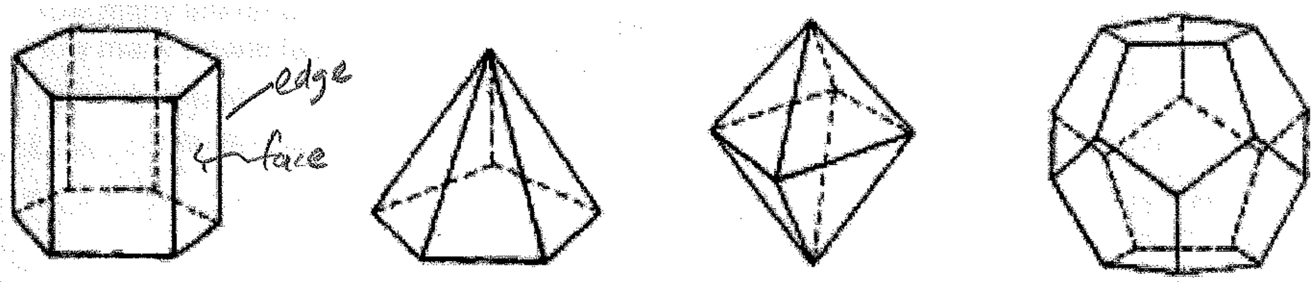
Geometry, 12.1: Surface area of prisms

Solid shapes with flat faces are called polyhedra

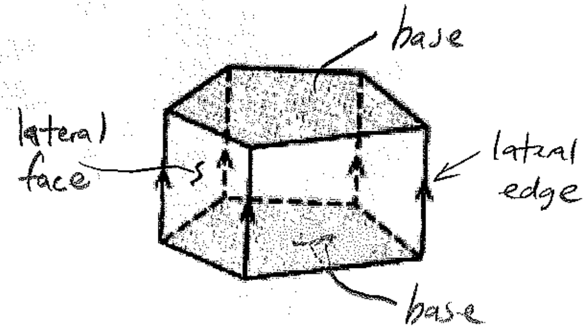
The surfaces are called faces and each is a polygon

The lines where the faces intersect are called edges



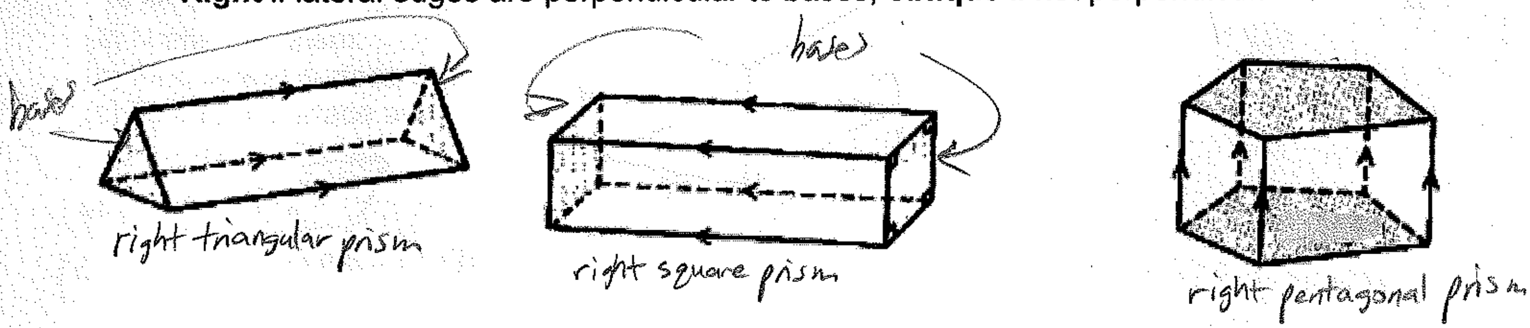
Prisms have:

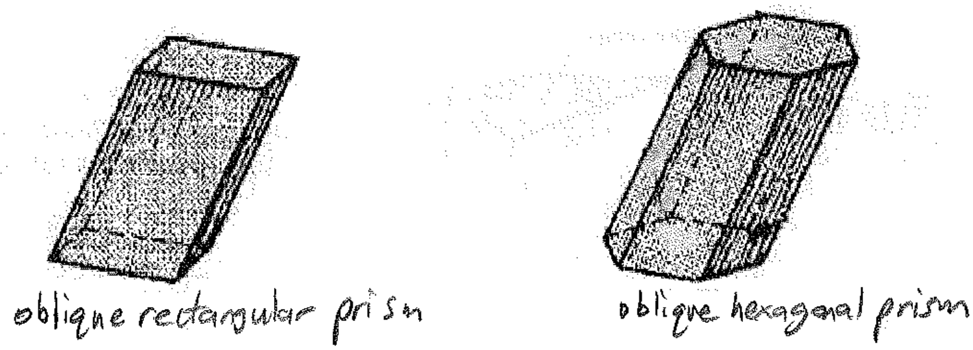
- 2 congruent, parallel faces called **bases**.
- parallel edges that connect corresponding vertices of the 2 parallel faces called **lateral edges.**
- The faces that are not bases are called **lateral faces.**



Naming prisms:

- Named by the shape of the bases.
- **Right** if lateral edges are perpendicular to bases, **oblique** if not perpendicular.

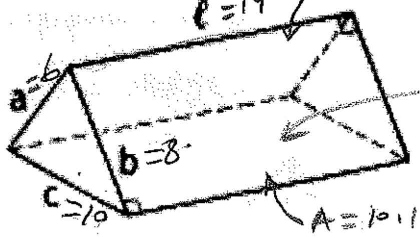




Lateral Surface Area – the sum of the areas of the lateral faces. Sum of
Total Surface Area – the sum of the areas of the lateral faces plus the areas of the 2 bases. (all faces)

Example: $l=14$, $a=6$, $b=8$, $c=10$

(a) Name the prism.
(b) Find lateral surface area.
(c) Find total surface area.
(d) How many lateral edges? $\boxed{3}$
(e) How many lateral faces? $\boxed{3}$



$A=6\cdot14=84$

$A=8\cdot14=112$

$A=10\cdot12=120$

(a) right triangular prism
(b) L.S.A. $=84+112+120=\boxed{316\,u^2}$
(c) T.S.A. $=\boxed{316+8\sqrt{15}\ u^2}$

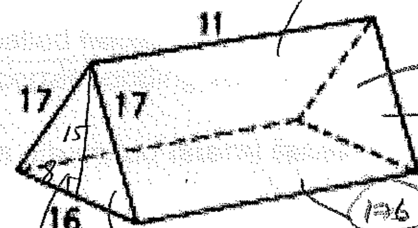
$A_\Delta=$ Hero's formula

$s=\frac{6+8+10}{2}=12$

$A=\sqrt{12(12-6)(12-8)(12-10)}$
$=\sqrt{12(6)(4)(2)}$
$=\sqrt{240}=\sqrt{4}\sqrt{60}=2\sqrt{60}$
$=2\sqrt{4}\sqrt{15}=4\sqrt{15}$

$A_{\text{both }\Delta's}=8\sqrt{15}$

Practice:
(a) Name the prism.
(b) Find lateral surface area.
(c) Find total surface area.



$11\cdot17=187$

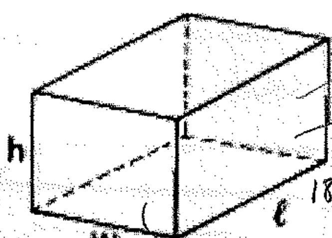
187

176

$A=\frac{1}{2}b\cdot h=\frac{1}{2}16\cdot15$
$=8\cdot15$
$=120$

(a) right triangular prism
(b) L.S.A $=187+187+176=\boxed{550u^2}$
(c) T.S.A $=550+120+120=\boxed{790u^2}$

Practice: $l=18$, $w=9$, $h=9$
(a) Name the prism.
(b) Find lateral surface area.
(c) Find total surface area.



each lat. face $A=9\cdot18$

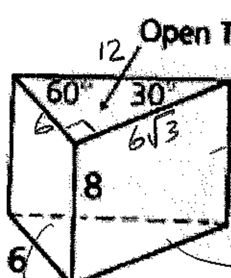
81

81

$162\times4=648$

(a) right square prism
(b) L.S.A $=\boxed{648\,u^2}$
(c) T.S.A. $=648+81+81=\boxed{810\,u^2}$

Practice:
(a) Name the prism.
(b) Find lateral surface area.
(c) Find total surface area.



