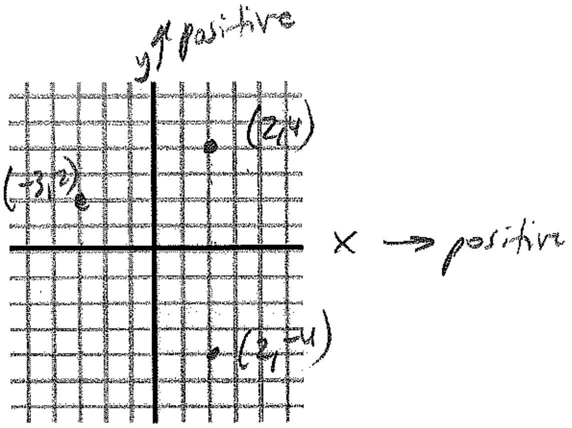
# Geometry, 13.1: Graphing Equations

**Plotting points: (x, y)**

Example: plot the points (2, 4), (-3, 2), (2, -4)



**Graphing a line: 3 methods...**

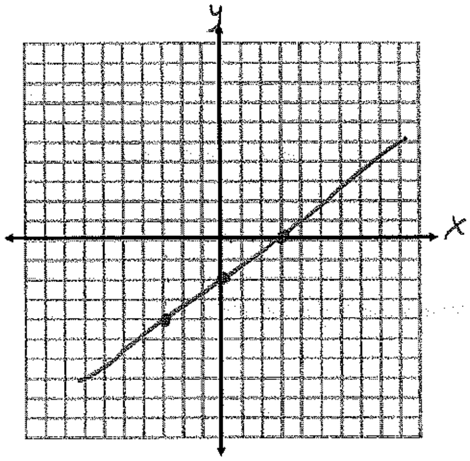
**1) Plot points from a table of x,y values**

Example: graph the line $2x - 3y = 6$

| x | o |
|---|---|
| 0 | -2 |
| 3 | 0 |
| -3 | -4 |

1) solve for y
2) make a table
   pick x's, find matching y values
3) plot points

$$2x - 3y = 6$$
$$\underline{-2x \qquad -2x}$$
$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$$
$$y = \frac{2}{3}x - 2$$

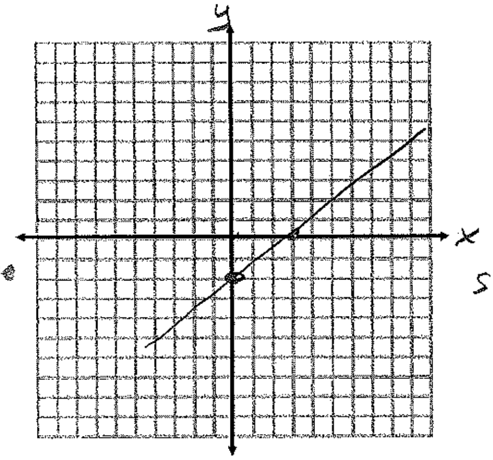


**2) Use y-intercept and slope**

Example: graph the line $2x - 3y = 6$

1) solve for y
2) $y = \underset{\substack{\uparrow \\ \text{slope}}}{m}x + \underset{\substack{\uparrow \\ \text{y-int}}}{b}$
3) plot y-int
4) from y-int, use slope to get 2nd point

$$y = \underset{\substack{\uparrow \\ \text{slope}}}{\frac{2}{3}}x - \underset{\substack{\uparrow \\ \text{y-int}}}{2}$$



$\text{slope} = \frac{\overset{+}{\text{up}}/\overset{-}{\text{down}}}{\text{right}}$

$= \frac{2}{3}$

**3) Use x-intercept and y-intercept**

Example: graph the line $2x - 3y = 6$

y-intercept when x = 0

x-intercept when y = 0

y-int: (x = 0)

