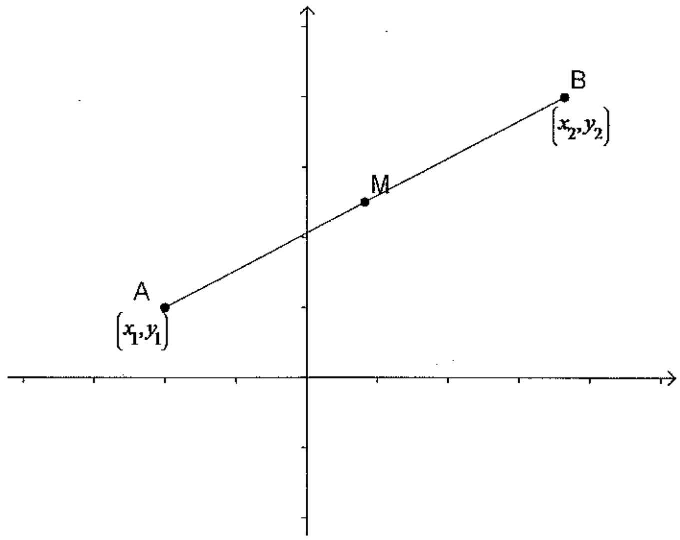
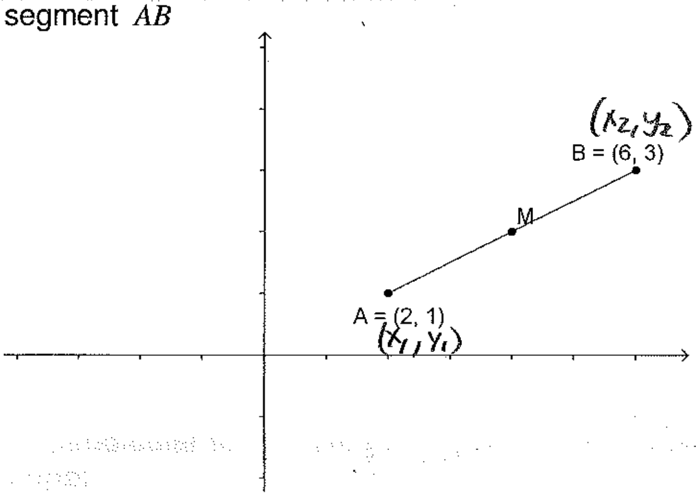
Geometry, 4.1 and 4.6 Notes – Midpoint and Slope

Midpoint formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Examples:
Find the midpoint of line segment $\overline{AB}$

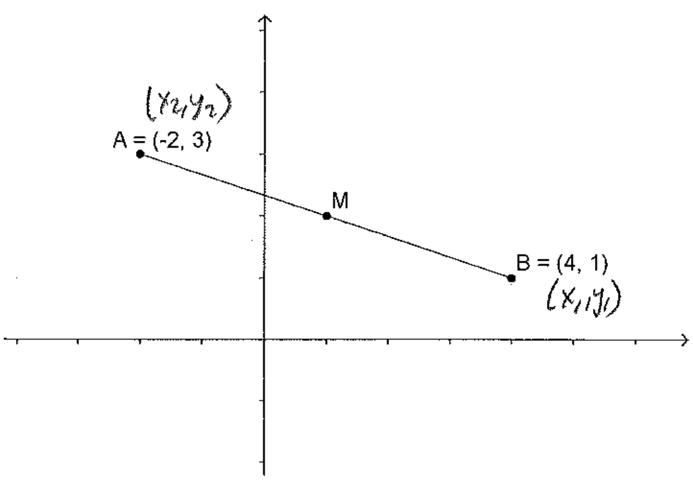


$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2+6}{2}, \frac{1+3}{2}\right)$$