back $A=8\cdot12=96$

$A=b\cdot h$
$8\cdot6\sqrt{3}=48\sqrt{3}$

48

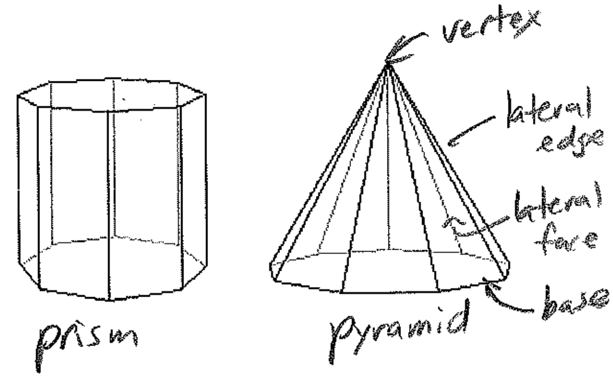
$A_\Delta=\frac{1}{2}bh=\frac{1}{2}6\cdot6\sqrt{3}=\frac{36}{2}\sqrt{3}=18\sqrt{3}$

(a) right triangular prism
(b) L.S.A $=\boxed{144+48\sqrt{3}\ u^2}$
(c) T.S.A. $=144+48\sqrt{3}+18\sqrt{3}$
$=\boxed{144+66\sqrt{3}\ u^2}$

add just 1 triangle, (top is open)

Geometry, ~~12.1~~ 12.2: Surface area of ~~prisms~~ Pyramids

Solid shapes with 2 congruent bases are called **prisms**
Solid shapes with only 1 base are called Pyramids



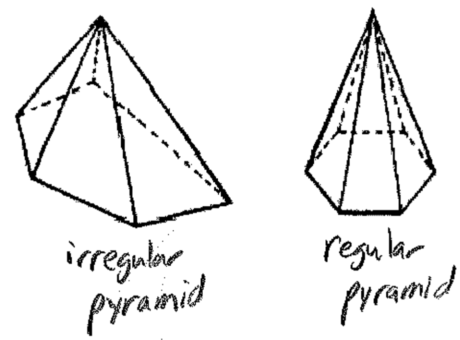
Pyramids have:

- 1 base.
- **Lateral edges** which meet at a point called the vertex.

A pyramid is a **regular pyramid** if:

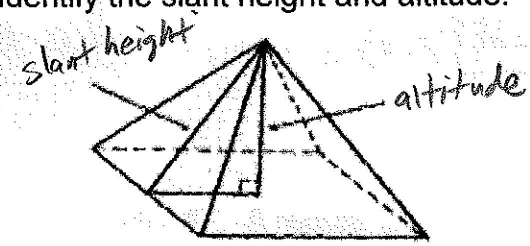
- The base is a regular polygon.
- The lateral edges are congruent.

Therefore, the lateral faces of a regular pyramid are isosceles triangles.

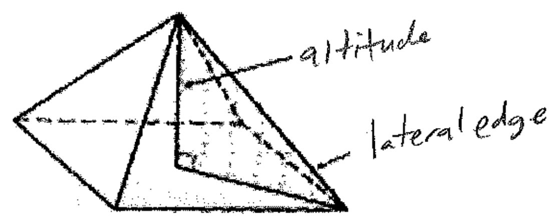


A quick review...

Identify the slant height and altitude:

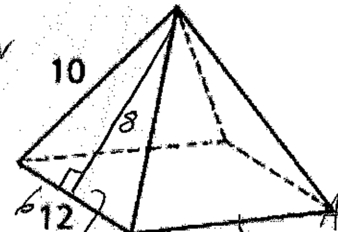


Identify the lateral edge and altitude:



What kinds of triangles are determined from the sides above? right triangles

Example: Find the lateral area and the total area of the regular pyramid:



lateral faces congruent ✓

L.A. = $4 \times 48 = \boxed{192 u^2}$

$$\begin{array}{r} 48 \\ \times 4 \\ \hline 192 \end{array}$$

T.A. = $192 + 144 = \boxed{336 u^2}$

$$\begin{array}{r} 192 \\ 144 \\ \hline 336 \end{array}$$

base = $12 \cdot 12 = 144 u^2$

$A = \frac{1}{2}bh$

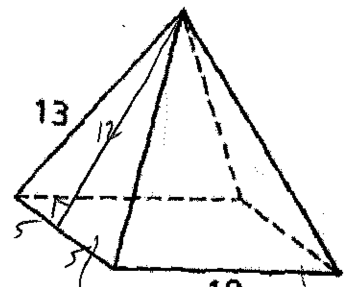
$A = \frac{1}{2} 12 \cdot 8 = 48 u^2$

Practice: For the regular, square pyramid:
(a) Find the area of each lateral face.
(b) Find the pyramid's lateral area.
(c) Find the pyramids total area.