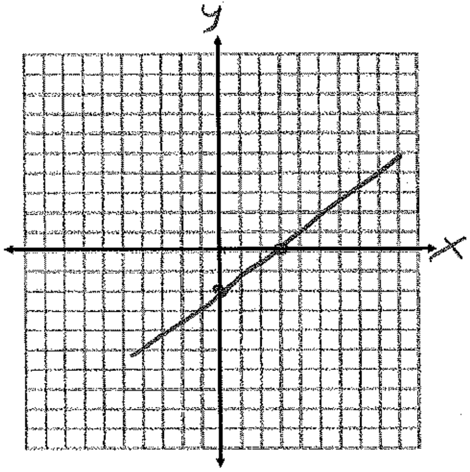
$$2(0) - 3y = 6$$
$$-3y = 6$$
$$y = -2$$

x-int (y = 0)

$$2x - 3(0) = 6$$
$$2x = 6$$
$$x = 3$$

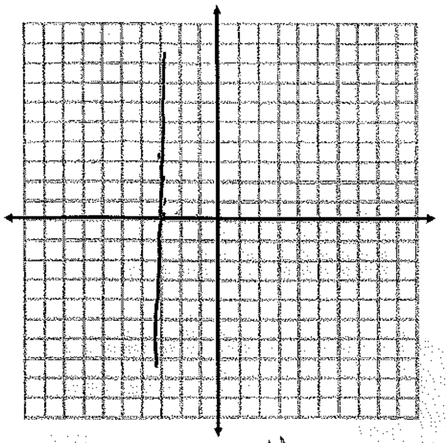
'Coverup method'

$$\cancel{2x} - 3y = 6$$
$$2x - \cancel{3y} = 6$$



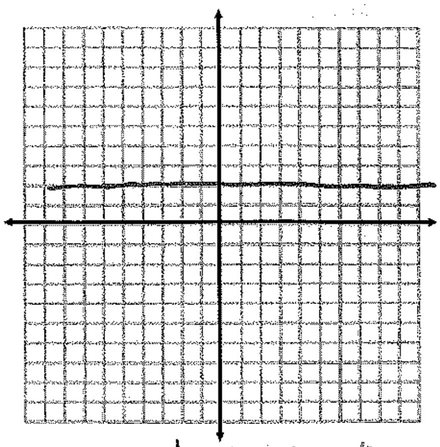
**Special cases: vertical and horizontal lines**

Example: graph the line $x = -3$



Vertical line

Example: graph the line $y = 2$



horizontal line

**Verifying that points are on the curve of an equation:**

Example: Verify that the points (0, 3), (-3, 0) and (3, 0) lie on the circle whose equation is $x^2 + y^2 = 9$

x y

$(0,3)$ $0^2 + 3^2 \overset{?}{=} 9$

$0 + 9 = 9$

$9 = 9$

✓

$(-3,0)$ $(-3)^2 + (0)^2 \overset{?}{=} 9$

$9 + 0 = 9$

$9 = 9$

✓

$(3,0)$ $(3)^2 + (0)^2 \overset{?}{=} 9$

$9 + 0 = 9$

$9 = 9$

✓

## Geometry, 13.2 ~~day 1~~: Equations of lines

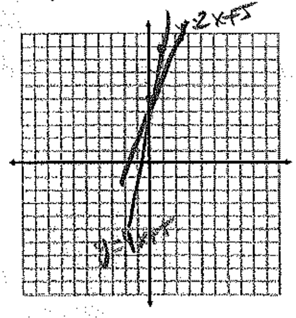
When an equation of a line is in the form: $y = mx + b$ we call this 'slope, y-intercept form' because we can read the slope (m) and y-intercept (b) directly from the equation.

Find the slope and y-intercept:

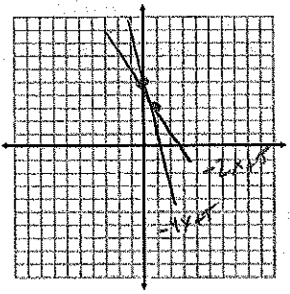
| $y = 2x + 5$ | $y = -\frac{2}{3}x + 5$ | $y = 32x - 14$ | $2x - 3y = 6$ | $4y - 3x = 7$ |
|---|---|---|---|---|
| $m = 2$<br>$b = 5$ | $m = \frac{-2}{3}$<br>$b = 5$ | $m = 32$<br>$b = -14$ | $\underline{-2x \qquad -3x}$<br>$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$<br>$y = \frac{2}{3}x - 2$<br>$m = \frac{2}{3}$<br>$b = -2$ | $\underline{+3x \qquad +3x}$<br>$\frac{4y}{4} = \frac{3x + 7}{4}$<br>$y = \frac{3}{4}x + \frac{7}{4}$<br>$m = \frac{3}{4}$<br>$b = \frac{7}{4}$ |

