Geometry, 11.1: Area - Rectangles

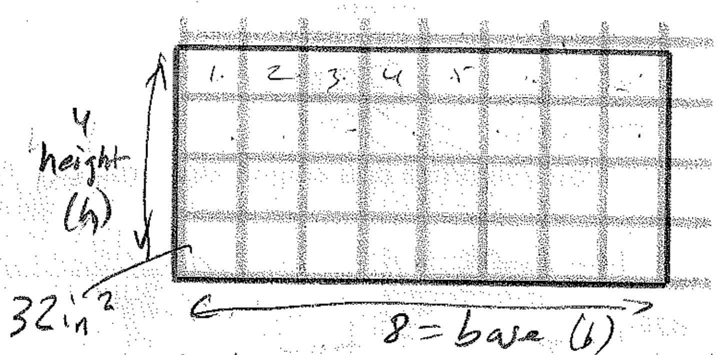
Measure length => distance

← linear units (ex: in, m, ft, miles, km)

Measure space inside a closed figure => area

1 square in
1 sq. in
1 in^2

square units
(ex. in^2, ft^2, m^2, km^2)

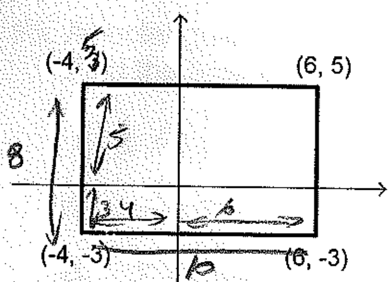


Count squares —or— multiply base x height

Area of a rectangle = $b \cdot h$

Examples:

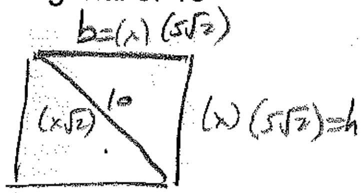
#1. Find the area



$A = (10)(8)$

$\boxed{A = 80\ u^2}$

#2. Find the area of a square with diagonal of 10



$x\sqrt{2} = 10$

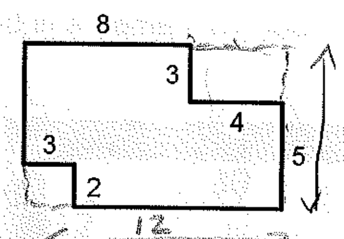
$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{10}{\sqrt{2}}$

$x = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

$A = (5\sqrt{2})(5\sqrt{2})$

$A = 25 \cdot 2 = \boxed{50\ u^2}$

#3. Find the area



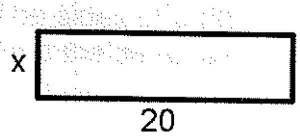
12
x 8
96
−18
78

$A = A_{big} - A_{corner_1} - A_{corner_2}$

$A = (12)(8) - (3)(2) - (3)(4)$

$A = 96 - 6 - 12 = \boxed{78\ u^2}$

#4. The area of the rectangle is 100.
Find x.

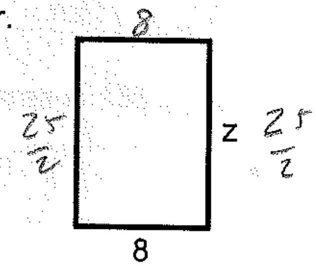


$A = 20x$

$\frac{100}{20} = \frac{20x}{20}$

$\boxed{5 = x}$

#5. The area of the rectangle is 100.
Find the perimeter.



$A = 8z$

$\frac{100}{8} = \frac{8z}{8}$

$\frac{25}{2} = \frac{50}{4} = \frac{100}{8} = z$

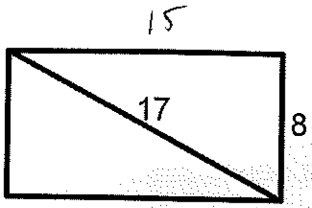
$P = 8 + 8 + \frac{25}{2} + \frac{25}{2}$

$P = 16 + \frac{50}{2}$

$P = 16 + 25 = \boxed{41}$

Practice:

#1. Find the area.

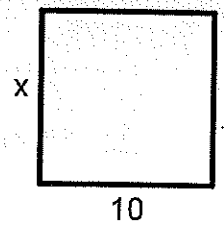


$15 \times 8 = 120$

$A = (8)(15)$

$\boxed{A = 120\,u^2}$

#2. Area = 100. Find x.

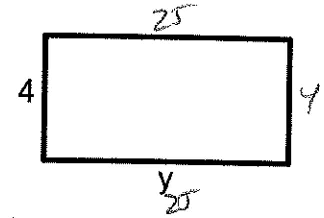


$A = 10x$

$100 = 10x$

$\boxed{x = 10}$

#3. Area = 100.
Find y and the perimeter.



$A = 4y$

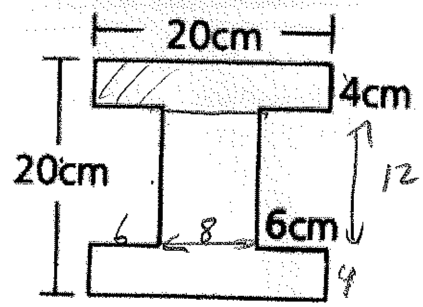
$100 = 4y$

$\boxed{25 = y}$

$P = 50 + 8$

$\boxed{P = 58}$

#4. A cross section of a steel I-beam is shown. Assume right angles and symmetry from appearances. Find the area of the cross section.



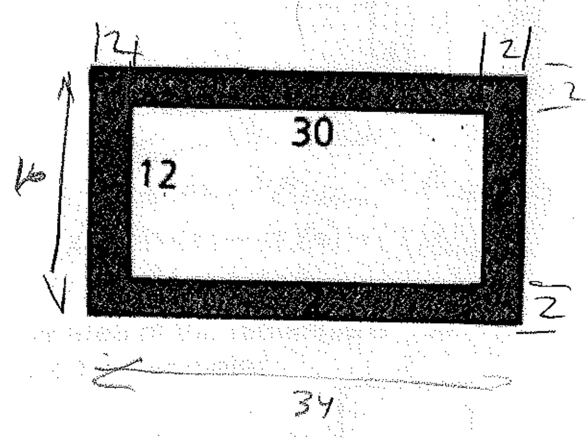
$A = (20)(4) + (20)(4) + (8)(12)$

$A = 80 + 80 + 96$

$\boxed{A = 256\ cm^2}$

$12 \times 8 = 96$, $96 + 80 + 80 = 256$

#5. A rectangular picture measures 12 cm by 30 cm. It is mounted in a frame 2 cm wide. Find the area of the frame.



$A = A_{total} - A_{inside}$

$A = (34)(16) - (30)(12)$

$A = 544 - 360$

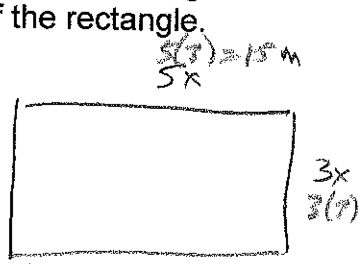
$\boxed{A = 184\ cm^2}$

$34 \times 16 = 204 + 340 = 544$, $544 - 360 = 184$

$30 \times 12 = 60 + 300 = 360$

One more example:

The sides of a rectangle ~~of a rectangle~~ are in a ratio 3:5, and the rectangle's area is 135 sq m. Find the dimensions of the rectangle.