(a) $\boxed{60u^2}$

(b) $L.A. = 4 \cdot 60 = \boxed{240u^2}$

(c) $T.A. = 240 + 100 = \boxed{340u^2}$



$A = \frac{1}{2}bh$

$A = \frac{1}{2} 10 \cdot 12 = 60u^2$

$A = b \cdot h = 10 \cdot 10 = 100u^2$

Practice: For the regular, triangular pyramid:
(a) Find the area of each lateral face.
(b) Find the area of the base.
(c) Find the total area.

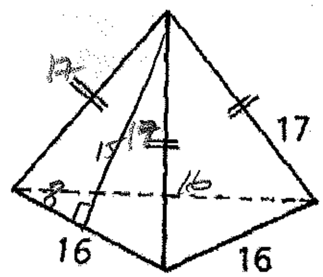
(a) $A = \frac{1}{2}bh = \frac{1}{2}(16)(15) = \boxed{120u^2}$

$\frac{15}{\times 8}{120}$

(b) $A = \frac{1}{2}bh = \frac{1}{2} 16 \cdot 8\sqrt{3} = \boxed{64\sqrt{3}u^2}$

16, 8, $8\sqrt{3}$, 60, 30, 16, 16

(c) $A_{total} = 3(120) + 64\sqrt{3} = \boxed{360 + 64\sqrt{3}\ u^2}$



Practice: HW #7. A regular pyramid has a slant height of 12 and lateral edge of 15.
What is...
(a)...the perimeter of the base?
(b)...the pyramid's lateral area?
(c)...the area of the base?
(d)...the pyramid's total area?

(a) $18 \cdot 4 = \boxed{72}$

(b) each $A = \frac{1}{2}bh = \frac{1}{2} 18 \cdot 12 = 108$

$\times 4$

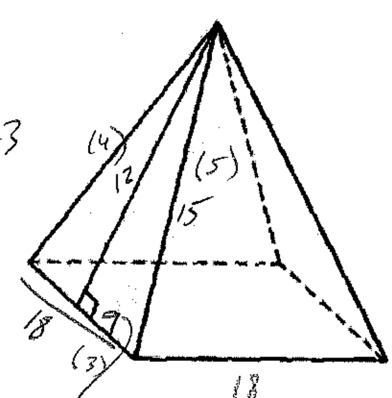
$\boxed{432u^2}$

(c) $18 \times 18 = 144, 18 \to \boxed{324u^2}$

(d) $432 + 324 = \boxed{756u^2}$

18×18

$\div 3$



$A = \frac{1}{2}bh$

$= \frac{1}{2} 18 \cdot 12 = 108$

$\times 4$

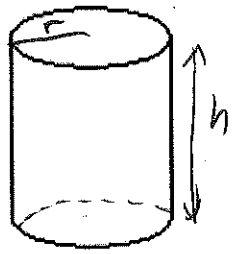
$108 \times 4 = 432$

$12 \times 9 = 108$

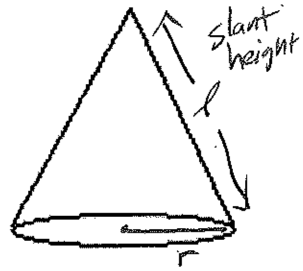
$18 \times 4 = 72$

Geometry, 12.3: Surface area of circular solids

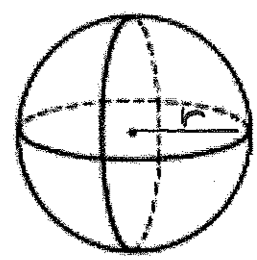
3 solids that include circles:



Cylinder



Cone



sphere

<u>Lateral Area</u>: $L.A._{cylinder} = C \cdot h = 2\pi rh$ $\quad L.A._{cone} = \frac{1}{2} C \cdot l = \pi rl$ $\quad$ none (no lateral edges)

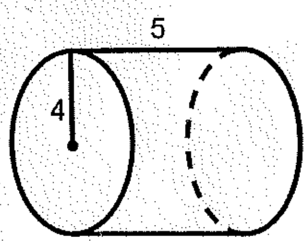
<u>Total Area</u>:

$$T.A._{cylinder} = L.A. + 2A_{base} = 2\pi rh + 2(\pi r^2)$$

$$T.A._{cone} = L.A. + A_{base} = \pi rl + \pi r^2$$

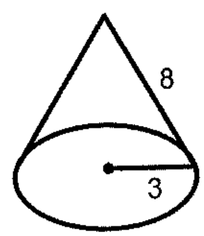
$$T.A._{sphere} = 4\pi r^2$$

Examples/Practice: Find the lateral area and the total area.