$$= \boxed{(4, 2)}$$

Find the midpoint of line segment $\overline{AB}$



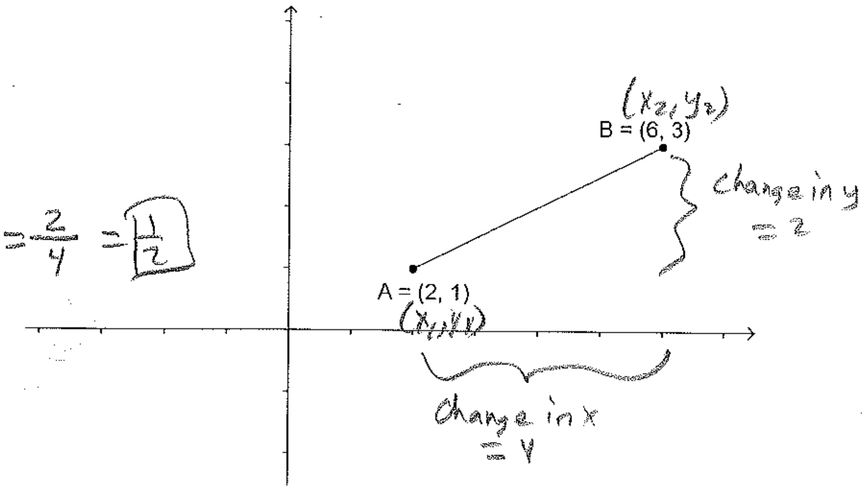
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{4+(-2)}{2}, \frac{1+3}{2}\right)$$

$$= \boxed{(1, 2)}$$

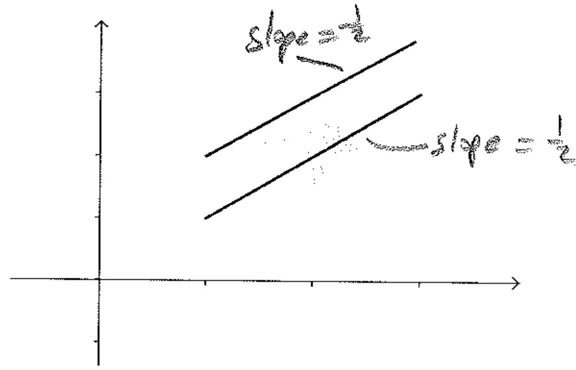
Slope:

$$slope = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

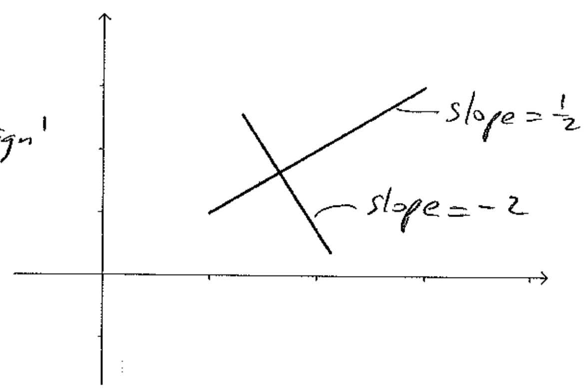


$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{6-2} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

Parallel lines have the same slope:



Perpendicular lines have the negative, reciprocal slope:



negative, reciprocal = 'flip and change sign'

examples $\frac{1}{2} \Rightarrow -2$

$4 \rightarrow -\frac{1}{4}$

$-\frac{3}{4} \rightarrow \frac{4}{3}$

Geometry, 4.2 Notes –Proofs with no Diagrams

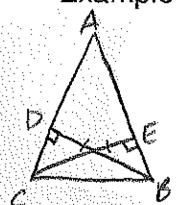
Sometimes, you need to prove something given only as a statement. You need to create a diagram and state the givens and what is to be proved.

General procedure:

1. Rewrite the statement in the form 'if (givens), then (what you want to prove')
   Some hints:
   - Sketch the problem out first to visualize what is being said.
   - Find the 'verb' or 'action word' (is, bisects, divides, etc.)
   - Look at the words after the action word. This is part of what you want to prove.
   - Find what is doing the action…this is also part of what you want to prove.
   - Look at the rest of the statement…everything else, is part of the givens.
2. Make up a diagram that will allow you to prove this.
3. Make up 'Given:' and 'Prove:' words that match your diagram and if/then statement.

givens prove

Example: If two altitudes of a triangle are congruent, then the triangle is isosceles.