$A = (5x)(3x)$

$135 = 15x^2$

$9 = x^2$

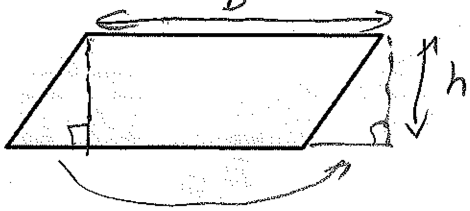
$3 = x$

$135 \div 15 = 9$

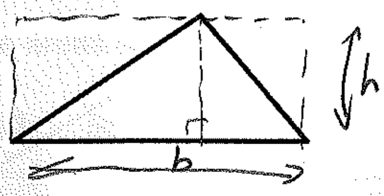
$\boxed{15m \times 9m}$

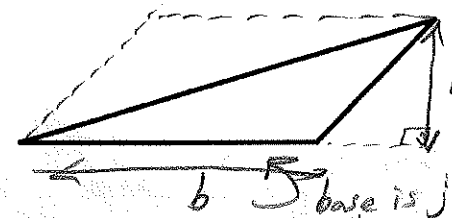
Geometry, 11.2: Area – Parallelograms and Triangles

Area of a parallelogram = $b \cdot h$ (base & height $\perp$)



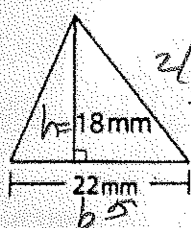
Area of a triangle = $\frac{1}{2} b \cdot h$ (base & height $\perp$)





Examples/Practice: Find the areas of the triangles.

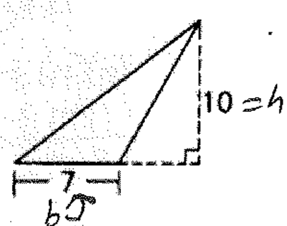
#1.



$A = \frac{1}{2} b h$

$A = \frac{1}{2}(22\,mm)(18\,mm) = \boxed{198\,mm^2}$

#2.

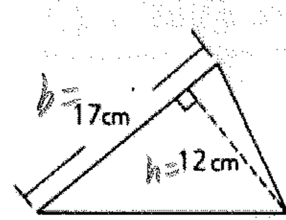


$A = \frac{1}{2} b h$

$A = \frac{1}{2}(7)(10)$

$\boxed{A = 35\,u^2}$

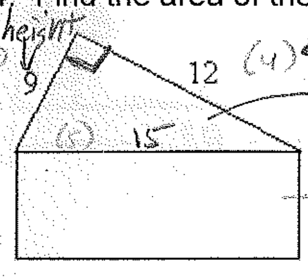
#3.



$A = \frac{1}{2}(17\,cm)(12\,cm)$

$\boxed{A = 102\,cm^2}$

#4. Find the area of the figure.



(4) ← base [* Find a base & height ⊥ to each other]

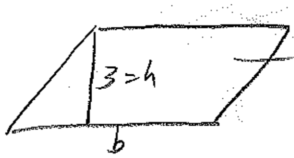
$A_{triangle} = \frac{1}{2}(12 \cdot 9) = 54\,u^2$

$A_{rect.} = 6 \cdot 15 = 90\,u^2$

$A_{total} = 54u^2 + 90u^2$

$\boxed{A_{total} = 144\,u^2}$

#5. Find the base of a parallelogram of height 3 and area 42.



Area $= b \cdot h$

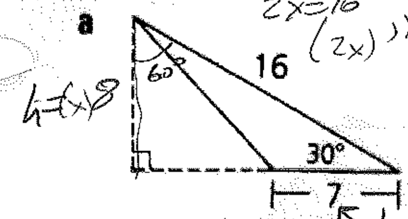
$\frac{42}{3} = \frac{b \cdot 3}{3}$

$\frac{42}{3} = b$

$\boxed{14 = b}$

#6. Find the areas of the triangles.

a



$2x=16$

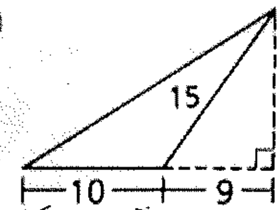
$(2x)\ x=8$

$(x\sqrt{3})$

$A=\frac{1}{2}bh$

$A=\frac{1}{2}(7)(8)=\frac{1}{2}56=\boxed{28u^2}$

b



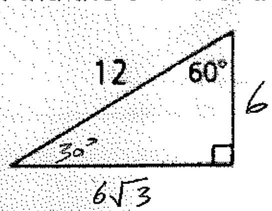
$12 = h$

$A=\frac{1}{2}bh$

$A=\frac{1}{2}(10)(12)=\frac{1}{2}120=\boxed{60u^2}$

#7. Find the areas of the triangles.

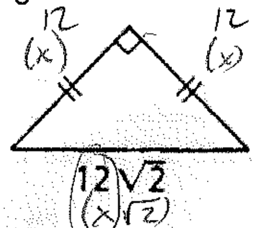
a



$A=\frac{1}{2}(6\sqrt{3})(6)$

$A=\frac{1}{2}36\sqrt{3}=\boxed{18\sqrt{3}\ u^2}$

b

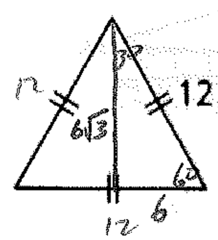


$A=\frac{1}{2}(12)(12)$

$A=\frac{1}{2}144$

$\boxed{A=72u^2}$

c

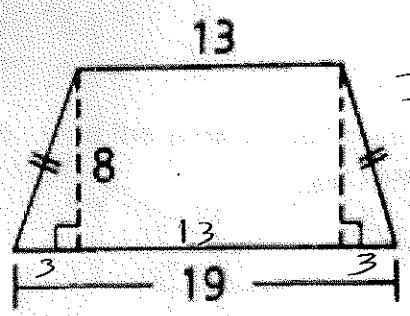


$A=\frac{1}{2}(12)(6\sqrt{3})$

$=\frac{1}{2}72\sqrt{3}$

$\boxed{A=36\sqrt{3}\ u^2}$

#8. Find the area of the trapezoid using triangles and rectangles.



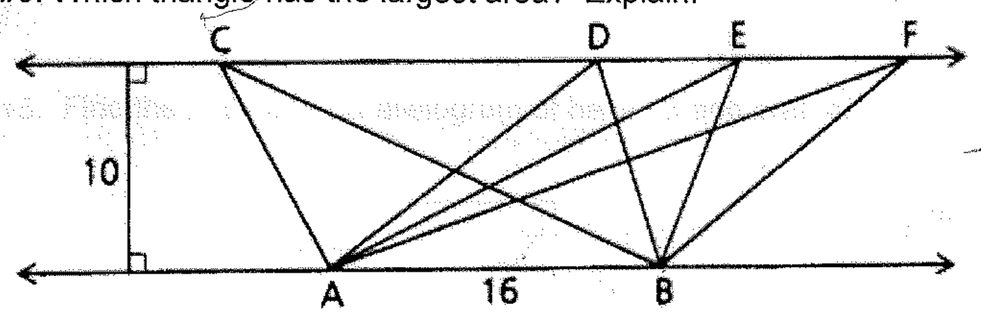
$19-13=6$

each triangle $A=\frac{1}{2}3\cdot 8=12u^2$

rectangle $A=13\cdot 8=104u^2$

$\boxed{\text{total } A = 128\ u^2}$

#9. Which triangle has the largest area? Explain.