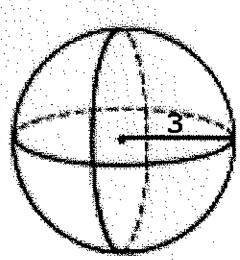
$L.A. = 2\pi rh$
$= 2\pi \, 4 \cdot 5$
$= \boxed{40\pi \ u^2}$

$T.A. = 40\pi + 2(\pi 4^2)$
$= 40\pi + 2(16\pi)$
$= 40\pi + 32\pi$
$= \boxed{72\pi \ u^2}$



$L.A. = \pi rl$
$= \pi \, 3 \cdot 8$
$= \boxed{24\pi \ u^2}$

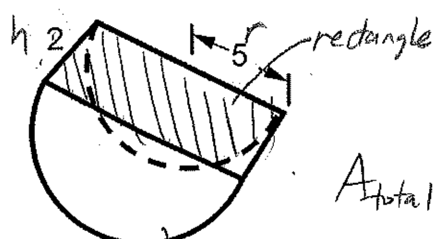
$T.A. = 24\pi + \pi r^2$
$= 24\pi + \pi 3^2$
$= 24\pi + 9\pi$
$= \boxed{33\pi \ u^2}$



L.A. (none)

$T.A. = 4\pi r^2$
$= 4\pi 3^2$
$= 4\pi \cdot 9$
$= \boxed{36\pi}$

Find total surface area:



$A = b \cdot h$
$A = 10 \cdot 2 = 20$

$A_{total} = \boxed{35\pi + 20 \ u^2}$

$\frac{1}{2}$ of a cylinder

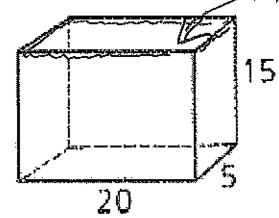
$LA_{cyl.} = 2\pi rh = 2\pi 5 \cdot 2 = 20\pi$

$TA_{cyl.} = 20\pi + 2(\pi r^2) = 20\pi + 2(\pi 5^2)$
$= 20\pi + 50\pi = 70\pi$

$T.A. \ \frac{1}{2}(70\pi) = 35\pi$

Geometry, 12.4: Volumes of Prisms and Cylinders

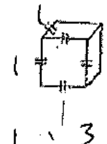
What is volume? Area = space inside a 2-D shape.
Volume = space inside a 3-D shape.



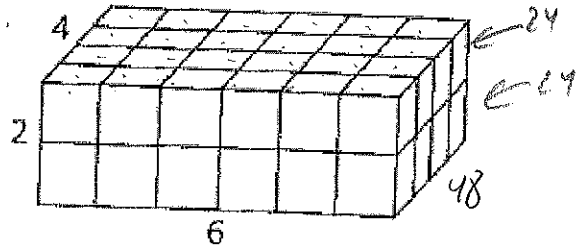
how much H_2O fits here? volume

$V = 20 \cdot 5 \cdot 15$
$= 100 \cdot 15$
$= \boxed{1500\ u^3}$

Volume of a rectangular box:



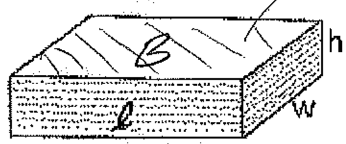
$1\ in^3$



$V_{rect.box} = \cancel{lwh}$
$= \boxed{6 \cdot 4 \cdot 2 = 48\ in^3}$

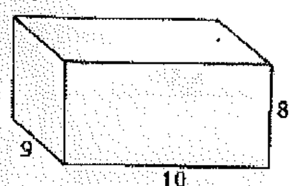
$B = lw$

$V_{rect.box} = Bh$



$V = lwh$
$V = Bh$

Try these...Find the volume.



