Geometry, 7.1 Notes –Triangle Application Theorems

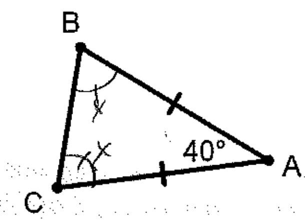
Theorems from group activity:

1) Interior angles of a triangle add to 180°.

2) Line joining midpoints of 2 sides (midline) is parallel to 3rd side and half as long as 3rd side.

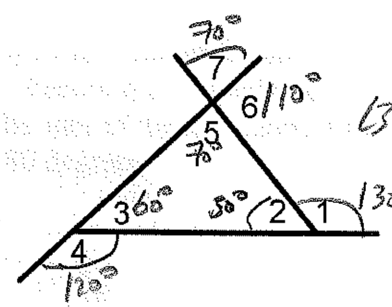
3) Exterior angle of a triangle = sum of remote interior angles.

#1. Given: diagram as marked
Find: $m\angle B$ = $\boxed{70°}$



$2x+40=180$
$2x=140$
$x=70$

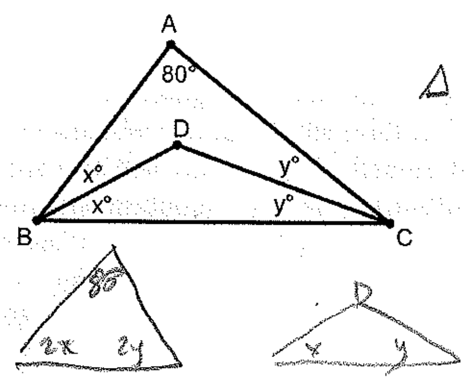
#2. Given: $m\angle 1 = 130°$
$m\angle 7 = 70°$
Find remaining angles



$\angle 3 = 180° - (70° + 50°)$
$180° - 120°$
$60°$

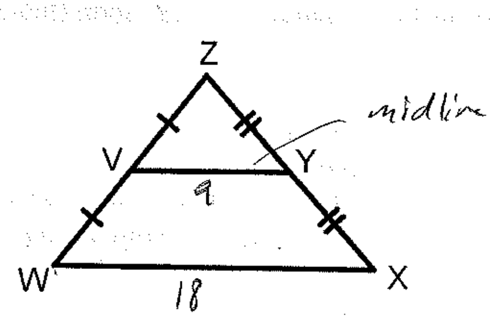
#3. Find $m\angle D$

$\triangle ABC$: $2x+2y+80=180$
$\frac{2x+2y=100}{2}$
$x+y=50$
→ Substitute



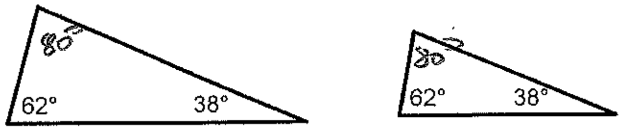
$\triangle DBC$: $D+\underbrace{x+y}_{50}=180$
$D+50=180$
$\boxed{D=130}$

#4. If WX=18, find VY = $\boxed{9}$



Geometry, 7.2 Notes –'No Choice' Theorem and AAS Triangle Congruency Shortcut

Find the missing angles in these two triangles:

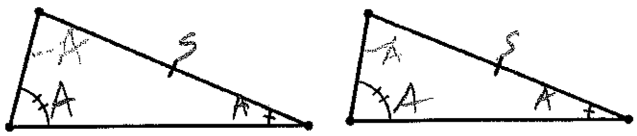


62
38
100

What can you conclude?

The 'no choice' theorem: If 2 angles of one triangle are congruent to 2 angles of another triangle, then the 3rd angles are congruent.

Now we can add a 5th triangle congruency shortcut: AAS

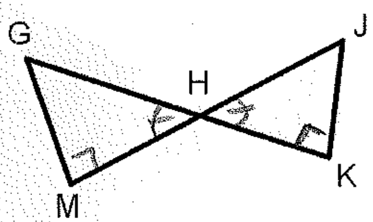


(because of 'no choice' theorem, AAS is really ASA)

#1. Given: $\overline{JM} \perp \overline{GM}$

$\overline{GK} \perp \overline{KJ}$

Prove: $\angle G \cong \angle J$



1. $\overline{JM} \perp \overline{GM}$	1. Given
2. $\overline{GK} \perp \overline{KJ}$	2. Given
3. $\angle M \cong \angle K$	3. all rt $\angle$s $\cong$
4. $\angle GHM \cong \angle JHK$	4. vert. $\angle$s $\cong$
5. $\angle G \cong \angle J$	5. no choice theorem

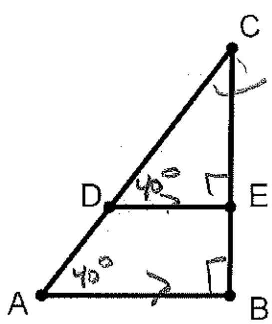
#2. Given: $\overline{CB} \perp \overline{AB}$