**How slope affect the graph of a line:**

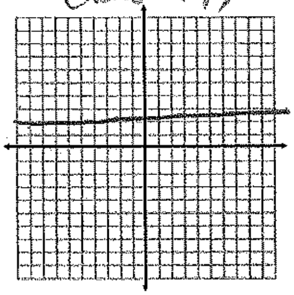
Positive slope:
$y = 2x + 5$
$y = 4x + 5$



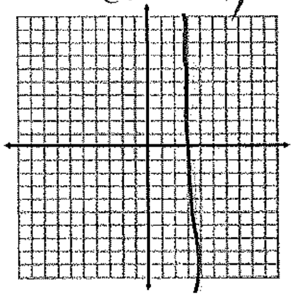
Negative slope:
$y = -2x + 5$
$y = -4x + 5$



Zero slope:
$y = 2$
(horizontal))



Undefined slope:
$x = 3$
(vertical)

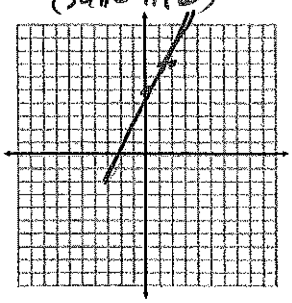


**Comparing lines by their slopes:**

Coinciding lines

same slope,
same y-intercept

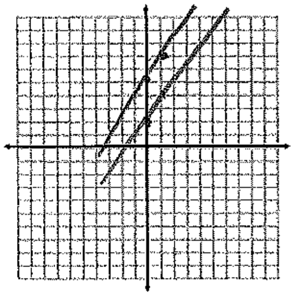
$y = 2x + 5$
$y = 2x + 5$
(same line)



Parallel lines

same slope,
different y-intercept

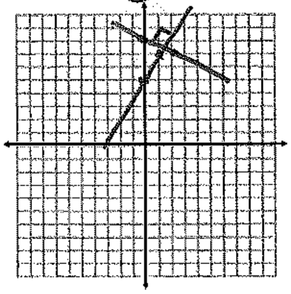
$y = 2x + 5$
$y = 2x + 2$



Perpendicular lines

negative, reciprocal
slopes

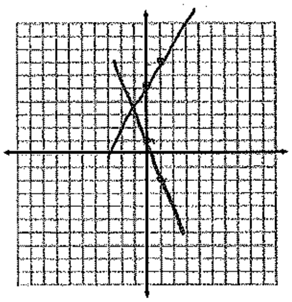
$y = 2x + 5$
$y = -\frac{1}{2}x + 8$



Intersecting lines

<-none of these

$y = 2x + 5$
~~[illegible]~~ $y = -3x + 1$



**Forms of line equations:**

Slope-intercept form / y-form: $y = mx + b$ ex. $y = 2x - 5$

Point-slope form: $y - y_1 = m(x - x_1)$ ex. $y - 2 = 2(x - 4)$

General form: $ax + by + c = 0$ ex. $6x - 4y - 2 = 0$

Examples:

Find the equation of the line that passes through these two points: (2, 4) and (6, 16)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16 - 4}{6 - 2} = \frac{12}{4} = 3$$

point-slope form: $\boxed{y - 4 = 3(x - 2)}$

Find the general form equation for the line $\left(y = \frac{2}{3}x - 8\right) 3$

$$3y = 2x - 24$$

$$-3y \quad -3y$$

$$\boxed{2x - 3y - 24 = 0}$$

Write a slope-intercept (y-form) equation for the line that is perpendicular to line $2y = x + 16$ and passes through (0, -5)