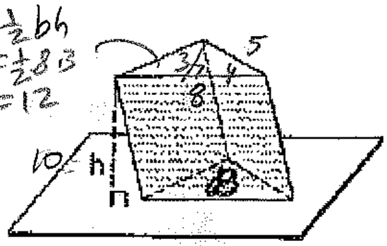
$V = lwh$
$V = 10 \cdot 9 \cdot 8 = 10 \cdot 72 = \boxed{720\ u^3}$

Rectangular box with dimensions of 2in x 3in x 5in

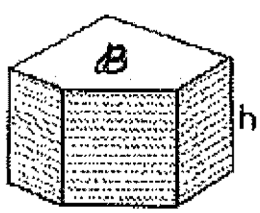
$V = lwh$
$V = 2 \cdot 3 \cdot 5 = \boxed{30\ in^3}$

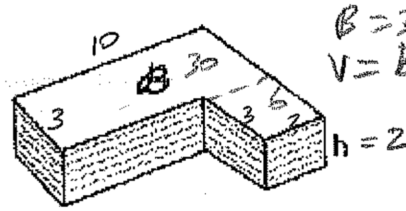
Volume of other prisms: $V_{prism} = Bh$ * B is called a <u>base</u> or <u>cross-section</u>

$A = \frac{1}{2}bh$
$= \frac{1}{2} 8 \cdot 3$
$= 12$



$V = Bh$
$V = 12 \cdot 10 = 120\ u^3$

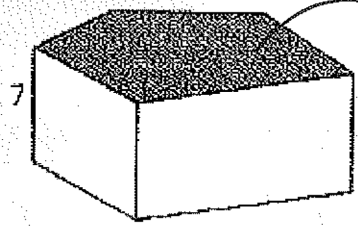




$B = 36$
$V = Bh = 36 \cdot 2$
$= \boxed{72\ u^3}$

Try these...

3 The area of the shaded face of the right pentagonal prism is 51. Find the prism's volume.



$B=51$

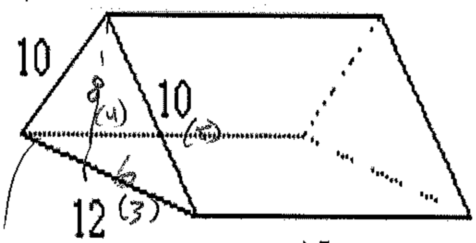
$V=Bh$

$V=51\cdot 7$

$\boxed{V=357\,u^3}$

$51 \times 7 = 357$

Find the volume of the triangular prism:



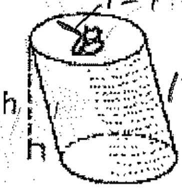
$B=\frac{1}{2}\,12\cdot 8=48$

$V=Bh=48\cdot 15$

48×15: 240, 48 → $\boxed{720\,u^3}$

Volume of a cylinder: $V_{cylinder}=\pi r^2 h$

$B=\pi r^2=16\pi$, $r=4$

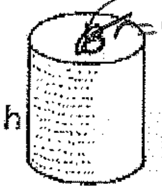


$10=h$

$V=16\pi\cdot 10$

$=\boxed{160\pi\,u^3}$

$B=\pi 4^2=16\pi$, $r=4$

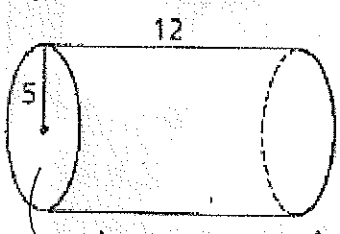


$10=h$

$V=16\pi\cdot 10$

$=\boxed{160\pi\,u^3}$

Try these:



$V=\pi r^2 h=\pi(5)^2 12=\pi\,25\cdot 12=\boxed{300\pi\,u^2}$

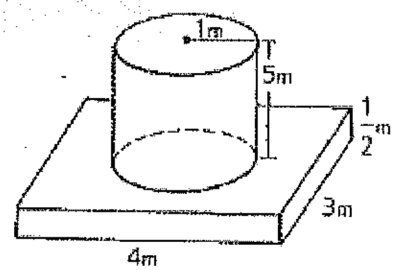
25×12: 50, 25 → 300

Find the volume of cement needed to form the concrete pedestal shown:

$V_{cyl}=\pi r^2 h=\pi(1)^2 5=5\pi\ m^3$

$+\ V_{box}=l\cdot w\cdot h=4\cdot 3\cdot \frac{1}{2}=6\ m^3$

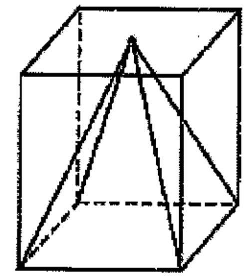
$\boxed{5\pi+6\ m^3}$



Geometry, 12.5: Volumes of Pyramids and Cones

Compare this pyramid and rectangular prism with same base:

$$\boxed{V_{pyramid} = \frac{1}{3}Bh}$$

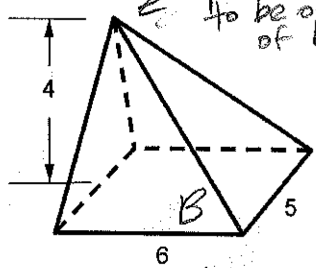


Not ~~1/2~~

1/3 ☆

Examples:

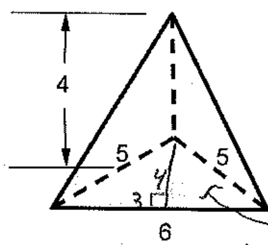
apex does not have to be over center of base



$V=\frac{1}{3}Bh=\frac{1}{3}(6\cdot5)(4)$

$=\frac{1}{3}(30)(4)$

$=10\cdot4=\boxed{40u^3}$

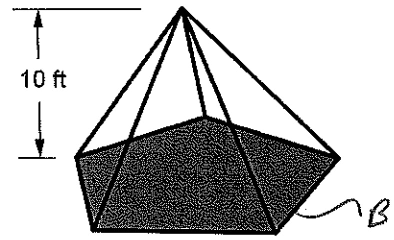


$B=12$

$B=\frac{1}{2}bh=\frac{1}{2}6\cdot4=12$

$V=\frac{1}{3}Bh=\frac{1}{3}(12)(4)$

$V=\boxed{16u^3}$

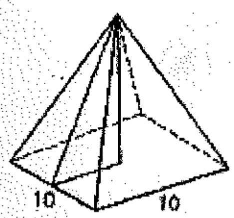


base area = 70 ft² $=B$

$V=\frac{1}{3}Bh=\frac{1}{3}(70ft^2)(10ft)$

$=\boxed{\frac{700}{3}ft^3}$

Try these. Find the volume...



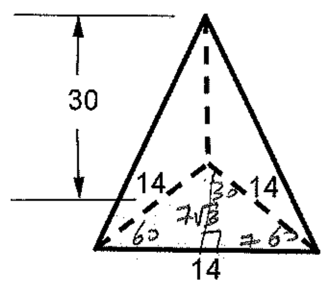
(height = 12)

$V=\frac{1}{3}Bh \quad B=10\cdot10$

$V=\frac{1}{3}(100)(12) \quad B=100$

$=4\cdot100$

$=\boxed{400u^3}$



$B=\frac{1}{2}bh=\frac{1}{2}(14)(7\sqrt{3})$

$=7\cdot7\sqrt{3}$

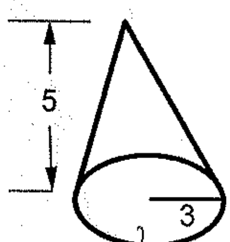
$=49\sqrt{3}$

$V=\frac{1}{3}Bh=\frac{1}{3}(49\sqrt{3})(30)$

$=\boxed{490\sqrt{3}\ u^3}$

Volume of a cone: $V_{cone}=\frac{1}{3}Bh=\frac{1}{3}\pi r^2 h$

base of a cone is a circle ($B=\pi r^2$)