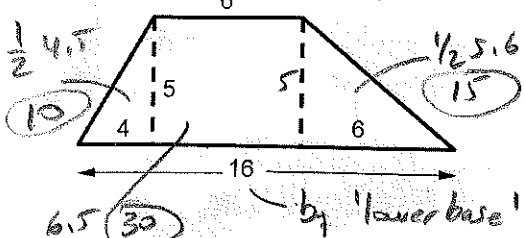
all have same heights so largest area $\Rightarrow$ largest base.

$\boxed{\triangle ACF \text{ or } \triangle BCF}$

Geometry, 11.3: Area – Trapezoids

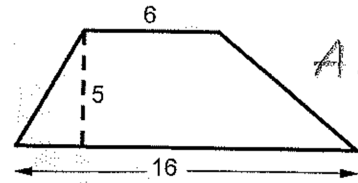
2 ways to find the area of a trapezoid:

'Pieces' way



$\boxed{55}$

'Formula' way $\boxed{A = \frac{1}{2}h(b_1+b_2)}$

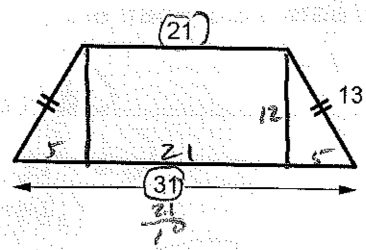


$A = \frac{1}{2}(5)(16+6)$

$= \frac{1}{2}5 \cdot 22$

$= \frac{5 \cdot 22}{2} = \frac{5 \cdot 11}{1}$ $\boxed{55}$

Example: Find the area



$A = \frac{1}{2}(12)(31+21)$

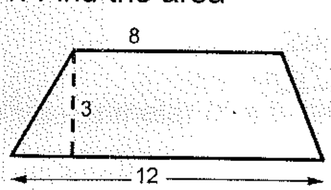
$\frac{1}{2}\,12 \cdot 52$

$6 \cdot 52$

52×6 $\boxed{312}$

Practice:

#1. Find the area

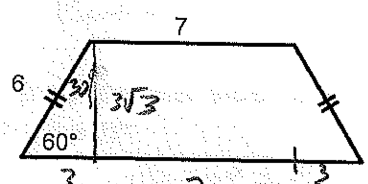


$\frac{1}{2}3(12+8)$

$\frac{1}{2}3(20)$

$3 \cdot 10 = \boxed{30}$

#2. Find the area

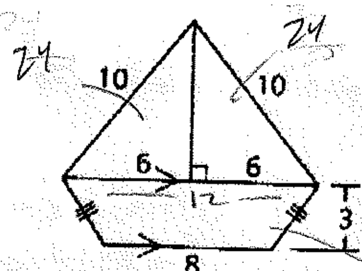


13

$\frac{1}{2}(3\sqrt{3})(7+13)$

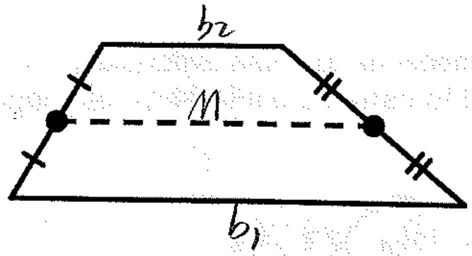
$\frac{1}{2}3\sqrt{3}(20) = \boxed{30\sqrt{3}}$

#3. Find area of figure.



$\boxed{78}$

Median of a Trapezoid = line connecting midpoints of the two non-parallel sides

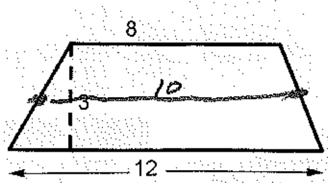


Two Formulas using <u>median</u> of a trapezoid:

$$M = \frac{1}{2}(b_1 + b_2)$$

$$Area_{trapezoid} = Mh$$

Example/Practice: Find the median and area of the trapezoid

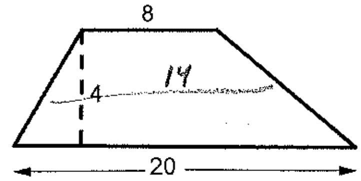


$m = \frac{1}{2}(b_1+b_2)$

$m = \frac{1}{2}(8+12)$

$\boxed{m = 10}$

$A = m \cdot h = (10)(3) = \boxed{30}$



$m = \frac{1}{2}(8+20)$

$\boxed{m = 14}$

$A = m \cdot h = (14)(4)$

$\boxed{A = 56}$

$14 \times 4 = 56$

The height of a trapezoid is 10, and the trapezoid's area is 130. If one base is 15, find the other base.

$A = \frac{1}{2}h(b_1+b_2)$

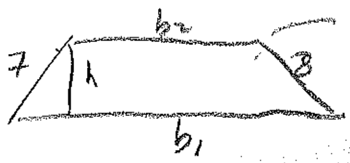
$130 = \frac{1}{2}(10)(15+b_2)$

$130 = 5(15+b_2)$

$26 = 15 + b_2$

$\boxed{b_2 = 11}$

The perimeter of a trapezoid is 35. The nonparallel sides are 7 and 8. Find the area if height = 5.



$P = 35$

$(b_1+b_2)+7+8 = 35$

$(b_1+b_2)+15 = 35$

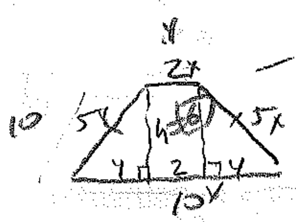
$b_1+b_2 = 20$

$A = \frac{1}{2}h(b_1+b_2)$

$A = \frac{1}{2}(5)(20)$

$A = \boxed{50}$

The consecutive sides of an isosceles trapezoid are in the ratio of 2:5:10:5 and the trapezoid's perimeter is 44. Find the area of the trapezoid.



