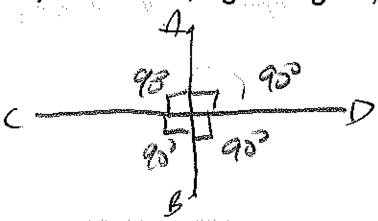
Geometry, 2.1 Notes – Perpendicularity

Parallel and perpendicular are opposite.

Parallel = Same direction, never intersect

Perpendicular = intersect at right angles

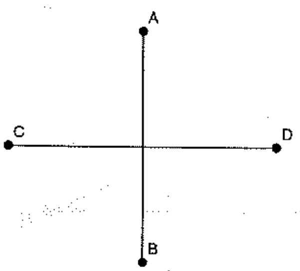
Perpendicular, *right angles*, 90° *angles*, all go together.



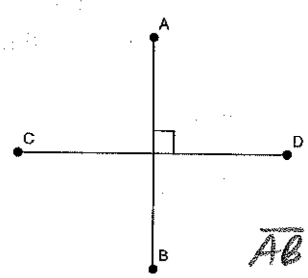
$AB \perp CD$

notation for perpendicular

Do not assume something is perpendicular on a diagram:

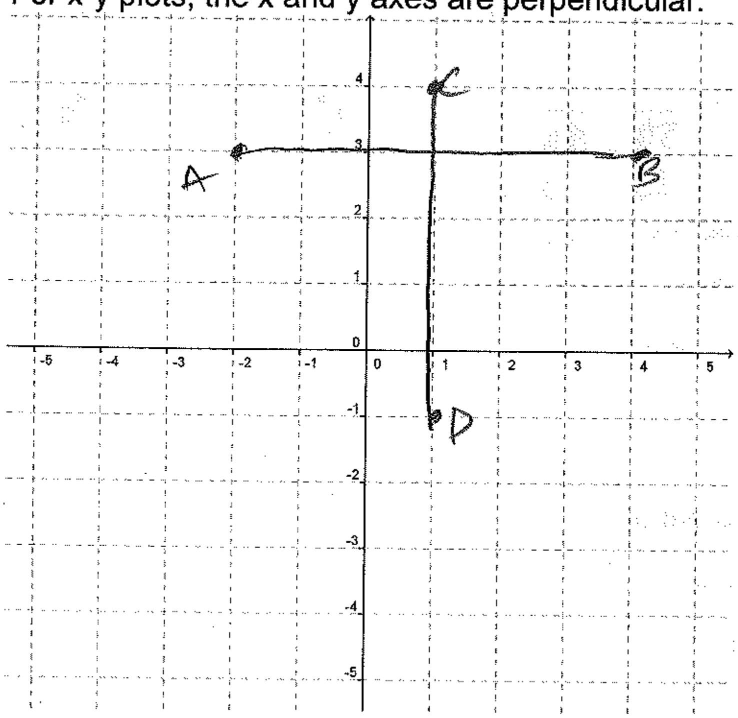


Not perpendicular



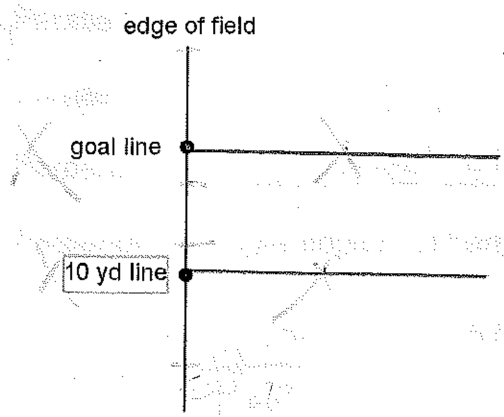
$\overline{AB} \perp \overline{CD}$

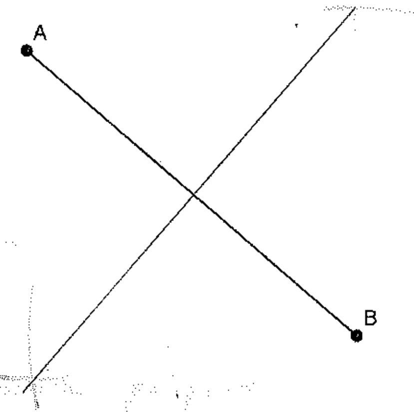
For x-y plots, the x and y axes are perpendicular:



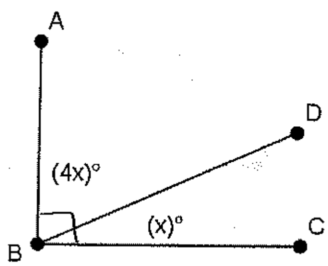
$AB \perp CD$

Practical use of Geometry: constructing perpendicular line segments, bisecting line segments…