Given: $\overline{BD}$ is altitude to $\overline{AC}$
$\overline{CE}$ is altitude to $\overline{AB}$
$\overline{BD} \cong \overline{CE}$

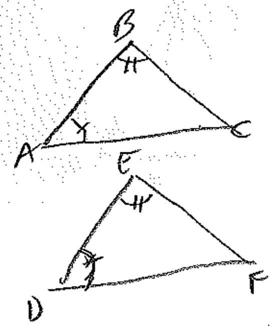
Prove: $\triangle ABC$ is isosceles

Statement	Reason
1. $\overline{BD}$ is alt. to $\overline{AC}$	1. Given
2. $\overline{CE}$ is alt. to $\overline{AB}$	2. Given
3. $\overline{BD} \cong \overline{CE}$	3. Given
4. $\overline{CB} \cong \overline{BC}$	4. Reflexive property
5. $\angle BDC$, $\angle CEB$ rt $\angle$'s	5. def. of altitude
6. $\triangle BDC \cong \triangle CEB$	6. HL
7. $\angle DCB \cong \angle EBC$	7. CPCTC
8. $\overline{AB} \cong \overline{AC}$	8. $\triangle \rightarrow \triangle$
9. $\triangle ABC$ is isosceles	9. def. of isosceles

givens

Example: If two angles of one triangle are congruent to two angles of another triangle, the remaining pair of angles is also congruent.

then



prove

Given: $\angle A \cong \angle D$
$\angle B \cong \angle E$

Prove: $\angle C \cong \angle F$

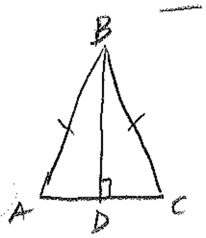
(need sum to 180° to prove)

what does the action

action

Example: The altitude to the base of an isosceles triangle bisects the vertex angle.

If an isosceles triangle then the altitude to the base bisects the vertex angle.



Given: $\triangle ABC$ isosceles with base $\overline{AC}$
$\overline{BD}$ is altitude to $\overline{AC}$

Prove: $\overline{BD}$ bisects $\angle ABC$

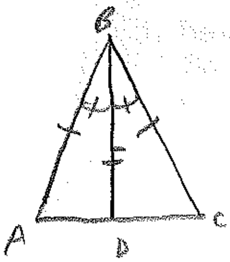
S	R
1, 2	Givens
3. $\overline{AB} \cong \overline{BC}$	3. def of isosceles $\triangle$
4. $\overline{BD} \cong \overline{BD}$	4. Reflexive property
5. $\triangle ABD \cong \triangle CBD$	5. HL
6. $\angle ABD \cong \angle CBD$	6. CPCTC
7. $\overline{BD}$ bis. $\angle ABC$	7. def. of bisector

action

Practice: The bisector of the vertex angle of an isosceles triangle is perpendicular to the base.

what is doing action

If isosceles triangle, then the bisector of vertex angle is perpendicular to the base.



Given: - $\triangle ABC$ is isosceles with base $\overline{AC}$
- $\overline{BD}$ bisects $\angle ABC$

Prove: $\overline{BD} \perp \overline{AC}$

S	R
1. $\triangle ABC$ is isos. w/ base $\overline{AC}$	1. Given
2. $\overline{BD}$ bisects $\angle ABC$	2. Given
3. $\angle ABD \cong \angle CBD$	3. def. of bisector
4. $\overline{BD} \cong \overline{BD}$	4. Reflexive property
5. $\overline{AB} \cong \overline{BC}$	5. def. of isosceles $\triangle$
6. $\triangle ABD \cong \triangle CBD$	6. SAS
7. $\angle ADB \cong \angle CDB$	7. CPCTC
8. $\angle ADB = \angle CDB = 90°$	8. Sum to 180°
9. $\overline{BD} \perp \overline{AC}$	9. $\perp$ lines meet at 90° $\angle$

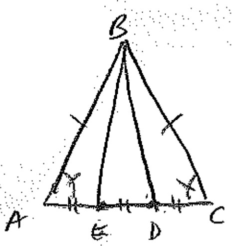
Practice: The line segments joining the vertex angle of an isosceles triangle to the trisection points of the base are congruent.

action

given

If triangle is isosceles, then line segments joining vertex angle to trisection points of base are congruent.

prove.