$P = 44$

$P = 2x+5x+10x+5x$

$44 = 22x$

$2 = x$

$h = 6$

$A = \frac{1}{2}(6)(4+20)$

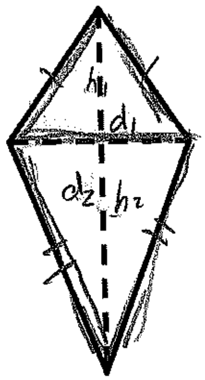
$= 3(24)$

$= \boxed{72}$

$24 \times 3 = 72$

Geometry, 11.4: Area – Kite and Rhombus

2 triangles with a common base (d_1)



$A_{top\,\Delta} = \frac{1}{2} d_1 h_1$

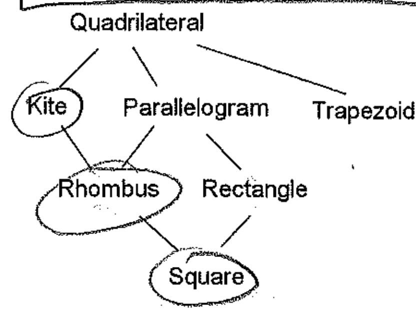
$A_{bottom\,\Delta} = \frac{1}{2} d_1 h_2$

$A = \frac{1}{2} d_1 h_1 + \frac{1}{2} d_1 h_2$

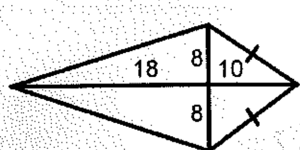
$A = \frac{1}{2} d_1 (h_1 + h_2)$

$A = \frac{1}{2} d_1 d_2$

$$\boxed{A_{\text{kite or rhombus or square}} = \frac{1}{2} d_1 d_2}$$

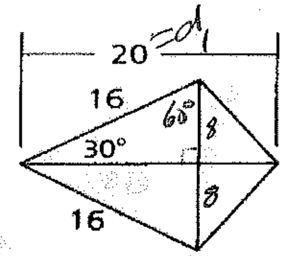


Examples: Find the area.



$d_1 = 28$
$d_2 = 16$
$A = \frac{1}{2}(28)(16)$
$A = \boxed{224}$

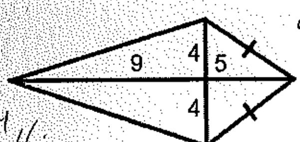
28 × 16 = 168 + 28 = 448



$d_2 = 16$

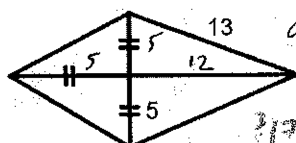
$A = \frac{1}{2}\, 20(16)$
$A = 10 \cdot 16$
$A = \boxed{160}$

Practice: Find the area.



$d_1 = 8$
$d_2 = 14$
$A = \frac{1}{2} d_1 d_2$
$A = \frac{1}{2}\, 8 \cdot 14$
$A = 56\ u^2$

16 × 7 = 112

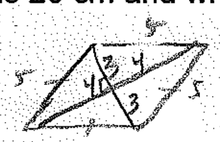


$d_1 = 17$
$d_2 = 10$

17 × 5 = 85

$A = \frac{1}{2}(17)(10)$
$A = 5 \cdot 17$
$A = \boxed{85}$

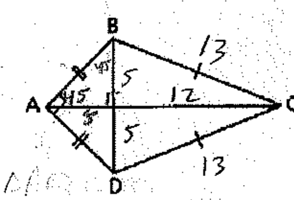
Examples:
Find the area of a rhombus whose perimeter is 20 cm and whole longer diagonal is 8.



$P = 20$ cm
side $= 5$ cm
$d_1 = 6$
$d_2 = 8$

$A = \frac{1}{2}(6)(8)$
$A = \boxed{24\ cm^2}$

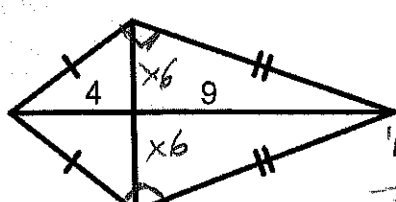
Given: ABCD is a kite.
∠BAD is a right ∠.
BD = 10, BC = 13
Find: The area of ABCD



$d_1 = 17$
$d_2 = 10$

$A = \frac{1}{2}\, 10 \cdot 17$
$A = 5 \cdot 17$
$A = \boxed{85}$

Find the area.

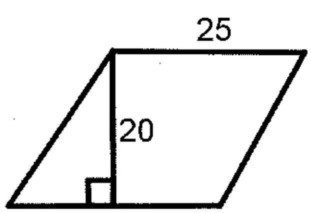


13 × 6 = 78

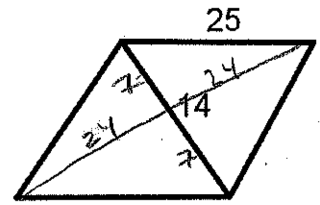
$x^2 = 4 \cdot 9$
$x^2 = 36$
$x = 6$

$d_1 = 13$
$d_2 = 12$

$A = \frac{1}{2}\, 12 \cdot 13$
$A = 6 \cdot 13 = \boxed{78}$



$A = b \cdot h$
$A = 25 \cdot 20 = \boxed{500}$



48 × 7 = 336

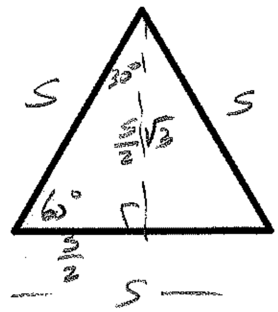
$A = \frac{1}{2}(14)(48)$
$A = 7 \cdot 48$
$A = \boxed{336}$

Geometry, 11.5: Area – Regular Polygons

A special formula for areas of equilateral triangles:

$$A = \frac{1}{2} b \cdot h$$
$$= \frac{1}{2} s\left(\frac{s}{2}\sqrt{3}\right)$$
$$= \frac{1}{4} s^2 \sqrt{3}$$

$$A_{equil.\Delta} = \frac{s^2}{4}\sqrt{3}$$

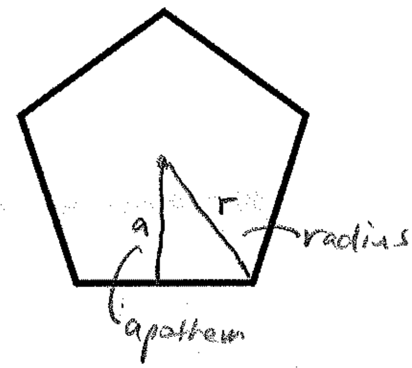


Regular Polygon Terms...

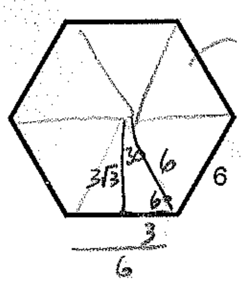
Regular means – Congruent sides and angles

Radius means – line segment center to corner (r)

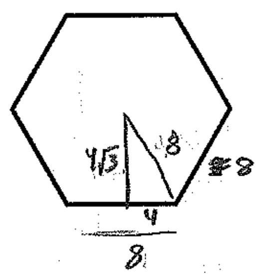
Apothem means – line segment center to midpt of side (a)
(divides side in half)
(perpendicular to side)



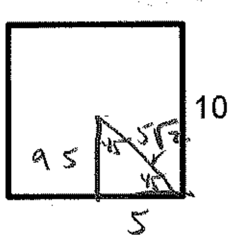
Example/Practice: Find the radius and apothem.