$B=\pi r^2$

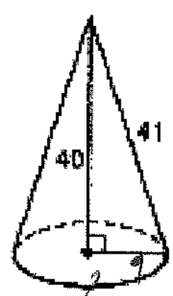
$=\pi 3^2$

$=9\pi$

$V=\frac{1}{3}Bh=\frac{1}{3}(9\pi)5$

$=3\pi 5$

$=\boxed{15\pi u^3}$

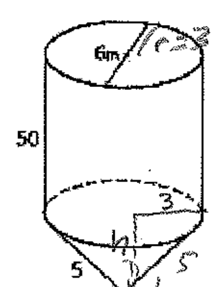


$B=\pi(9)^2=81\pi$

$V=\frac{1}{3}Bh=\frac{1}{3}81\pi 40$

$=\boxed{1080\pi u^3}$

27
3)81
27
×40
1080



$V_{cyl.}=Bh$

$=9\pi 50=450\pi$

$B=\pi r^2=\pi 3^2=9\pi$

$V_{cone}=\frac{1}{3}Bh=\frac{1}{3}9\pi(4)$

$=3\pi 4$

$=12\pi$

$V_{total}=V_{cyl}+V_{cone}$

$=450\pi+12\pi$

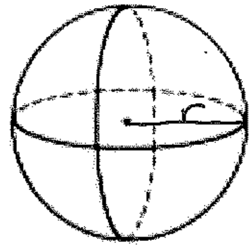
$=462\pi\ m^3$

$(\approx 1451.4\ m^3)$

Geometry, 12.6: Volumes of Spheres

Volume of a sphere:

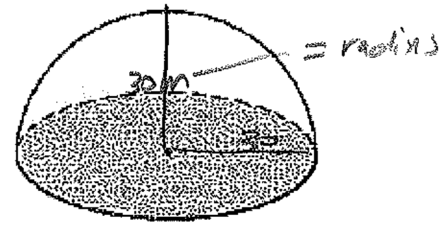
$$V_{sphere} = \frac{4}{3}\pi r^3$$



Examples:
Find the volume of a sphere with radius = 3

$V = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi 27 = \frac{4\cdot 27^9}{3_1}\pi$ $\boxed{36\pi\ u^3}$

8. A hemispherical dome has a height of 30 m.



(a) Find the total volume enclosed.
(b) The the area of ground covered by the dome.
(c) How much more paint is needed to paint the dome than to paint the floor?
(d) Find the radius of a dome that would cover double the ground area.

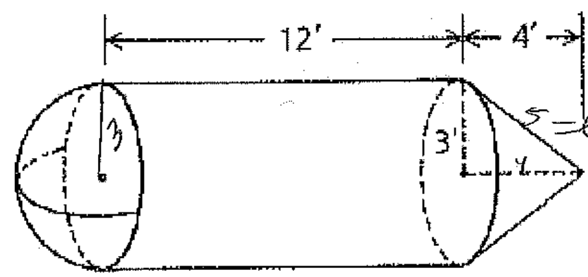
(a) half a sphere:
$\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{1}{2}\left(\frac{4}{3}\pi 30^3\right)$
$= \boxed{18000\pi\ m^3}$

(b) $A = \pi r^2 = \pi(30)^2$
$= \boxed{900\pi\ m^2}$

(c) $SA_{dome} = \frac{1}{2}(4\pi r^2)$
$= \frac{1}{2}(4\pi 30^2)$
$= 1800\pi\ m^2$
$\boxed{\text{twice as much}}$

(d) double $A = 1800\pi m^2$
$1800\pi = \pi r^2$
$1800 = r^2$
$r = \sqrt{1800}$
$\sqrt{100}\ \sqrt{9}\ \sqrt{2}$
$r = 10\cdot 3\sqrt{2} = \boxed{30\sqrt{2}\ m}$

11. A minisubmarine has the dimensions shown.
(a) What is the sub's total volume?
(b) What is the sub's total surface area?



(a) $V_{cyl} = \pi r^2 h$
$\pi(3)^2(12)$
$108\pi\ ft^3$

$V_{sphere} = \frac{4}{3}\pi r^3$
$\frac{1}{2}\left(\frac{4}{3}\pi(3)^3\right)$
$\frac{2\cancel{4}\cdot \cancel{27}^9}{{}_1\cancel{2}\cdot\cancel{3}_1}\pi = 18\pi\ ft^3$

$V_{cone} = \frac{1}{3}\pi r^2 h$
$\frac{1}{3}\pi(3)^2\cdot 4$
$\frac{{}^3\cancel{9}\cdot 4}{\cancel{3}_1}\pi = 12\pi$

108
18
12
138

$\boxed{V = 138\pi\ ft^3}$

(b) $L.A._{cyl} = 2\pi rh$
$2\pi(3)(12)$
72π

S.A. sphere
$4\pi r^2$
$\frac{1}{2}(4\pi 3^2)$
$\frac{\cancel{4}^2\pi 9}{\cancel{2}_1}$
18π

$L.A._{cone} = \pi r l$
$\pi(3)5$
15π

72
+18
15
105

$\boxed{S.A. = 105\pi\ ft^2}$

12 × 9 = 108
36 / 2