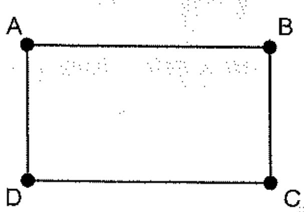
Example problems:



Given $\overline{AB} \perp \overline{CB}$
Find $m\angle ABD$

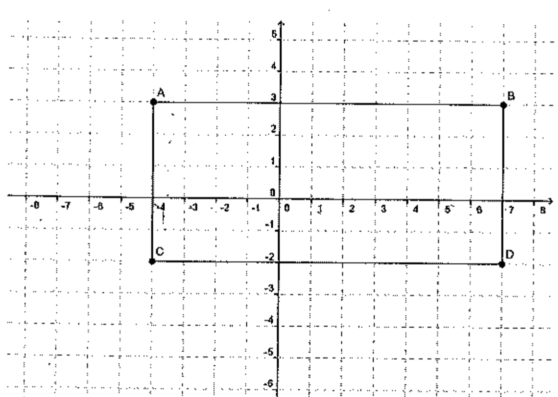
$4x + x = 90$
$5x = 90$
$x = 18$

$m\angle ABD = 4x = 4(18) = 72°$



Given: $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$
Prove: $\angle B \cong \angle C$

S	R
1. $\overline{AB} \perp \overline{BC}$	1. Given
2. $\overline{DC} \perp \overline{BC}$	2. Given
3. $\angle B$ is rt angle	3. If segments $\perp$, they form a rt $\angle$
4. $\angle C$ is rt angle	4. If segments $\perp$, they form a rt $\angle$
5. $\angle B \cong \angle C$	5. rt angles are $\cong$.



Find the area of ~~rectangle~~ ~~shape~~ figure CABD.

- corners on grid, so sides are $\perp$
- rectangle
- $AC = 5$, $AB = 11$

Area $= (5)(11) = \boxed{55\ u^2}$

Geometry, 2.2 Notes – Complementary and Supplementary Angles

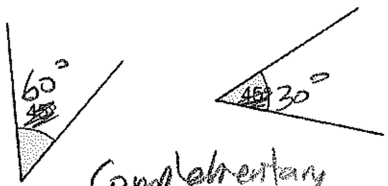
Complementary angles are two angles that add to 90°

(Each angle is called the complement of the other angle.)

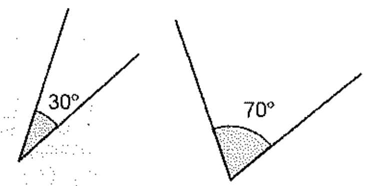
Supplementary angles are two angles that add to 180°

(Each angle is called the supplement of the other angle.)

Are these pairs of angles complementary, supplementary or neither?



Complementary



neither

supplementary

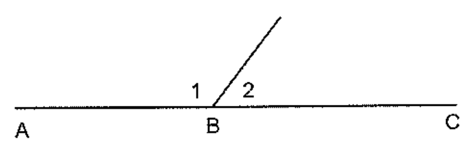
Example: $\angle A = 40°$
What is the complement of $\angle A$? $90 - 40 = \boxed{50°}$
What is the supplement of $\angle A$? $180 - 40 = \boxed{140°}$

Example: One of two supplementary angles is 3 times as big as the other. Find the measures of the two angles.

$x + 3x = 180$
$4x = 180$
$x = 45$

45°, 135°

Example:
Given: Diagram as shown.
Conclusion (prove): $\angle 1$ is the supplement of $\angle 2$



1. Diagram as shown.	1. Given
2. $\angle ABC$ is a straight angle	2. assumed from diagram
3. Sum of $\angle 1 + \angle 2 = 180°$	3. straight angle = 180°
4. $\angle 1$ is supplement of $\angle 2$	4. supplementary angles add to 180°

Left

One of two complementary angles is 10° greater than the other. Find the measure of the larger angle.

$$x + (x+10) = 90°$$

$x + x + 10 = 90$

$2x + 10 = 90$

$2x = 80$

$x = 40$

$x + 10 = \boxed{50°}$

Middle

Two supplementary angles are in the ratio 11:7. Find the measure of each.

$$11x + 7x = 180$$

$11x + 7x = 180$

$18x = 180$

$x = 10$

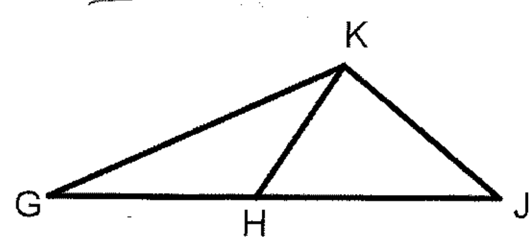
$11(10) = \boxed{110°}$

$7(10) = \boxed{70°}$

Right

Given: Diagram as shown

Prove: $\angle GHK$ *and* $\angle KHJ$ are supplementary



S	R
1. diagram	1. given
2. $m\angle GHJ = 180°$	2. diagram
3. $m\angle GHK + m\angle KHJ = 180°$	3. addition
4. $\angle GHK$ and $\angle KHJ$ are supplementary	4. supplementary angles add to 180°