$\overline{DE} \parallel \overline{AB}$

$m\angle CDE = 40°$

Find: $m\angle A$, $m\angle C$, $m\angle CED$

$\boxed{40°}$ 50° 90°



180 − 130 = 50°

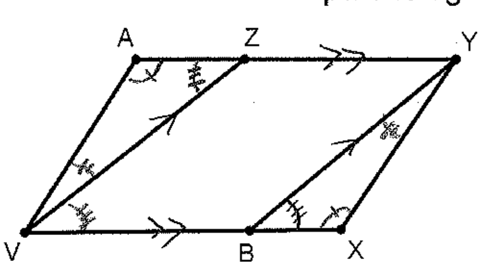
#3. Given: $\angle A \cong \angle X$

$\angle AVZ \cong \angle XYB$

$\angle ZVB \cong \angle YBX$

Prove:

VBYZ is a parallelogram



S	R
1. $\angle A \cong \angle X$	1. Given
2. $\angle AVZ \cong \angle XYB$	2. Given
3. $\angle ZVB \cong \angle YBX$	3. Given
4. $\overline{VZ} \parallel \overline{BY}$	4. corr. $\angle$s $\cong$ ⟹ ∥ lines
5. $\angle VZA \cong \angle YBX$	5. No choice theorem
6. $\angle VZA \cong \angle ZVB$	6. substitution
7. $\overline{ZY} \parallel \overline{VB}$	7. alt int $\angle$s $\cong$ ⟹ ∥ lines
8. VBYZ is ▱	8. both pairs opp sides ∥

Geometry, 7.3 Notes –Polygon Formulas

Names of polygons:

number of sides (n)	name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
15	pentadecagon
n	n-gon

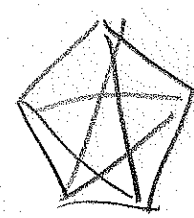
Sum of interior angles of a polygon: Sum of interior angles = $S_i = (n-2)180$

Examples:
- triangle (n=3) $S_i = (3-2)180 = 180°$
- quadrilateral (n=4) $S_i = (4-2)180 = 360°$
- pentagon (n=5) $S_i = (5-2)180 = 540°$
- 27-gon (n=27) $S_i = (27-2)180 = 4500°$

Sum of exterior angles of a polygon: Sum of exterior angles = $S_e = 360°$

Examples:
- triangle (n=3) $S_e = 360°$
- pentagon (n=5) $S_e = 360°$
- 27-gon (n=27) $S_e = 360°$

Number of diagonals of a polygon: number of diagonals = $d = \dfrac{n(n-3)}{2}$



Examples:
- triangle (n=3) $d = \dfrac{3(3-3)}{2} = 0$
- pentagon (n=5) $d = \dfrac{5(5-3)}{2} = \dfrac{5(2)}{2} = 5$
- 27-gon (n=27) $d = \dfrac{27(27-3)}{2} = \dfrac{27(24)}{2} = 324$

#1. Find the polygon whose sum of interior angles is 900°

$Si = (n-2)180$

$900 = (n-2)180$

$900 = 180n - 360$

$+360 \quad +360$

$\dfrac{1260}{180} = \dfrac{180n}{180}$

$\boxed{7 = n}$

#2. What polygon has 35 diagonals?

$d = \dfrac{n(n-3)}{2}$

$(2)35 = \dfrac{n(n-3)}{2}(2)$

$70 = n(n-3)$

$n^2 - 3n = 70$

$n^2 - 3n - 70 = 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\dfrac{3 \pm \sqrt{9+4(70)}}{2} = \dfrac{3 \pm 17}{2}$

$\dfrac{20}{2}$ or $\dfrac{-14}{2}$

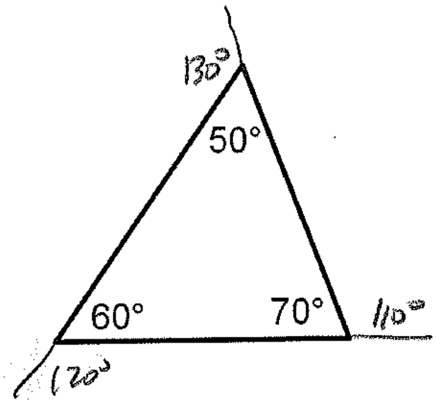
$\boxed{10}$ or -7

#2. What is the sum of interior angles and sum of exterior angles for an 18-sided polygon?

$\boxed{S_e = 360°}$

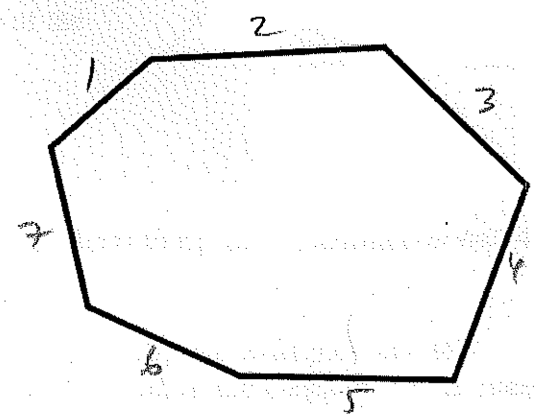
$S_i = (n-2)180$
$= (18-2)180$
$= 16(180)$
$\boxed{S_i = 2880°}$

#3. Find one exterior angle for each vertex of the polygon and find the sum of these exterior angles.