$$\frac{2y}{2} = \frac{x + 16}{2}$$

$$y = \frac{1}{2}x + 8$$

$\perp$, slopes negative, reciprocal

$$m = -2$$

$$y = -2x + b$$

$$-5 = -2(0) + b$$

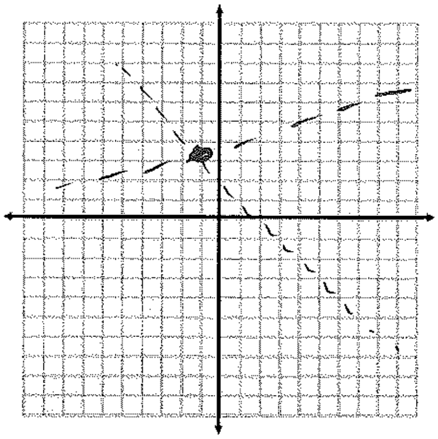
$$-5 = b$$

$$\boxed{y = -2x - 5}$$

## Geometry, 13.3: Systems of Equations

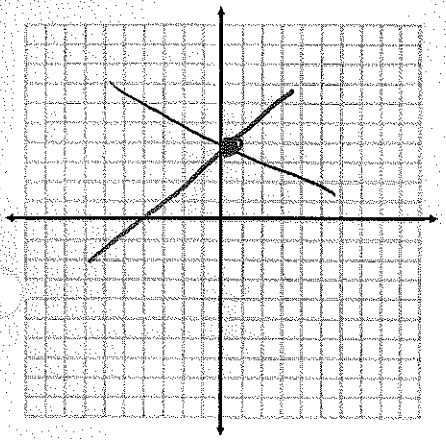
A system of equations has 2 lines. Each line contains points that make that line equation true.

The solution of a system of equations are all the points that make **both** equations true at the same time.

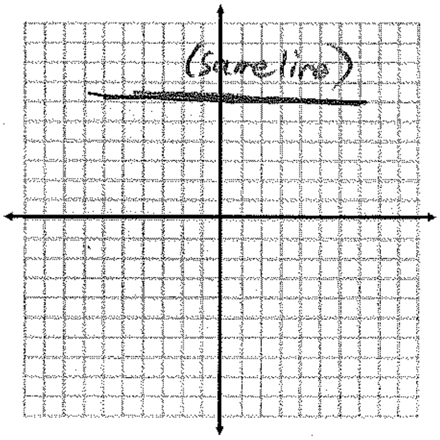


**3 possibilities:**

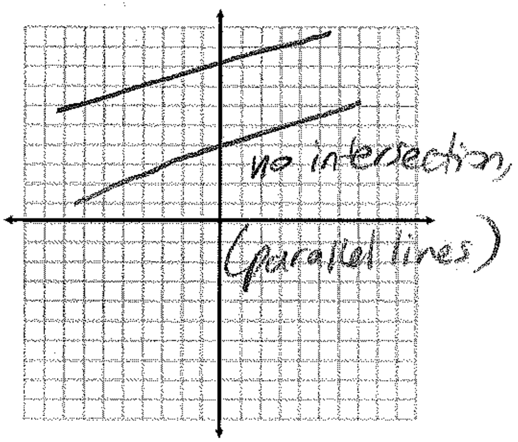
A point



A line



No solution



**2 methods to find the intersection point:**

### Substitution

$$\begin{cases} x+2y=7 \\ 4x \ -y=10 \end{cases}$$

1) Solve one equation for a variable:

$x+2y=7$
$-2y \quad -2y$
$x=7-2y$

2) substitute expression for variable in other equation, and solve that eqn.

$4x-y=10$
$4(7-2y)-y=10$
$28-8y-y=10$
$28-9y=10$
$-9y=-18$
$\boxed{y=2}$

3) plug answer into other equation and solve

$x=7-2y$
$x=7-2(2)$
$x=7-4$
$\boxed{x=3}$

$\boxed{(3,2)}$

### Elimination

$$\begin{cases} x+2y=7 \\ 4x \ -y=10 \end{cases}$$

1) multiply one or both equations by numbers so one term is equal/opposite:

$-4(x+2y=7)$
$4x-y=10$

$-4x-8y=-28$
$4x-y=10$

2) add equations, solve result eqn

$-9y=-18$
$y=2$

3) plug answer into either original eqn and solve:

$x+2(2)=7$
$x+4=7$
$x=3$

$\boxed{(3,2)}$

$$\begin{cases} 2x \;\; + y = 10 \\ 8x + 4y = 17 \end{cases}$$

$y = 10 - 2x$

$8x + 4(10 - 2x) = 17$

$\cancel{8x} + 40 - \cancel{8x} = 17$

$40 = 17$

not true

this is the
parallel line case

$\boxed{\text{no solution}}$

$$\begin{array}{l} 2x + y = 10 \\ -2x \qquad -2x \end{array}$$

$y = \boxed{-2x} + 10$

$8x + 4y = 17$

$4y = -8x + 17$

$y = -\frac{8}{4}x + \frac{17}{4}$

$y = \boxed{-2x} + \frac{17}{4}$

same slope,
parallel

$$\begin{cases} y = 3x + 1 \\ 6x - 2y = -2 \end{cases}$$

$6x - 2(3x + 1) = -2$

$\cancel{6x} - \cancel{6x} - 2 = -2$

$-2 = -2$

true

this is the same line case

$y = 3x + 1$

$$\begin{array}{l} 6x - 2y = -2 \\ -6x \qquad\quad -6x \end{array}$$

$\frac{-2y}{-2} = \frac{-6x - 2}{-2}$

$y = 3x + 1$

same line

## Geometry, 13.6: Circles

Equation of a circle:

$$(x-h)^2+(y-k)^2=r^2$$

$$where \quad r = radius$$

$$(h,k) = center\ of\ circle$$

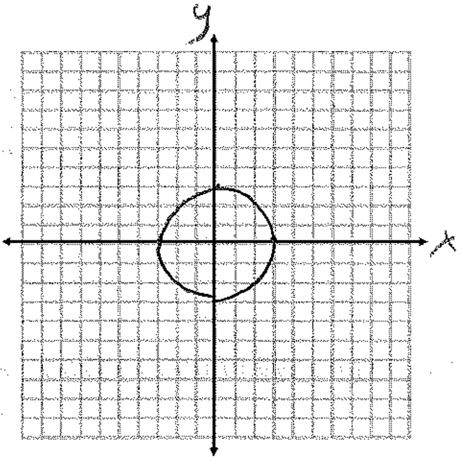
Examples:

Graph the circle: $x^2+y^2=9$

$(x-0)^2+(y-0)^2=3^2$

center $(0,0)$

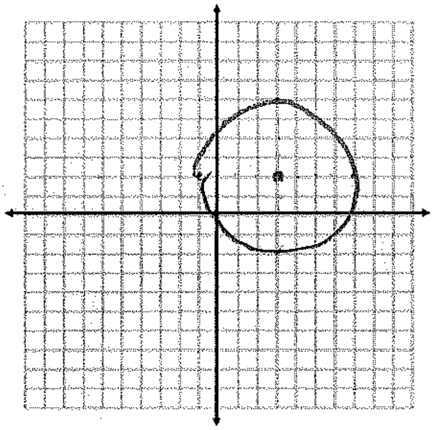
$r=3$



Graph the circle: $(x-3)^2+(y-2)^2=16$

Center $(3,2)$

$r=4$

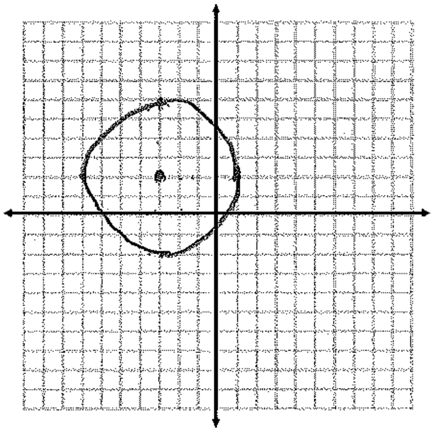


Graph the circle: $(x+3)^2+(y-2)^2=16$

$(x-(-3))^2+(y-2)^2$

Center $(-3,2)$

$r=4$



Find the center and radius, circumference and area of the circle $(x-2)^2+(y+7)^2=64$

$C: (2,-7)$    $C=2\pi r$    $A=\pi r^2$

$r=8$    $C=2\pi 8$    $A=\pi 8^2$

$C=16\pi$    $A=64\pi\ u^2$

Write the equation of a circle with radius of 5 and center at (-2, 1)

$(x+2)^2+(y-1)^2=25$

Is the point (4, 2) on the graph of $(x-3)^2+(y+2)^2=17$

$(4-3)^2+(2+2)^2 \overset{?}{=} 17$

$1^2+4^2 \overset{?}{=} 17$

$1+16 \overset{?}{=} 17$

$17=17$ ✓ yes

Write an equation for a circle with center at (-1, 7) that passes through the origin. $(0,0)$

$(x+1)^2+(y-7)^2=r^2$

$(0+1)^2+(0-7)^2=r^2$

$1^2+(-7)^2=r^2$

$1+49=r^2$

$50=r^2$

$\boxed{(x+1)^2+(y-7)^2=50}$

Is $x^2-8x+y^2-10y=8$ the equation of a circle?

complete the square

$(x^2-8x+16)+(y^2-10y+25)=8+16+25$

$(x-4)^2+(y-5)^2=49$

$\boxed{\text{yes}}$

$$\begin{array}{r} 8 \\ 16 \\ 25 \\ \hline 49 \end{array}$$