Geometry, 2.3 Notes – Making conclusions

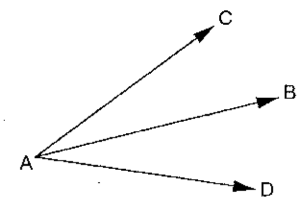
Procedure for making new conclusions…

1. Memorize, keep lists of theorems, definitions, and postulates (things you know).

2. Look for key words and symbols given in the problem.
3. Think of all the theorems, definitions and postulates (things you know) that mention those key words or symbols.
4. Decide which theorem, definition or postulate (thing you know) allows you to make a conclusion.
5. Make the conclusion and give reasons to justify the conclusion.

teacher

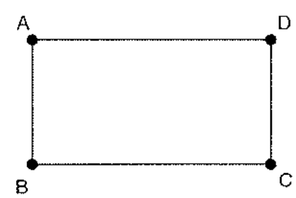
Example:
Given: $\overrightarrow{AB}$ bisects $\angle CAD$
Conclusion: ?



1. $\overrightarrow{AB}$ bisects $\angle CAD$	1. Given
2. $\angle CAD \cong \angle DAB$	2. A bisector divides an angle into 2 $\cong$ angles.

teacher

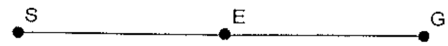
Example:
Given: $\angle A$ is a right angle.
$\angle B$ is a right angle.
Conclusion: ?



1. $\angle A$ is rt $\angle$.	1. Given.
2. $\angle B$ is rt. $\angle$.	2. Given.
3. $\angle A \cong \angle B$	3. If 2 $\angle$'s are rt $\angle$'s, they are $\cong$.

students

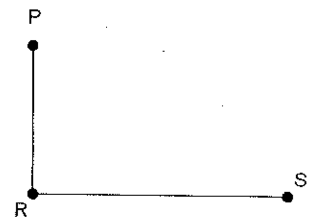
Example:
Given: E is the midpoint of $\overline{SG}$
Conclusion: ?



1. E is midpt of $\overline{SG}$	1. Given
2. $\overline{SE} \cong \overline{EG}$	2. midpt divides segment into 2 ≅ segments

students

Example:
Given: $\angle PRS$ is a right angle.
Conclusion: ?



1. $\angle PRS$ is a rt $\angle$.	1. Given
2. $\overline{PR} \perp \overline{RS}$	2. 2 lines intersect to form a rt$\angle$ are perpendicular.
—or—	
2. $m\angle PRS = 90°$	3. Rt $\angle = 90°$

Some theorems, definitions and postulates you already know:

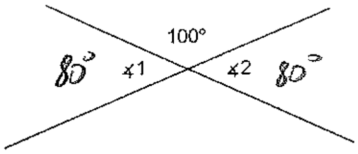
Definitions:

- An acute angle is an angle with measure between 0 and 90 degrees.
- A right angle is an angle whose measure is 90 degrees.
- An obtuse angle is an angle whose measure is between 90 and 180 degrees.
- A straight angle is an angle whose measure is 180 degrees.
- Congruent angles have the same angle measure.
- Congruent line segments have the same length.
- Points that lie on the same line are called collinear.
- A midpoint bisects a line segment into 2 congruent segments.
- An angle bisector bisects an angle into 2 congruent angles.
- A trisector divides a line segment or angle into 3 congruent pieces.
- Lines, rays, or line segments that intersect at right angles are perpendicular.
- Complementary angles are 2 angles that add to 90 degrees.
- Supplementary angles are 2 angles that add to 180 degrees.

Theorems:

- If 2 angles are right angles, then they are congruent.
- If 2 angles are straight angles, then they are congruent.
- If a conditional statement is true, then the contrapositive of the statement is true.

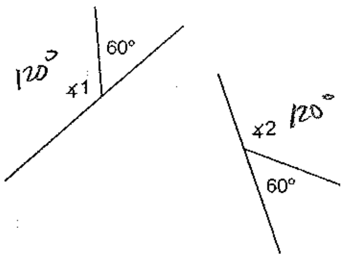
Geometry, 2.4 Notes – Congruent Supplements and Compliments



Find $\angle 1$ and $\angle 2$

(theorem)
If angles are supplementary to the same angle,

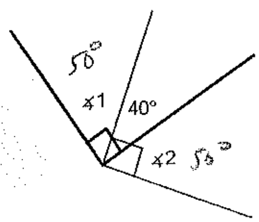
then they are congruent



Find $\angle 1$ and $\angle 2$

(theorem)
If angles are supplementary to congruent angles,

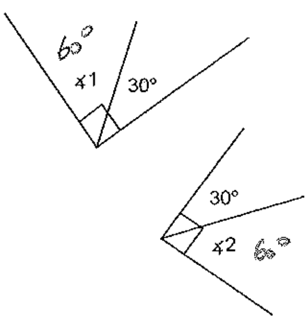
then they are congruent.