$130° + 110° + 120° = 360°$

#4. How many diagonals does this polygon have?



$d = \frac{n(n-3)}{2}$

$n = 7$

$d = \frac{7(7-3)}{2}$

$d = \frac{7(4)}{2} = \boxed{14}$

#5. What polygon has 35 diagonals?

$d = \frac{n(n-3)}{2}$

$35 = \frac{n(n-3)}{2}$

$70 = n(n-3)$

$70 = n^2 - 3n$

$0 = n^2 - 3n - 70$

$a = 1 \quad b = -3 \quad c = -70$

quadratic equation:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$n = \frac{3 \pm \sqrt{9 + 4(1)(70)}}{2(1)}$

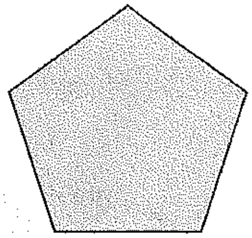
$= \frac{3 \pm 17}{2}$

$= \frac{20}{2}$ or $\frac{-14}{2}$

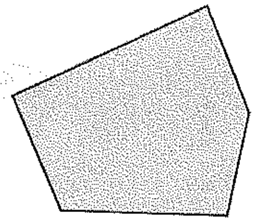
$= \boxed{10}$ or -7

Geometry, 7.4 Notes –Regular Polygons

'Regular' = equilateral and equiangular



regular pentagon



not regular pentagon,
irregular pentagon

External angle of a polygon:

external angle and internal angle are supplementary

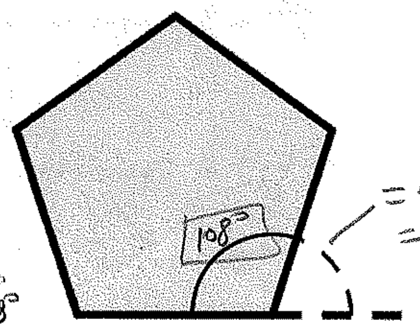
$n=5$

$S_i=(n-2)180$

$=(5-2)180$

$=540$

each int. $\angle = \frac{540}{5} = 108°$



$=180-108°$

$=\boxed{72°}$

External angles of a regular polygon:

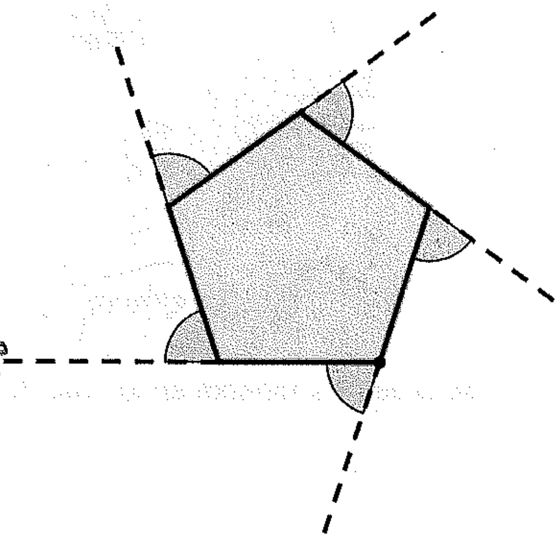
$S_E=360°$

so E is 360 divided by number of sides

$$E=\frac{360}{n}$$

$n=5$

$E=\frac{360}{5}=72°$



Examples:

#1. Find the measure of an exterior angle of a regular hexagon:

$n=6$ $\quad E=\frac{360}{n}$

$E=\frac{360}{6}$ $\quad 6\overline{)360}$ = 60

$E=\boxed{60^\circ}$

#2. Find the measure of each angle of an equiangular nonagon: $n=9$

if it doesn't say 'exterior' then it means 'interior'

Find exterior angle: $E=\frac{360}{9}=40$ $\quad 9\overline{)360}$ = 40

... then take supplement to find interior angle:

$180-40=\boxed{140^\circ}$

#3. If each angle of a polygon is 108° how many sides does the polygon have?

(interior)

interior $=108^\circ$, so exterior $= 180 - 108 = 72$

72°

$E=\frac{360}{n}$ $\quad n=\frac{360}{72}$ $\quad 72\overline{)360}$ = 5

$72=\frac{360}{n}$ $\quad \boxed{n=5}$

$72n=360$ $\quad$ (pentagon)

#4. Find the number of sides of an equiangular polygon if each of its exterior angles is 36°:

$E=\frac{360}{n}$

$\frac{36}{1}=\frac{360}{n}$

$36n=360$

$n=\frac{360}{36}$

$\boxed{n=10}$ (decagon)