Given: $\triangle ABC$ isosceles w/ base $\overline{AC}$

E, D, trisect $\overline{AC}$

Prove: $\overline{BE} \cong \overline{BD}$

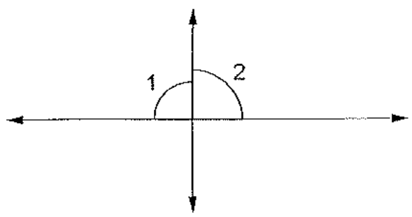
S	R
1. $\triangle ABC$ isos. w/ base $\overline{AC}$	1. Given
2. E, D trisect $\overline{AC}$	2. Given
3. $\overline{AB} \cong \overline{BC}$	3. def of isosceles
4. $\overline{AE} \cong \overline{ED} \cong \overline{DC}$	4. def of trisect
5. $\angle A \cong \angle C$	5. $\triangle \Rightarrow \triangle$
6. $\overline{AD} \cong \overline{CE}$	6. Addition property
7. $\triangle ABD \cong \triangle CBE$	7. SAS
8. $\overline{BE} \cong \overline{BD}$	8. CPCTC

Geometry, 4.3 Notes –Right angle Theorem

Theorem: If two angles are both supplementary and congruent, then they are right angles.

Given: $\angle 1 \cong \angle 2$

Prove: $\angle 1$ *and* $\angle 2$ are right angles.

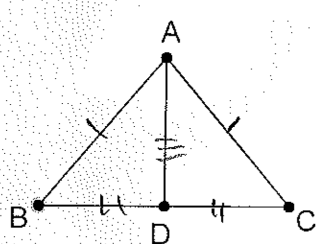


S	R
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 1, \angle 2$ form straight line	2. diagram
3. $m\angle 1 + m\angle 2 = 180°$	3. def of straight $\angle$
4. $m\angle 1 + m\angle 1 = 180°$	4. Substitution
5. $m\angle 1 = 90°$	5. division
6. $\angle 1, \angle 2$ are rt angles	6. rt $\angle$'s $= 90°$

Now we can say: $\angle 1, \angle 2$ rt angles Reason: 2 $\angle$'s supp. & $\cong$ are rt $\angle$'s

Example: Given: $\overline{AB} \cong \overline{AC}$ and $\overline{BD} \cong \overline{CD}$

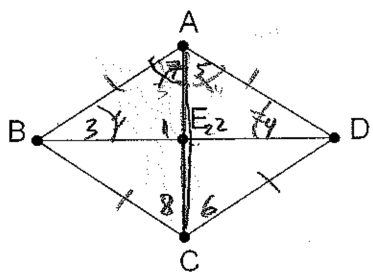
Prove: $\overline{AD}$ is an altitude.



1. $\overline{AB} \cong \overline{AC}$, $\overline{BD} \cong \overline{CD}$	1. Given
2. $\overline{AD} \cong \overline{AD}$	2. Reflexive prop.
3. $\triangle ABD \cong \triangle ACD$	3. SSS
4. $\angle ADB \cong \angle ADC$	4. CPCTC
5. $\angle ADB, \angle ADC$ rt $\angle$'s	5. 2 $\angle$'s supp. & $\cong$ are rt $\angle$'s
6. $\overline{AD}$ is an altitude	6. def of altitude

Example: Given: $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ (ABCD is a rhombus)

Prove: $\overline{AC} \perp \overline{BD}$



1. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$	1. Given
detour 2. $\overline{AC} \cong \overline{AC}$	2. Reflexive prop.
3. $\triangle BAC \cong \triangle DAC$	3. SSS
4. $\angle 7 \cong \angle 5$	4. CPCTC
5. $\angle 3 \cong \angle 4$	5. $\triangle \rightarrow \triangle$
6. $\triangle ABE \cong \triangle ADE$	6. ASA
7. $\angle 1 \cong \angle 2$	7. CPCTC
8. $\angle 1, \angle 2$ rt $\angle$'s	8. 2 $\angle$'s supp & $\cong$ are rt $\angle$'s
9. $\overline{AC} \perp \overline{BD}$	9. $\perp$ lines meet at rt $\angle$'s

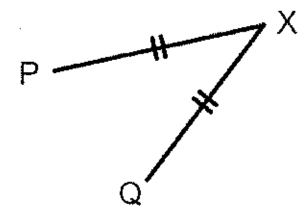
Geometry, 4.4 Notes – Equidistance Theorems

Definition: distance = length of the shortest path between objects.