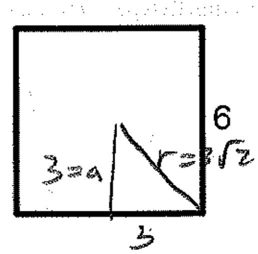
$\boxed{r = 6,\ a = 3\sqrt{3}}$



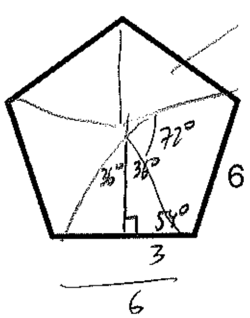
$\boxed{r = 8,\ a = 4\sqrt{3}}$



$\boxed{a = 5,\ r = 5\sqrt{2}}$



$\boxed{a = 3,\ r = 3\sqrt{2}}$



$\frac{360°}{5} = 72°$

$\frac{72}{2} = 36$

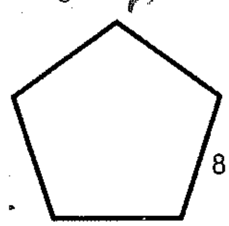
$\tan 54° = \frac{a}{3}$

$a = 4.13$

$\cos 54° = \frac{3}{r}$

$r = 5.104$

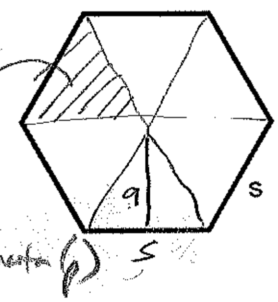
(skip)



Area of a regular polygon:

$A_{1part} = \frac{1}{2}bh$
$= \frac{1}{2}sa = \frac{1}{2}as$

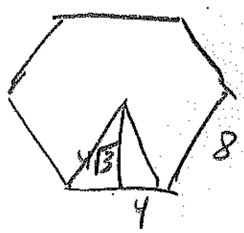
$A_{polygon} = 6 \cdot A_{1part}$
$= 6 \cdot \frac{1}{2}as$
$= \frac{1}{2}a(6s)$ ← $6s = \text{perimeter } (p)$



$$A_{regular\ polygon} = \frac{1}{2}ap$$

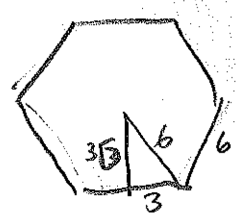
Examples/Practice: Find the area.

Regular hexagon, side = 8.



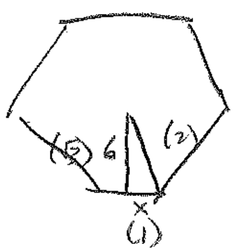
$a = 4\sqrt{3}$
$p = 6 \cdot 8 = 48$
$A = \frac{1}{2}ap = \frac{1}{2}4\sqrt{3} \cdot 48$
$= \boxed{96\sqrt{3}}$

Regular hexagon, perimeter = 36



$p = 36$
$s = 6$
$a = 3\sqrt{3}$
$A = \frac{1}{2}ap = \frac{1}{2}3\sqrt{3}\ 36$
$= \boxed{54\sqrt{3}}$

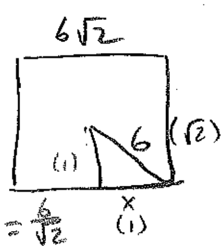
Regular hexagon, apothem = 6.



$\frac{x}{1} = \frac{6}{\sqrt{3}}$
$\sqrt{3}x = 6$
$x = \frac{6}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$
So side $= 4\sqrt{3}$
$p = 24\sqrt{3}$
$a = 6$

$A = \frac{1}{2}(6)(24\sqrt{3})$
$= \boxed{72\sqrt{3}}$

Square, radius=6.

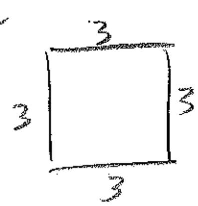


$\frac{x}{1} = \frac{6}{\sqrt{2}}$
$\sqrt{2}x = 6$
$x = \frac{6}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

$A = (6\sqrt{2})(6\sqrt{2})$
$= 36 \cdot 2$
$= \boxed{72}$

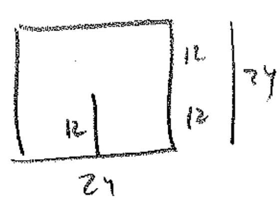
Square, perimeter = 12.

$p = 12$



$A = 3 \cdot 3 = \boxed{9}$

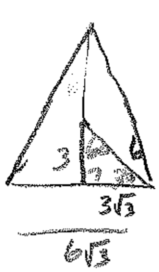
Square, apothem=12.



$A = (24)(24)$
$A = \boxed{576}$

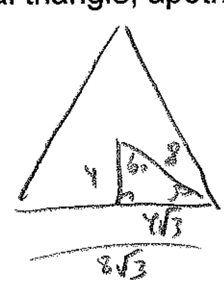
$$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 48 \\ \hline 576 \end{array}$$

Equilateral triangle, apothem=3.



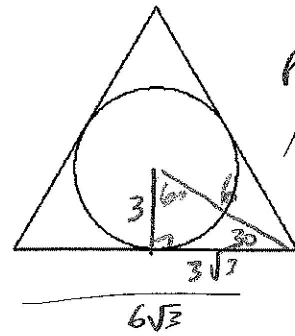
$A = \frac{1}{2}ap$
$p = 3 \cdot 6\sqrt{3} = 18\sqrt{3}$
$A = \frac{1}{2}\ 3 \cdot 18\sqrt{3}$
$= \boxed{27\sqrt{3}}$

Equilateral triangle, apothem=4



$p = 24\sqrt{3}$
$A = \frac{1}{2}ap$
$= \frac{1}{2}\ 4 \cdot 8\sqrt{3}$
$= \boxed{16\sqrt{3}}$

More examples:
Find the area of an equilateral triangle if the radius of its inscribed circle is 3.



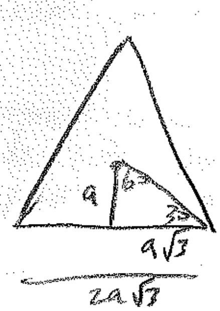
$p = 18\sqrt{3}$

$A = \frac{1}{2}ap$

$= \frac{1}{2}(3)(18\sqrt{3})$

$= \boxed{27\sqrt{3}}$

Find the side of an equilateral triangle whose area is $9\sqrt{3}$ sq. km.



$A = \frac{1}{2}ap$

$9\sqrt{3} = \frac{1}{2}ap$

$9\sqrt{3} = \frac{1}{2}a\,6a\sqrt{3}$

$9\cancel{\sqrt{3}} = 3\cancel{\sqrt{3}}\,a^2$

$9 = 3a^2$

$3 = a^2$

$a = \sqrt{3}$

$p = 3(2a\sqrt{3})$

$= 6a\sqrt{3}$

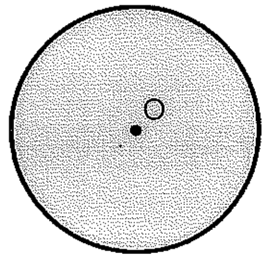
side $= 2(\sqrt{3})\sqrt{3}$

$= 2 \cdot 3$

$= \boxed{6}$

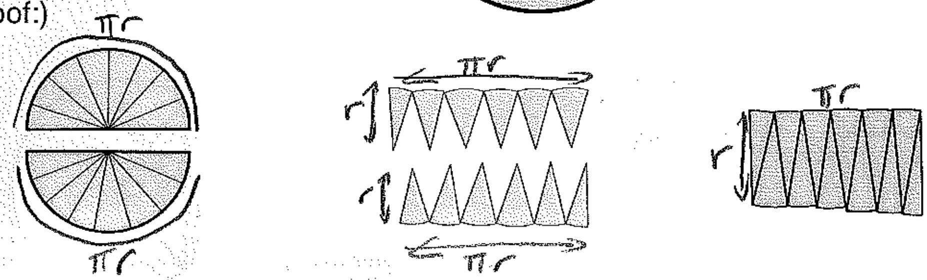
Geometry, 11.6: Area – Circles, Sectors, Segments

