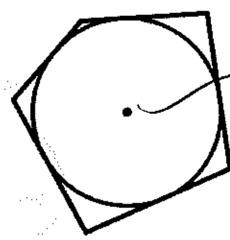
a quadrilateral circumscribed about a circle



a circle circumscribed about a triangle

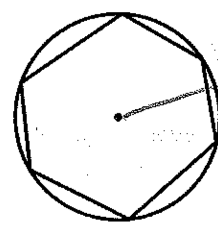
Incenter and Circumcenter of a polygon:

Incenter = center of circle inscribed in a polygon



in center of polygon

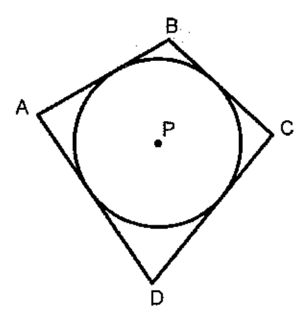
Circumcenter = center of circle circumscribed about a polygon



circumcenter of polygon

Example: Is P an incenter or a circumcenter of polygon ABCD?

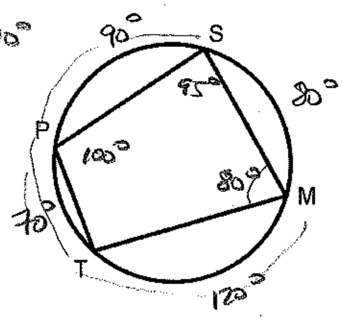
(circle is 'in' the polygon)



Example: Given the diagram, and $\widehat{SM} = 80°$, $\widehat{PS} = 90°$, $\widehat{PT} = 70°$ find:

a) TM $= 120°$

b) the 4 angles of the polygon



$$\left.\begin{array}{l} m\angle M = \tfrac{1}{2}(90+70) = \tfrac{1}{2}(160) = 80° \\ \angle P = \tfrac{1}{2}(80+120) = \tfrac{1}{2}(200) = 100° \end{array}\right] + 180°$$

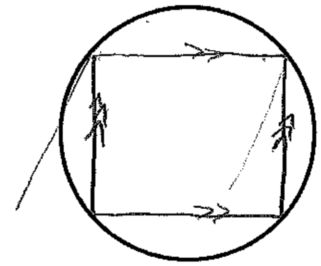
$$\left.\begin{array}{l} m\angle S = \tfrac{1}{2}(120+70) = \tfrac{1}{2}(190) = 95° \\ m\angle T = \tfrac{1}{2}(90+80) = \tfrac{1}{2}(170) = 85° \end{array}\right] + 180°$$

$$90 + 80 + 70 = 240° \qquad 360 - 240 = 120$$

What do you notice about the sum of opposite angles? they add to 180°

Theorem: If a quadrilateral is inscribed in a circle, its opposite angles are supplementary (add to 180°).

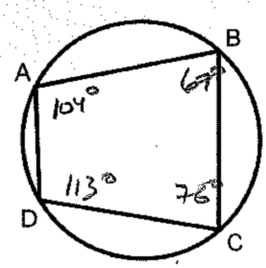
Try this: Draw a parallelogram inscribed in a circle:



Theorem: If a parallelogram is inscribed in a circle, it must be a rectangle

Practice:

#1. Given: $\angle A = 104°, \angle B = 67°$
Find: $m\angle C$ and $m\angle D$



$m\angle C = 180 - 104 = \boxed{76°}$

$m\angle D = 180 - 67° = \boxed{113°}$

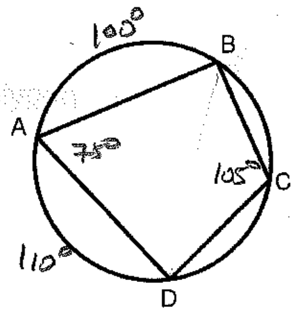
$\begin{array}{r} 180 \\ -104 \\ \hline 76 \end{array}$ $\begin{array}{r} 180 \\ -67 \\ \hline 113 \end{array}$

#2. Given: $m\widehat{AD} = 110°, m\widehat{AB} = 100°$
Find: $m\angle C$ and $m\angle A$

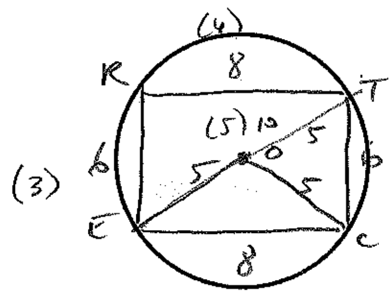
$m\angle C = \frac{1}{2}(100 + 110) = \boxed{105°}$

$m\angle A = 180 - 105 = \boxed{75°}$

$2\overline{)210}$ 105



#3. Parallelogram RECT is inscribed in circle O. If RE=6 and EC=8, find the perimeter of triangle ECO.