Find $\angle 1$ and $\angle 2$

(theorem)
If angles are complementary to the same angle,

then they are congruent

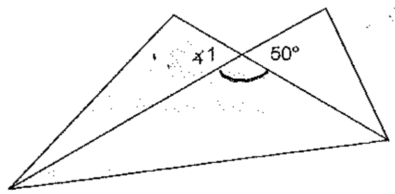


Find $\angle 1$ and $\angle 2$

(theorem)
If angles are complementary to congruent angles,

then they are congruent.

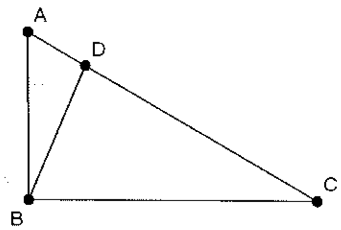
Example:



Find $\angle 1$

50°

Example:



Given: $\angle A$ is complementary to $\angle C$

$\angle DBC$ is complementary to $\angle C$

Conclusion: ?

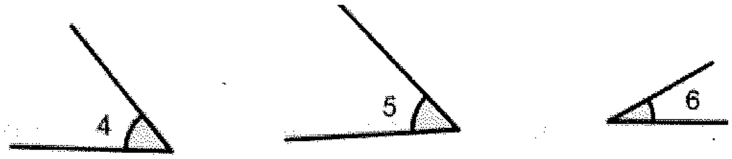
S	R
1. ∠A is comp. to ∠C	1. Given
2. ∠DBC is comp. to ∠C	2. Given
3. ∠A ≅ ∠DBC	3. Angles comp. to same angle are congruent

Try it...

Left

Given: $\angle 4$ is complementary to $\angle 6$
$\angle 5$ is complementary to $\angle 6$

Prove: $\angle 4 \cong \angle 5$



Middle

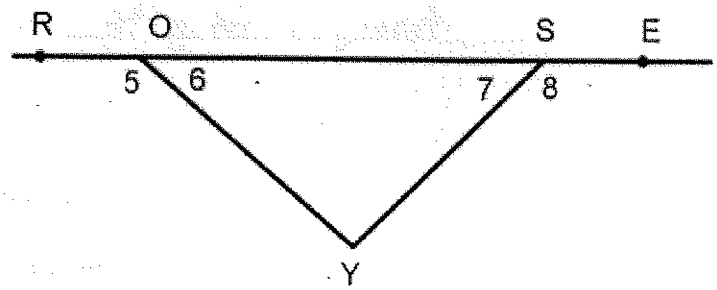
Given: $\angle 4$ is supplementary to $\angle 6$
$\angle 5$ is complementary to $\angle 7$
$\angle 4 \cong \angle 5$

Conclusion: ?

Right

Given: Diagram as shown.
$\angle 6 \cong \angle 7$

Prove: $\angle 5 \cong \angle 8$



left

S	R
1. $\angle 4$ comp. $\angle 6$	1. Given
2. $\angle 5$ comp. $\angle 6$	2. Given
3. $\angle 4 \cong \angle 5$	3. If angles are $\cong$ to same $\angle$, they are $\cong$

middle

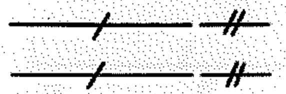
S	R
1. $\angle 4$ supp $\angle 6$	1. Given
2. $\angle 5$ supp $\angle 8$	2. Given
3. $\angle 4 \cong \angle 5$	3. Given
4. $\angle 6 \cong \angle 7$	4. $\angle$'s supp to $\cong$ $\angle$'s are $\cong$

right

S	R
1. diagram	1. given
2. $\angle 6 \cong \angle 7$	2. Given
3. $\angle 5$ supp. to $\angle 6$	3. diagram, st $\angle$
4. $\angle 8$ supp. to $\angle 7$	4. diagram, st $\angle$
5. $\angle 5 \cong \angle 8$	5. $\angle$'s supp. to $\cong$ $\angle$'s are $\cong$

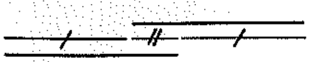
Geometry, 2.5 Notes – Addition and Subtraction Properties

Addition Properties:



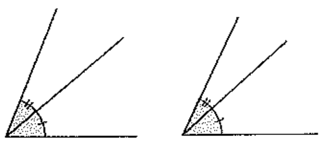
If Congruent segments are added to congruent segments

the sums are congruent



If a segment is added to congruent segments

the sums are congruent

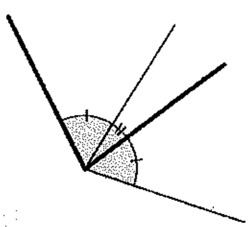


If congruent angles are added to congruent angles

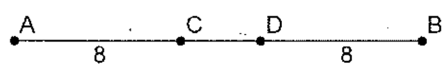
the sums are congruent

If an angle is added to congruent angles

the sums are congruent



Example: In the diagram below, does AD = CB? Yes (does not matter what CD is)



Subtraction Properties:

If congruent segments (or angles) are subtracted from congruent segments (or angles)

the differences are Congruent

If a segment (or angle) is subtracted from congruent segments (or angles)

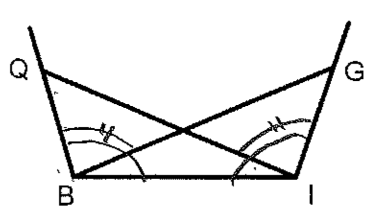
the differences are Congruent

Examples:

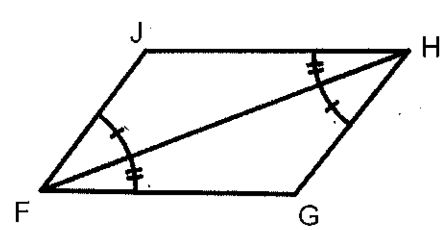
Given: $\angle QBI \cong \angle GIB$
$\angle GIQ \cong \angle QBG$

Name the angles congruent by the Addition Property

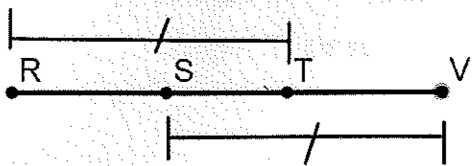
Is $\angle GBI \cong \angle QIB$? Yes



$\angle JFG \cong \angle JHG$



Name the segments that are congruent by the Subtraction Property:

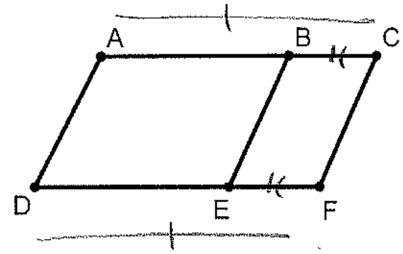


$\overline{RS} \cong \overline{TV}$

Given: $\overline{AC} \cong \overline{DF}$

$\overline{BC} \cong \overline{EF}$

Prove: $\overline{AB} \cong \overline{DE}$

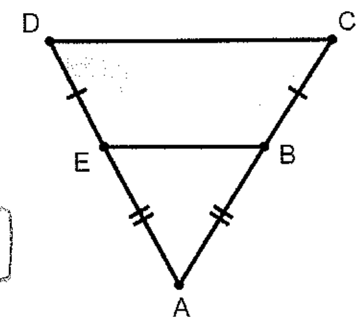


1. $\overline{AC} \cong \overline{DF}$	1. Given
2. $\overline{BC} \cong \overline{EF}$	2. Given
3. $\overline{AB} \cong \overline{DE}$	3. subtraction property

subtraction property because you have to subtract to get the result

Try it...

Name the segments congruent by the Addition Property



$\boxed{\overline{AD} \cong \overline{AC}}$

Given: $\overline{GH} \cong \overline{JK}$

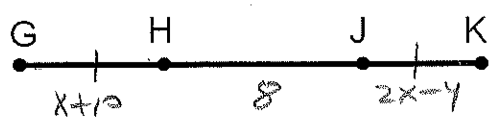
$GH = x+10$

$HJ = 8$

$JK = 2x-4$

Find: GJ

$= \boxed{32}$



$(14+10) = 24$

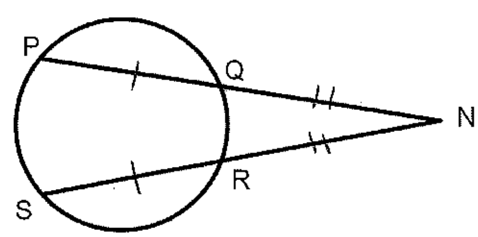
$x+10 = 2x-4$

$14 = x$

Given: $\overline{PQ} \cong \overline{SR}$

$\overline{QN} \cong \overline{RN}$

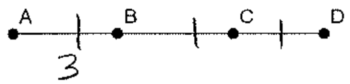
Prove: $\overline{PN} \cong \overline{SN}$

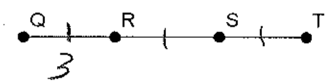


S	R
1. $\overline{PQ} \cong \overline{SR}$	1. Given
2. $\overline{QN} \cong \overline{RN}$	2. Given
3. $\overline{PN} \cong \overline{SN}$	3. Addition property