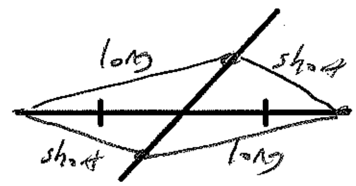
A line segment is the shortest path between two points.

If 2 points P and Q are the same distance from a 3rd point X,

then X is equidistant from P and Q.



Except for the midpoint, points on a segment bisector are not the same distance from the segment endpoints:

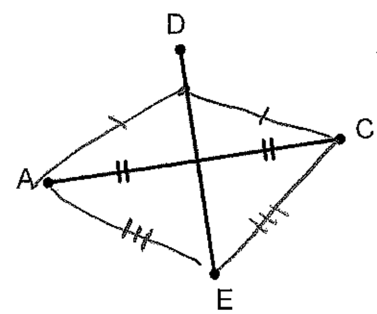


Definition: a perpendicular bisector of a line segment:

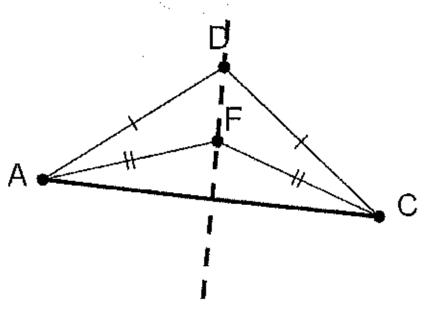
- Bisects the line
- Is perpendicular to the line
- All points on the perpendicular bisector are equidistant from the segment endpoints.

$\overline{DE}$ is a perpendicular bisector to $\overline{AC}$

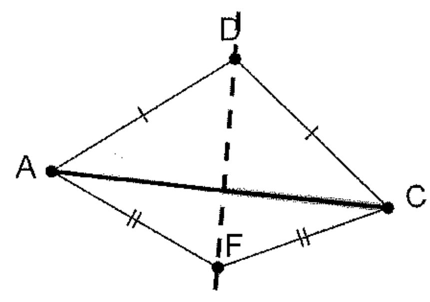
$\overline{DE}$ is $\perp$ bis. to $\overline{AC}$



Theorem: If 2 points are equidistant from the endpoints of a segment, then the 2 points are on the perpendicular bisector. (It doesn't matter which side of the line segment the points are on.)

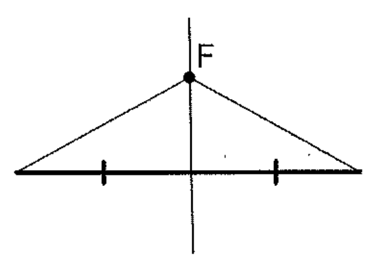


points D and F are equidistant from A and C so $\overline{DF}$ is $\perp$ bis to $\overline{AC}$



Theorem: If a point is on the perpendicular bisector of a line segment, then it is equidistant from the endpoints of that segment.

If F is is on $\perp$ bisector, then F is equidistant from A and B.

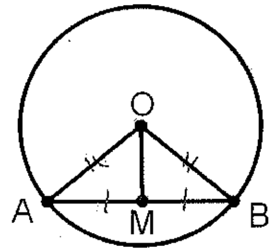


Example Proof:

Given: $\odot O$

M is the midpoint of $\overline{AB}$

Prove: $\overline{OM} \perp \overline{AB}$



(without the equidistant theorems).....

Statement	Reason
1. $\odot O$	1. Given
2. M is midpt of $\overline{AB}$	2. Given
3. $\overline{MA} \cong \overline{MB}$	3. def. of midpt
4. $\overline{OA} \cong \overline{OB}$	4. radii $\cong$
5. $\overline{OM} \cong \overline{OM}$	5. Reflexive property
6. $\triangle AOM \cong \triangle BOM$	6. SSS
7. $\angle OMA \cong \angle OMB$	7. CPCTC
8. $\angle OMA, \angle OMB$ are rt $\angle$'s	8. 2 $\angle$'s supp. and $\cong$ are rt. $\angle$'s
9. $\overline{OM} \perp \overline{AB}$	9. $\perp$ lines meet at rt $\angle$'s.