$P = 5 + 5 + 8 = \boxed{18}$

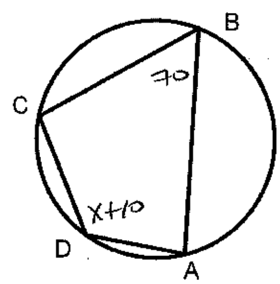
#4. Given: $m\angle D = x + 10, m\angle B = 70$
Find: $m\angle D$

$x + 10 + 70 = 180$

$x + 80 = 180$

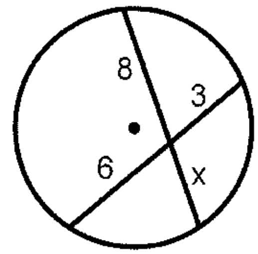
$x = 100$

$m\angle D = x + 10 = 100 + 10 = \boxed{110°}$



Geometry, 10.8 day 1: The Power Theorems

Chord-Chord Power Theorem: If 2 chords of a circle intersect:
the 2 pieces of one chord multiplied = the 2 pieces of the other chord multiplied



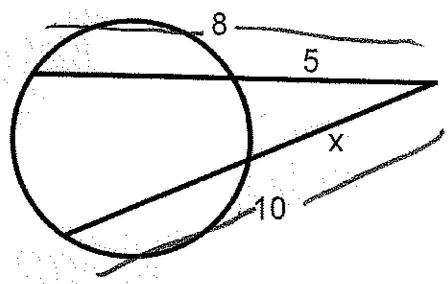
$8 \cdot x = 6 \cdot 3$

$8x = 18$

$x = \frac{18}{8}$

$x = \boxed{\frac{9}{4}}$

Secant-Secant Power Theorem: For 2 secants from same external point:
(whole secant) x (external part) = (whole secant) x (external part)

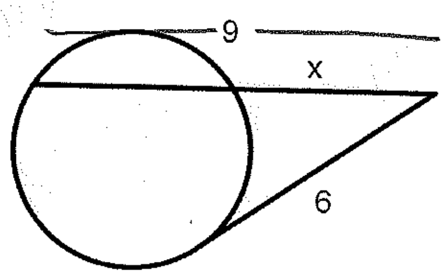


$8 \cdot 5 = 10 \cdot x$

$40 = 10x$

$\boxed{4 = x}$

Tangent-Secant Power Theorem: For tangent and secant from same external point:
(whole secant) x (external part of secant) = (tangent)^2



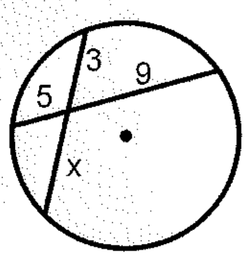
$9 \cdot x = 6^2$

$9x = 36$

$\boxed{x = 4}$

Practice: Find x

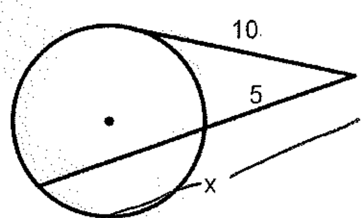
Find radius of circle:



$3x = 9 \cdot 5$

$x = \frac{\cancel{3}\,9 \cdot 5}{\cancel{3}}$

$\boxed{x = 15}$

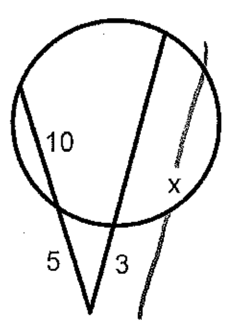


$5 \cdot x = 10^2$

$5x = 100$

$x = \frac{100}{5}$

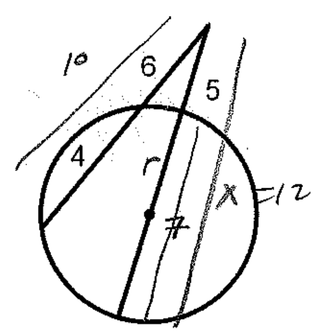
$\boxed{x = 20}$



$10 \cdot 5 = 3 \cdot x$

$50 = 3x$

$\boxed{\frac{50}{3} = x}$



$6 \cdot 10 = 5 \cdot x$

$60 = 5x$

$\frac{60}{5} = x$

$12 = x$

$r = \boxed{\frac{7}{2}}$

Geometry, 10.9: Circumference and Arc Length

Circumference of a circle = distance around the circle: $C = 2\pi r$

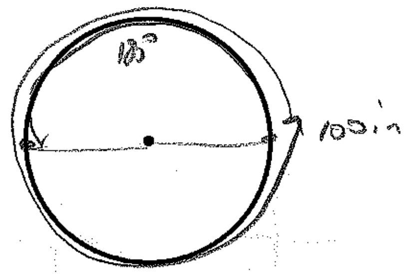
Arc length = a fraction of the circumference = (fraction) x (circumference)

Example: If a circle's circumference is 100 inches, what is the arc length of an arc with measure of 180°?

formula

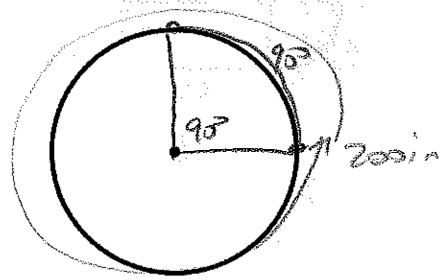
$$\text{arc length} = \frac{arc}{360} 2\pi r$$

$$\text{arc length} = (\text{fraction}) \times (\text{circumference})$$
$$= \left(\frac{180}{360}\right) \times (100 \text{ in})$$
$$= \left(\frac{1}{2}\right) \times 100 \text{ in}$$
$$= \boxed{50 \text{ in}}$$



Example: If a circle's circumference is 200 inches, what is the arc length of an arc with measure of 90°?

$$\text{arc length} = (\text{fraction}) \times (\text{circumference})$$
$$= \left(\frac{90}{360}\right) \times (200 \text{ in})$$
$$= \left(\frac{1}{4}\right) \times (200 \text{ in})$$
$$= \frac{1}{4} \cdot \frac{200}{1} = \frac{200}{4} = \boxed{50 \text{ in}}$$

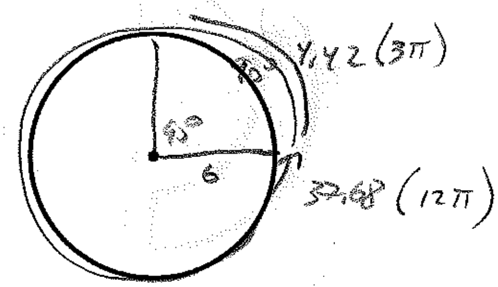


Example: If a circle's radius is 6, what is the arc length of a 90° arc?

$$\text{arc length} = (\text{fraction}) \times (\text{circumference})$$
$$= \left(\frac{90}{360}\right) \times (2\pi r) \rightarrow \frac{12\pi}{4} = 3\pi$$
$$= \left(\frac{1}{4}\right) \times (2\pi 6)$$
$$= \left(\frac{1}{4}\right) \times (12\pi) = \left(\frac{1}{4}\right)(12 \cdot 3.14)$$
$$= \left(\frac{1}{4}\right) \times (37.68) = \frac{37.68}{4} = \boxed{4.42}$$

3.14 × 12: 628, 3140, 37.68

$37.68 \div 4 = 4.42$



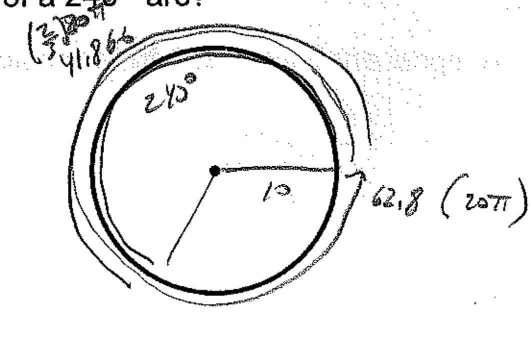
Example: If a circle's radius is 10, what is the arc length of a 240° arc?