Geometry, 2.6 Notes – Multiplication and Division Properties

Example:

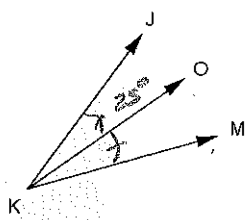


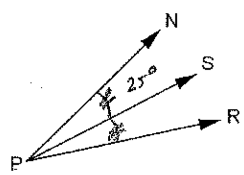


B, C trisect $\overline{AD}$ R, S trisect $\overline{QT}$

If AB=3 and QR=3, what can we say about $\overline{AD}$ and $\overline{QT}$? $\overline{AD} \cong \overline{QT}$

Example:





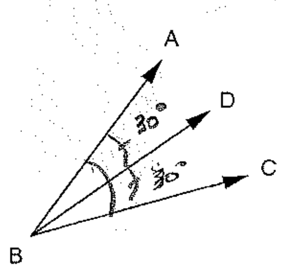
$\overrightarrow{KO}$ bisects $\angle JKM$
$\overrightarrow{PS}$ bisects $\angle NPR$
If $m\angle JKO = 25°$ and $m\angle NPS = 25°$
What can we say about $\angle JKM$ and $\angle NPR$

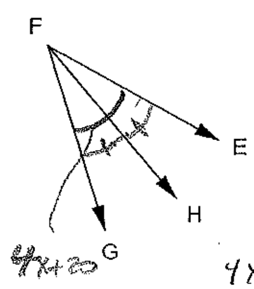
$\angle JKM \cong \angle NPR$

Multiplication Property:
If segments (or angles) are congruent, their like multiples are congruent.

Division Property:
If segments (or angles) are congruent, their like divisions are congruent.

Example:





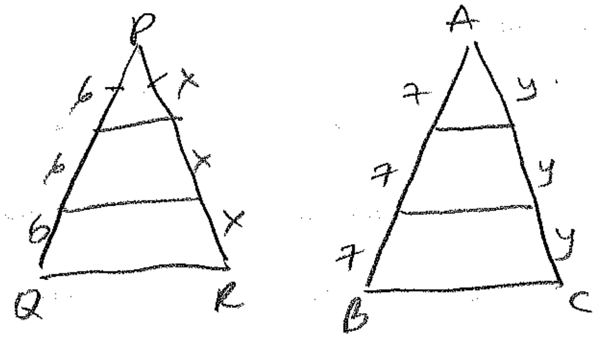
$\angle ABC \cong \angle EFG$
$\overrightarrow{BD}$ and $\overrightarrow{FH}$ are bisectors
$m\angle ABD = 30°$
$m\angle EFG = 4x + 20$
Find x

$$4x + 20 = 60$$
$$-20 \quad -20$$
$$\frac{4x}{4} = \frac{40}{4}$$
$$\boxed{x = 10}$$

Example

a) If $\overline{PQ} \cong \overline{PR}$ in $\triangle PQR$
what can we conclude?

$\boxed{x = 6}$

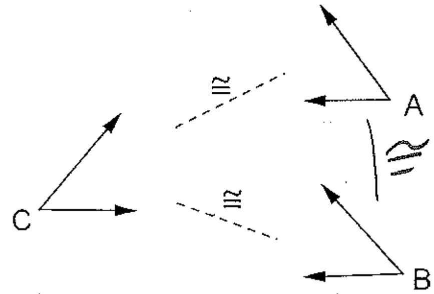


b) If $AC = AB + 3$ in $\triangle ABC$
what can we conclude?

$$AC = AB + 3$$
$$3y = 3(7) + 3$$
$$3y = 21 + 3$$
$$\frac{3y}{3} = \frac{24}{3}$$
$$\boxed{y = 8}$$

Geometry, 2.7 Notes – Transitive and Substitution Properties

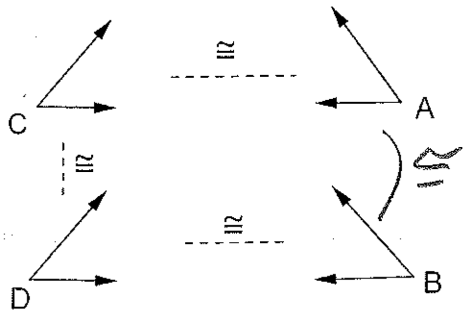
Transitive Property:



If angles (or segments) are congruent to the same angle (or segment), they are congruent to each other.

If $\angle A \cong \angle C$ and $\angle B \cong \angle C$

then $\angle A \cong \angle B$



If angles (or segments) are congruent to congruent angles (or segments), they are congruent to each other.