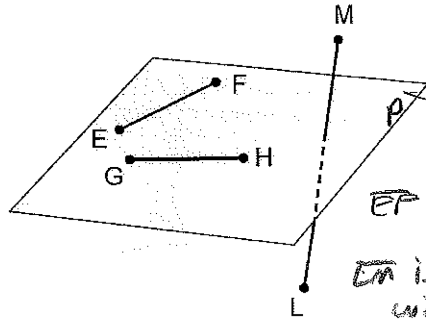
(with the equidistant theorems).....

Statement	Reason
1. $\odot O$	1. Given
2. M is midpt of $\overline{AB}$	2. Given
3. $\overline{MA} \cong \overline{MB}$	3. def. of midpt
4. $\overline{OA} \cong \overline{OB}$	4. radii $\cong$
5. $\overline{OM} \perp \overline{AB}$	5. 2 pts equidistant from endpts of a segment are on $\perp$ bisector.

Geometry, 4.5 Notes –Parallel Lines

Lots of new definitions….

Plane = a flat 2 dimensional surface
Coplanar = in the same plane
Noncoplanar = not in the same plane

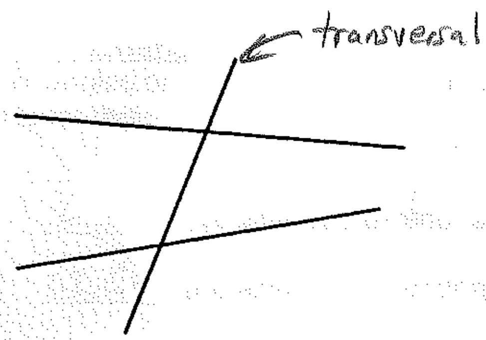


plane P

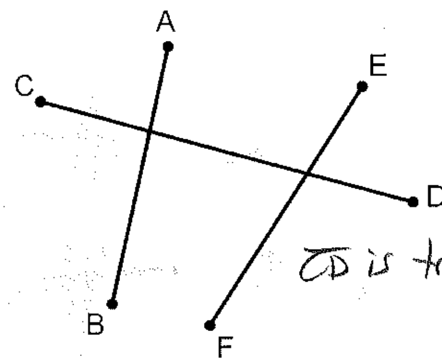
$\overline{EF}$ and $\overline{GH}$ are coplanar

$\overline{LM}$ is not coplanar with $\overline{EF}$ or $\overline{GH}$

Transversal = a line that intersects 2 other coplanar lines.

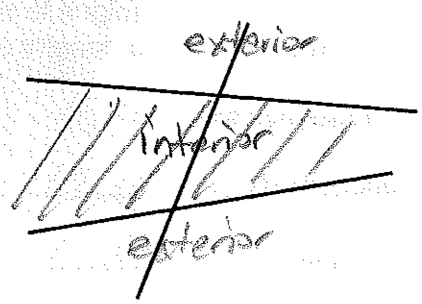


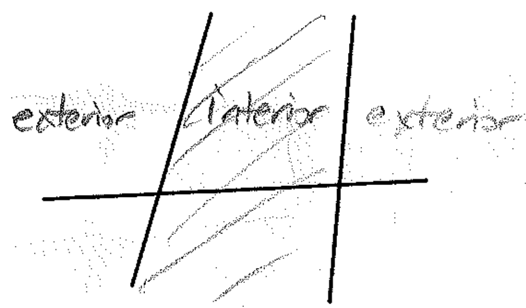
Which one is the transversal?



$\overline{CD}$ is transversal

Transversal Regions:

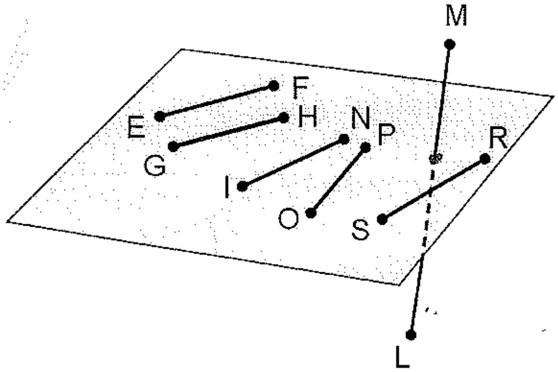




Interior = region between the 2 non-transversal lines.