Area of a circle: $A_{circle} = \pi r^2$



(Visual proof:)

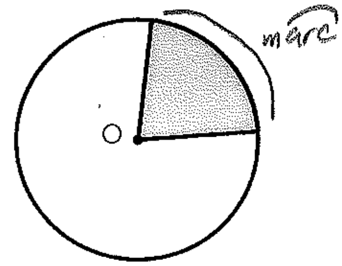
$C = 2\pi r$



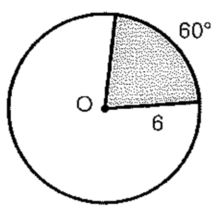
$A = b \cdot h$
$A = \pi r \cdot r$
$A = \pi r^2$

Area of a sector:

$$A_{sector} = (fraction) \cdot (circle\,area) = \left(\frac{m\overset{\frown}{arc}}{360°}\right) \cdot (\pi r^2)$$



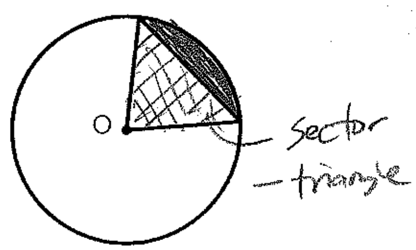
Example: Find the area of the sector.



$A = \left(\frac{60}{360}\right)(\pi 6^2)$
$= \frac{1}{6} \cdot \pi 36$
$= \frac{36\pi}{6}$
$= 6\pi$

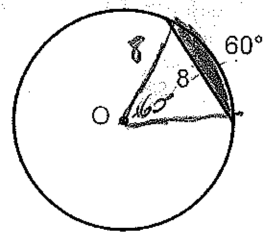
Area of a segment:

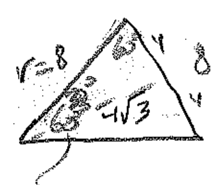
$$A_{segment} = (area\ of\ sector) - (area\ of\ triangle)$$



Example: Find the area of the segment:

$A_{sector} = \left(\frac{60}{360}\right)(\pi 8^2)$
$= \frac{1}{6} \cdot 64\pi = \frac{64\pi}{6}$
$= \frac{32\pi}{3}$



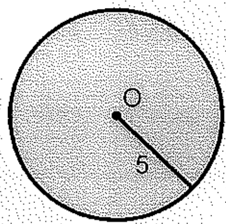


$A_{triangle} = \frac{1}{2} bh = \frac{1}{2} 8 \cdot 4\sqrt{3}$
$= 16\sqrt{3}$

$A_{segment} = \boxed{\frac{32\pi}{3} - 16\sqrt{3}}$

Practice: Find the area of the shaded part of the circles.

#1.

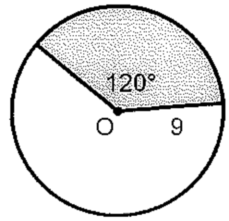


$A = \pi r^2$

$A = \pi 5^2$

$A = \boxed{25\pi}$

#2.

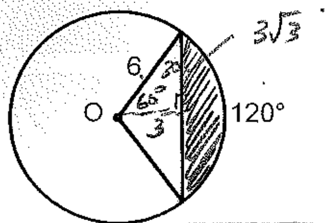


$A = \left(\frac{120}{360}\right)(\pi 9^2)$

$A = \frac{1}{3} \cdot \frac{81\pi}{1} = \frac{81\pi}{3}$

$A = \boxed{27\pi}$

#3.



$A_{sect} = \frac{120}{360} \pi 6^2$

$= \frac{1}{3} 36\pi$

$= 12\pi$

$A_{tri} = \frac{1}{2}(6\sqrt{3})(3)$

$= 9\sqrt{3}$

$A_{seg} = \boxed{12\pi - 9\sqrt{3}}$

#4. Find the area of circle with radius = 4.

$A = \pi 4^2 = \boxed{16\pi}$

More examples:

Find the radius of a circle with area of 169π

$A = \pi r^2$

$169\cancel{\pi} = \cancel{\pi} r^2$

$169 = r^2$

$\sqrt{169} = r$

$\boxed{r = 13}$

Find the area whose circumference is 16π cm.

$C = 2\pi r$

$16\cancel{\pi} = 2\cancel{\pi} r$

$16 = 2r$

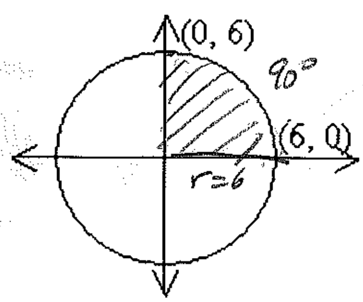
$8 = r$

$A = \pi r^2$

$A = \pi (8)^2$

$A = \boxed{64\pi}$

Find the area of one sector:



$A = \frac{90}{360} \pi 6^2$

$= \frac{1}{4} 36\pi$

$= \boxed{9\pi}$

Geometry, 11.7: Area – Ratios of Areas

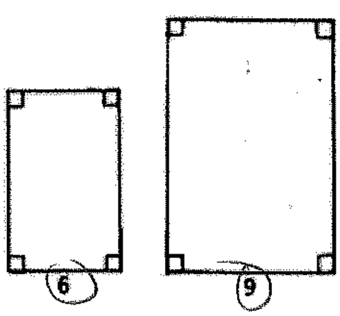
Small shape side	Big shape side	Scale factor (ratio of sides)	How much bigger is perimeter?	How much bigger is area?
3	**6**	$\frac{6}{3}=2$	$\frac{20}{10}=2$	$\frac{24}{6}=4$
2	**4**	$\frac{4}{2}=2$	$\frac{28}{14}=2$	$\frac{28}{7}=4$
2	**6**	$\frac{6}{2}=3$	$\frac{24}{8}=3$	$\frac{27}{3}=9$

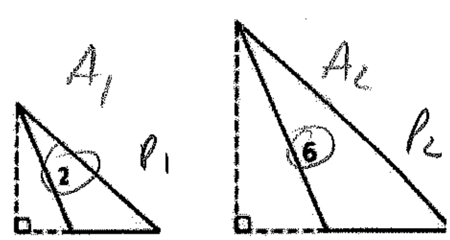
Ratio of perimeters = scale factor (ratio of sides)

Ratio of areas = $(\text{scale factor})^2$ (ratio of sides squared)