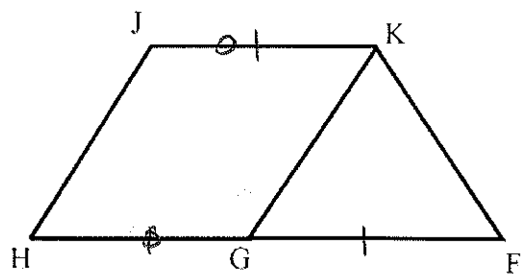
If $\angle A \cong \angle C$, $\angle B \cong \angle D$, and $\angle C \cong \angle D$

then $\angle A \cong \angle B$

Example:

Given: $\overline{FG} \cong \overline{KJ}$
$\overline{GH} \cong \overline{KJ}$

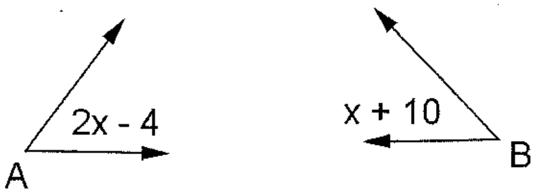
Prove: $\overleftrightarrow{KG}$ bisects $\overline{FH}$



1. $\overline{FG} \cong \overline{KJ}$	1. Given
2. $\overline{GH} \cong \overline{KJ}$	2. Given
3. $\overline{FG} \cong \overline{GH}$	3. Segments ≅ to same segment are ≅ (transitive property)
4. $\overleftrightarrow{KG}$ bisects $\overline{FH}$	4. a bisector divides a segment into 2 ≅ pieces.

Substitution Property:

Examples:



If $\angle A \cong \angle B$, find $m\angle A$

$2x-4 = x+10$

$-x \quad -x$

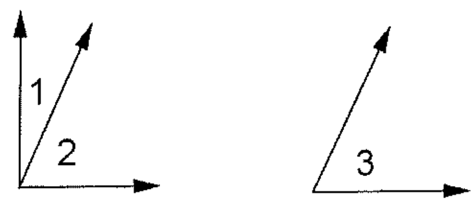
$x-4=10$

$+4 \quad +4$

$x = 14$

substitute

$m\angle A = 2x-4 = 2(14)-4 = \boxed{24}$



If $\angle 1$ is complementary to $\angle 2$ and $\angle 2 \cong \angle 3$ then:

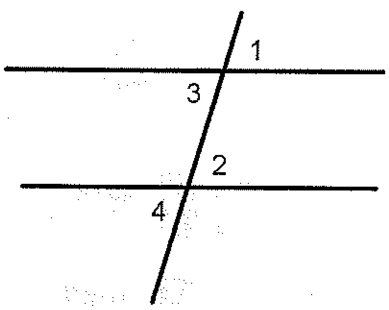
$\angle 1$ is complementary to $\angle 2$

$\angle 2 \cong \angle 3$

substitute

$\angle 1$ is complementary to $\angle 3$

Given: $\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 3$, $\angle 2 \cong \angle 4$ Prove: $\angle 1 \cong \angle 4$



S	R
1. $\angle 1 \cong \angle 3$	1. Given
2. $\angle 2 \cong \angle 3$	2. Given
3. $\angle 2 \cong \angle 4$	3. Given
4. $\angle 1 \cong \angle 2$	4. Substitution property
5. $\angle 1 \cong \angle 4$	5. Substitution property

#8. The complement of an angle is 24 degrees greater than twice the angle. Find the measure of the complement.

x = 'the angle'

comp of $x = 90 - x$

twice $x = 2x$

comp. of angle − twice angle = 24°

$(90-x) - (2x) = 24$

$90 - x - 2x = 24$

$90 - 3x = 24$

$-90 \quad -90$

$-3x = -66$

$x = \frac{-66}{-3} = 22$

comp of $x = 90 - 22 = \boxed{68°}$

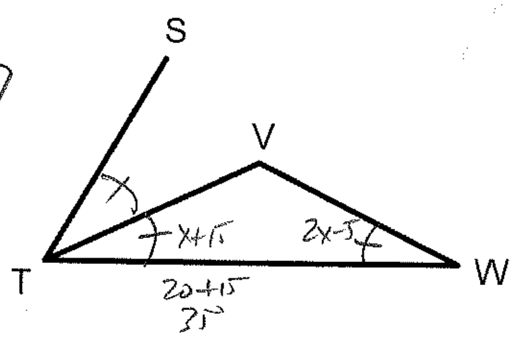
#9. $\angle W \cong \angle STV$

$\overrightarrow{TV}$ bisects $\angle STW$

$\angle W = (2x-5)°$

$\angle VTW = (x+15)°$

Find: $m\angle STW$ $= \boxed{70°}$



$2x-5 = x+15$

$-x \quad -x$

$x-5 = 15$

$+5 \quad +5$

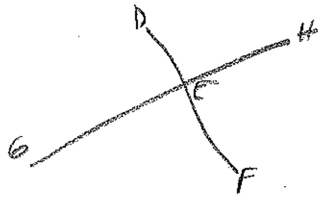
$x = 20$

Geometry, 2.8 Notes – Opposite Rays and Vertical Angles

Opposite Rays: have a common endpoint and extend in opposite directions.