$$\text{arc length} = (\text{fraction}) \times (\text{circumference})$$
$$= \left(\frac{240}{360}\right) \times (2\pi 10)$$
$$= \left(\frac{2}{3}\right) \times (20\pi) \rightarrow \frac{2}{3} \cdot \frac{20\pi}{1} = \frac{40\pi}{3}$$
$$= \left(\frac{2}{3}\right) \times (62.8)$$
$$= 41.866$$

3.14 × 20 = 6280

62.8 × 2 = 125.6

$125.6 \div 3 = 41.866$

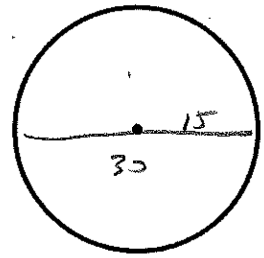


HW #4. A bicycle has wheels 30 cm in diameter. Find, the to nearest tenth of a centimeter, the distance that the bicycle moves forward during:
a) 1 revolution b) 10 revolutions

dist. = circumf.

$\boxed{94.2\text{ cm}}$

$10 \times 94.2 = \boxed{942.0\text{ cm}}$



$C = 2\pi r$
$= 2\pi(15)$
$= 30\pi$
$= 30(3.14)$
$C = 94.20$

$3.14 \times 30 = 94.20$

HW #5a. Find the complete perimeter of the figure.

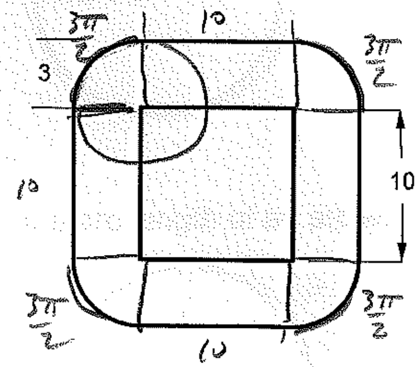
each corner:

$\frac{1}{4}$

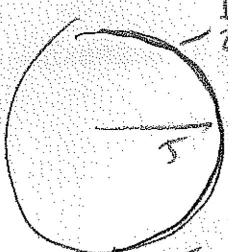
$\frac{6\pi}{4} = \frac{3\pi}{2}$

$C = 2\pi 3$
$= 6\pi$

$P = 40 + 4\cdot\left(\frac{3\pi}{2}\right)$
$= \boxed{40 + 6\pi}$

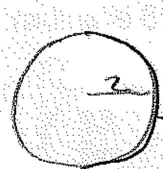


HW #5d. Find the complete perimeter of the figure.



$\frac{1}{2} 10\pi$
5π

$C = 2\pi 5$
$= 10\pi$

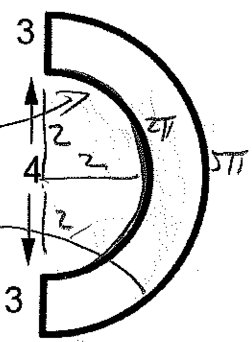


$\frac{1}{2} 4\pi$
$= \boxed{2\pi}$

$C = 2\pi 2$
$= 4\pi$

circle $r = 2$

circle $r = 2 + 3$ $= 5$



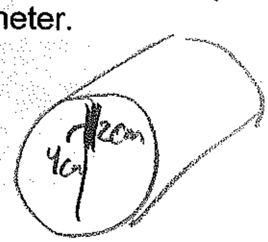
2π

5π

$P = 2\pi + 5\pi + 3 + 3 = \boxed{6 + 7\pi}$

HW #10. There are 100 turns of thread on a spool with a diameter of 4 cm. Find the length of the thread to the nearest centimeter.

$3.14 \times 4 = 12.56$



$C = 2\pi r$
$= 2\pi 2$
$= 4\pi$ cm
$= 4(3.14)$
$= 12.56$ cm

100 turns $\times$ 12.56 cm $= \boxed{1256\text{ cm}}$