1	**4**	$\frac{4}{1}=4$	4	16
2	**3**	$\frac{3}{2}$	$\frac{3}{2}$	$\left(\frac{3}{2}\right)^2=\frac{3^2}{2^2}=\frac{9}{4}$
8	$\frac{8}{1}\cdot\frac{5}{4}=\frac{40}{4}=10$	$\frac{5}{4}$	$\frac{5}{4}$	$\frac{5^2}{4^2}=\frac{25}{16}$
12	$\frac{12}{1}\cdot\frac{7}{6}=14$	$\frac{7}{6}$	$\frac{7}{6}$	$\frac{7^2}{6^2}=$ $\frac{49}{36}$
6 $9\cdot\frac{2}{3}=\frac{18}{3}$	**9** smaller	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3^2}{2^2}=\frac{9}{4}$

More examples: Find the ratio of the areas, and the ratio of the perimeters, of each pair of similar figures.





ratio of perimeters $=\frac{9}{6}=\frac{3}{2}$

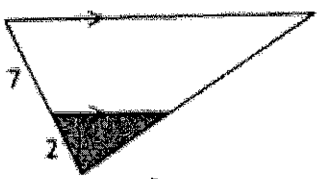
ratio of areas $=\left(\frac{3}{2}\right)^2=\frac{9}{4}$

$\frac{P_2}{P_1}=\frac{6}{2}=3$

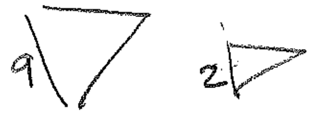
$\frac{A_2}{A_1}=\left(\frac{6}{2}\right)^2=\frac{36}{4}=9$

Examples/Practice:

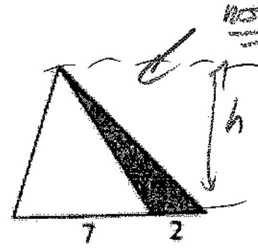
Find the ratio of the area of the shaded triangle to the area of the whole triangle.



overlapping – draw separate triangles:



$\frac{A_{shaded}}{A_{whole}} = \left(\frac{2}{9}\right)^2 = \boxed{\frac{4}{81}}$



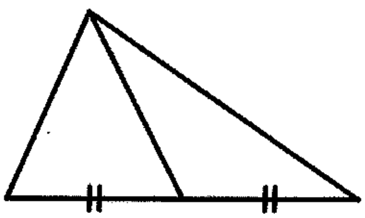
not similar, but has same height

$A_{shaded} = \frac{1}{2}(2)h$

$A_{whole} = \frac{1}{2}(9)h$

$\frac{A_{shaded}}{A_{whole}} = \frac{\cancel{\frac{1}{2}}(2)\cancel{h}}{\cancel{\frac{1}{2}}(9)\cancel{h}} = \boxed{\frac{2}{9}}$

A median of a triangle divides the triangle into 2 triangles with __equal__ **areas.**



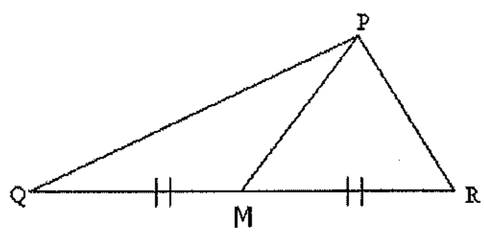
Example/Practice:

3. Given: $\overline{PM}$ is a median.

Find: a. $A_{\Delta PQM} : A_{\Delta PRM}$ $\boxed{1:1}$

b. $A_{\Delta PQM} : A_{\Delta PQR}$ $\boxed{1:2}$

c. QR : MR $\boxed{2:1}$



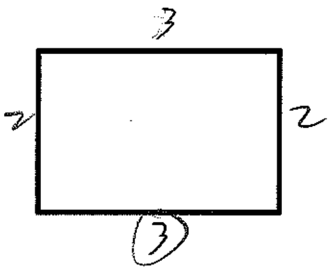
5. If the ratio of the areas of two similar polygons is 9:16, find the ratio of a pair of corresponding altitudes.

$\frac{A_2}{A_1} = \frac{16}{9} = \frac{4^2}{3^2}$ so side ratio is (or altitude) $\boxed{\frac{4}{3}}$

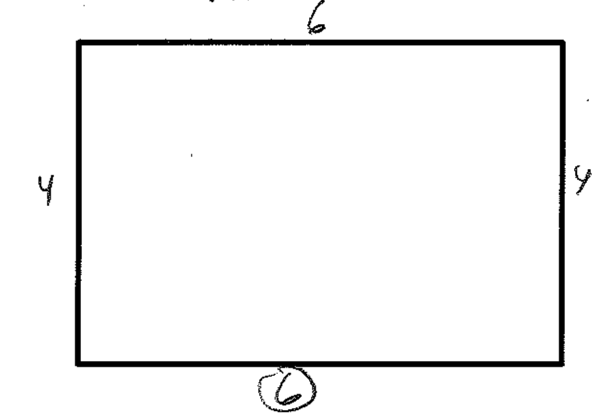
Geometry Activity – Side ratio and Perimeter, Area of Similar Shapes

Small shape side	Big shape side	Scale factor (ratio of sides)	How much bigger is perimeter? $= \text{scale factor}$	How much bigger is area? $= (\text{scale factor})^2$
3	6	$\frac{6}{3} = 2$	$\frac{20}{10} = 2$	$\frac{24}{6} = 4$
2	4	$\frac{4}{2} = 2$	$\frac{28}{14} = 2$	$\frac{28}{7} = 4$
2	6	$\frac{6}{2} = 3$	$\frac{24}{8} = 3$	$\frac{27}{3} = 9$