Examples: 

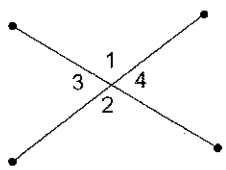
$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays



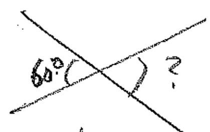
Name opposite rays:
$\overrightarrow{ED}$ and $\overrightarrow{EF}$,
$\overrightarrow{EG}$ and $\overrightarrow{EH}$

Vertical Angles: pairs of angles on opposite sides of intersection, where 2 lines cross.

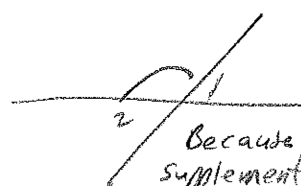
Example:



$\angle 1$ and $\angle 2$ are vertical angles
$\angle 3$ and $\angle 4$ are vertical angles

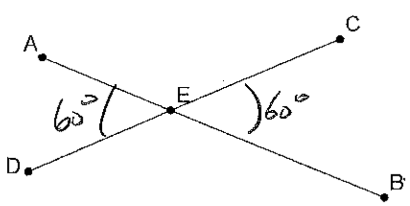


Vertical Angles are Congruent **Why?**



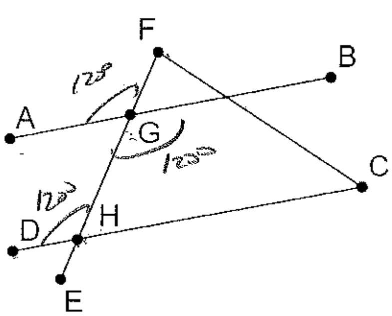
Because $\angle$'s supplementary to same $\angle$ are $\cong$

Examples:



$m\angle AED = 60°$

Find $m\angle BEC$ $= \boxed{60°}$



$m\angle DHG = 120°$
$\angle DHG \cong \angle AGF$

Find $m\angle BGH$ $= \boxed{120°}$

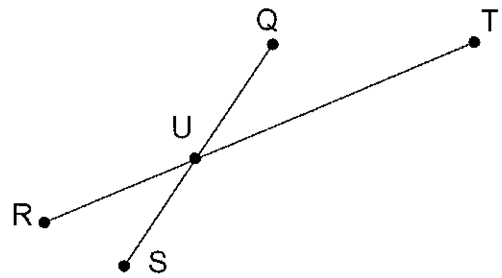
Practice Problems:

#1. Name 2 pairs of vertical angles:

$\angle RUS, \angle QUT$

or

$\angle RUQ, \angle SUT$

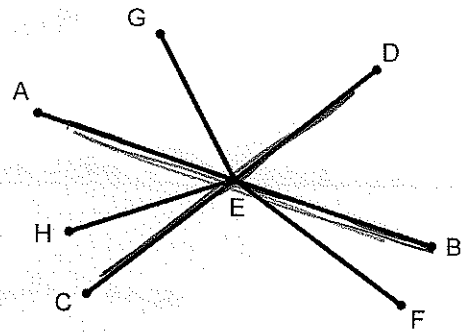


#2. $m\angle RUS = 42°$, Find $m\angle QUT$ $= 42°$

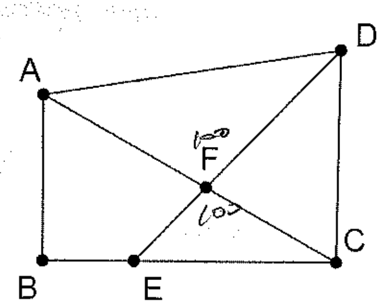
#3. Name 2 pairs of vertical angles:

$\angle AEC, \angle DEB$

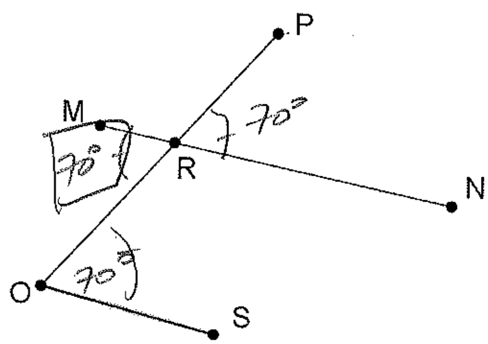
$\angle AED, \angle BEC$



#4. $m\angle EFC = 100°$, Find $m\angle AFD$ $= 100°$



#5. $m\angle SOR = 70°$, $\angle PRN \cong \angle SOR$
Find $m\angle MRO$



#6. $m\angle DBE = 40°$, Find $m\angle ABC$ $= 50°$