Parallel Lines = coplanar lines that do not intersect

symbol for parallel



$\overline{EF} \parallel \overline{GH}$

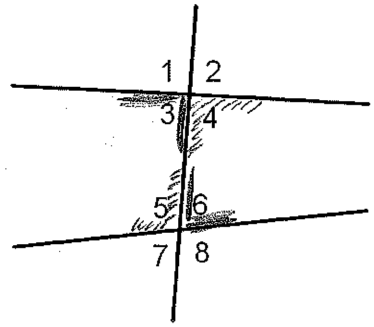
$\overline{IN}$ not parallel $\overline{OP}$ (intersects)

$\overline{SR}$ not parallel $\overline{LM}$ (not coplanar)

Angle pairs formed by transversal....

Alternate Interior Angles

Pair of angles on opposite sides of transversal, in interior region.

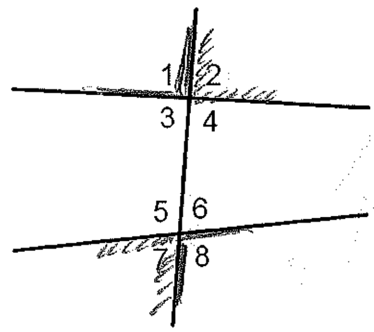


$\angle 3, \angle 6$ are alternate interior angles.

$\angle 4, \angle 5$ are alternate interior angles.

Alternate Exterior Angles

Pair of angles on opposite sides of transversal, in exterior region.

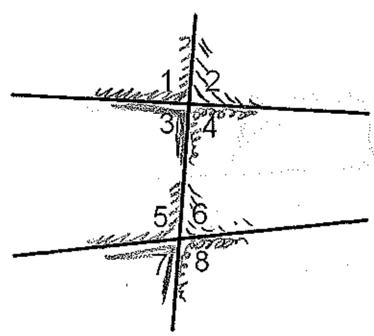


$\angle 1, \angle 8$ are alternate exterior angles.

$\angle 2, \angle 7$ are alternate exterior angles.

Corresponding Angles

Pair of angles on same side of transversal, one interior, one exterior, with different vertices.



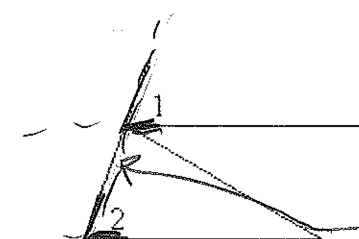
$\angle 1, \angle 5$ are corresponding angles.

$\angle 2, \angle 6$ are corresponding angles.

$\angle 3, \angle 7$ are corresponding angles.

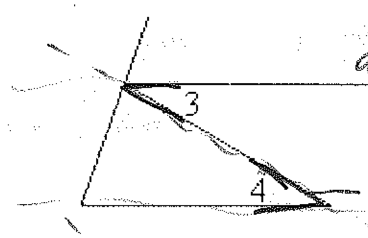
$\angle 4, \angle 8$ are corresponding angles.

Are these angles alternate interior angles, alternate exterior angles or corresponding angles?



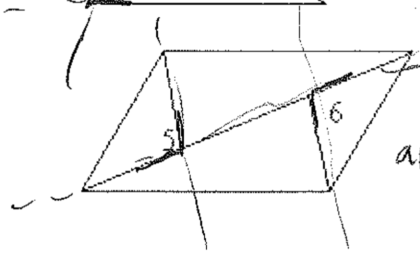
corresponding

transversal



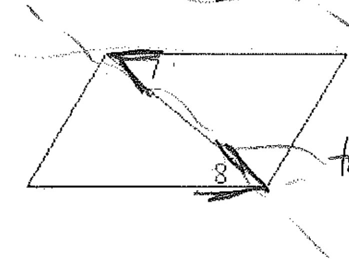
alternate interior

transversal



transversal

alternate exterior



transversal

alternate interior