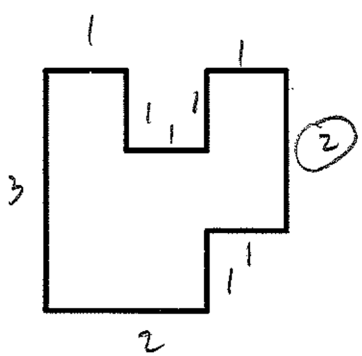
P = 10
A = 6



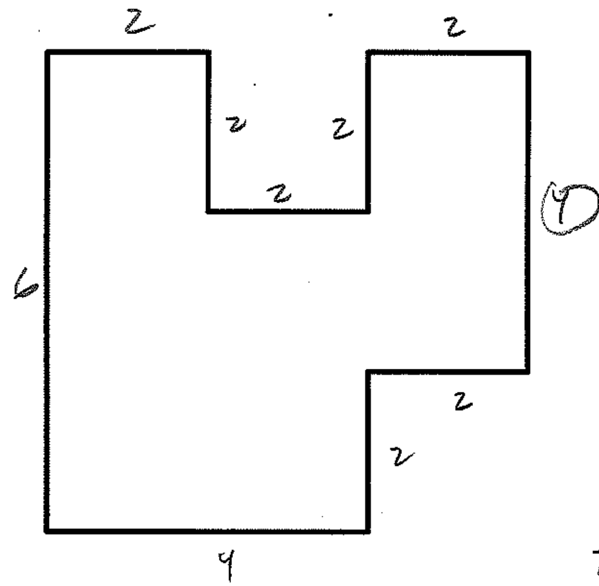
P = 20
A = 24



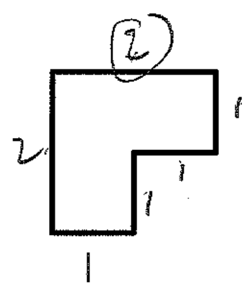
P = 14
A = 7



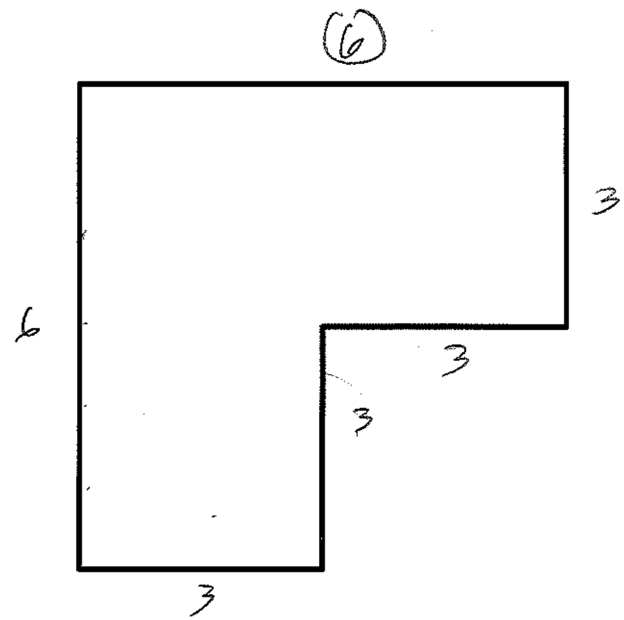
P = 28
A = 28



P = 8
A = 3

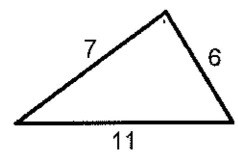


P = 24
A = 27



Geometry, 11.8: Area – Hero and Brahmagupta area formulas

Could we find the area of this triangle?

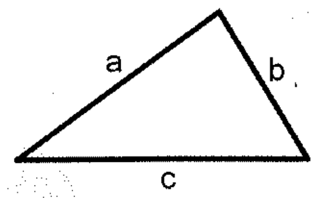


—no angles
—no triples
—can't use 30/60, 45/45 patterns

Hero's (or Heron's) Formula for triangle areas:

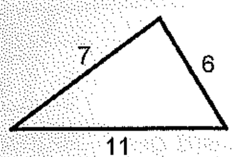
$$A_{triangle}=\sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s=\underset{\text{(half)}}{semiperimeter}=\frac{a+b+c}{2}$$



(Use Hero's formula for any triangle where you know all 3 sides)

Examples/Practice: Find the area of the triangles.



$S=\frac{7+6+11}{2}=\frac{24}{2}=12$

$A=\sqrt{12(12-7)(12-6)(12-11)}$

$=\sqrt{12(5)(6)(1)}$

$=\sqrt{360}$

$\sqrt{36}\cdot\sqrt{10}$

$\boxed{6\sqrt{10}}$

$30 \times 12 = 60,\ 30,\ 360$

Sides: 5, 6, 9.

$S=\frac{5+6+9}{2}=\frac{20}{2}=10$

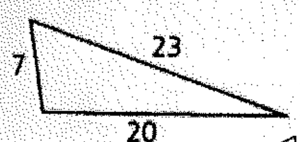
$A=\sqrt{10(10-5)(10-6)(10-9)}$

$=\sqrt{10(5)(4)(1)}$

$=\sqrt{10\cdot 20}$

$=\sqrt{200}$

$\sqrt{100}\ \sqrt{2}$ $=\boxed{10\sqrt{2}}$



$S=\frac{7+20+23}{2}=\frac{50}{2}=25$

$A=\sqrt{25(25-7)(25-23)(25-20)}$

$=\sqrt{25(18)(2)(5)}$

$=\sqrt{25(18)10}$

$=\sqrt{25(180)}$

$=\sqrt{4500}$

$\sqrt{100}\ \sqrt{45}$

$10\ \sqrt{9}\ \sqrt{5}$

$10\cdot 3\sqrt{5}$

$\boxed{30\sqrt{5}}$

$180 \times 25 = 900,\ 360,\ 4500$

Sides: 3, 7, 8.

$S=\frac{3+7+8}{2}=\frac{18}{2}=9$

$A=\sqrt{9(9-3)(9-7)(9-8)}$

$=\sqrt{9(6)(2)(1)}$

$=\sqrt{9\cdot 12}$

$=\sqrt{108}$

$\sqrt{4}\ \sqrt{27}$

$2\sqrt{27}$

$2\sqrt{9}\ \sqrt{3}$

$2\cdot 3\sqrt{3}$

$\boxed{6\sqrt{3}}$

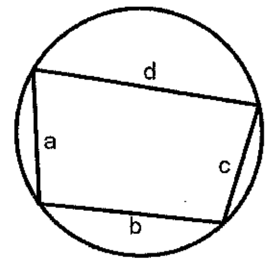
$12 \times 9 = 108$

$108 \div 4 = 27,\ 8,\ 28$

Brahmagupta's formula for inscribed (cyclic) quadrilateral areas:

$$A_{cyclic\,quadrilateral} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$where\ s = semiperimeter = \frac{a+b+c+d}{2}$$



Examples/Practice: Find the area of inscribed quadrilaterals with the following side lengths:

1, 5, 9, 11

$S = \frac{1+5+9+11}{2} = \frac{26}{2} = 13$

$A = \sqrt{(13-1)(13-5)(13-9)(13-11)}$

$= \sqrt{(12)(8)(4)(2)}$

$= \sqrt{96 \cdot 8}$

$= \boxed{\sqrt{768}}$

$12 \times 8 = 96$, $96 \times 8 = 768$

3, 5, 9, 5

$S = \frac{3+5+9+5}{2} = \frac{22}{2} = 11$

$A = \sqrt{(11-3)(11-5)(11-9)(11-5)}$

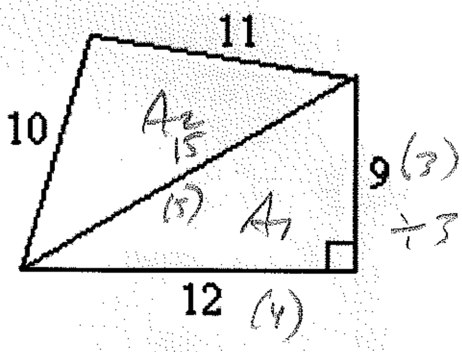
$= \sqrt{(8)(6)(2)(6)}$

$= \sqrt{48 \cdot 12}$

$= \boxed{\sqrt{576}}$

$48 \times 12 = 576$

Example: Find the area of the quadrilateral:



2 triangles added together

$A_1 = \frac{1}{2}bh = \frac{1}{2}(12)(9) = 6 \cdot 9 = 54$

$A_2 = \sqrt{s(s-a)(s-b)(s-c)}$

$S = \frac{10+11+15}{2} = \frac{36}{2} = 18$

$A_2 = \sqrt{18(18-10)(18-11)(18-15)}$

$= \sqrt{18(8)(7)(3)}$

$= \sqrt{144 \cdot 21}$

$\sqrt{144}\ \sqrt{21}$

$\boxed{12\sqrt{21}}$

$18 \times 8 = 144$, $3 \times 7 = 21$