Geometry, 9.1: Algebra Review – Radicals and Quadratic Equations

Some common perfect squares:

$1^2 = 1$	$11^2 = 121$
$2^2 = 4$	$12^2 = 144$
$3^2 = 9$	$13^2 = 169$
$4^2 = 16$	$14^2 = 196$
$5^2 = 25$	$15^2 = 225$
$6^2 = 36$	$20^2 = 400$
$7^2 = 49$	$25^2 = 625$
$8^2 = 64$	$30^2 = 900$
$9^2 = 81$	$40^2 = 1600$
$10^2 = 100$	$50^2 = 2500$

Simplifying Radicals:

A radical is simplified if:
1. Smallest number possible under the radical
2. no fractions under the radical
3.. no radicals in denominator of fraction

Examples:

$\sqrt{48}$ (factor out perfect squares)

$\sqrt{48}$
$\sqrt{16} \cdot \sqrt{3}$
$4 \cdot \sqrt{3} = \boxed{4\sqrt{3}}$

$\frac{1}{\sqrt{3}}$ (rationalize denominator) $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$ $\left(\sqrt{3} \cdot \sqrt{3} = \sqrt{3 \cdot 3} = \sqrt{3^2} = 3\right)$

$\sqrt{\frac{1}{3}}$ (break into separate radicals) $\frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

Practice: Simplify the following...

$\sqrt{20}$
$\sqrt{4}\sqrt{5}$
$\boxed{2\sqrt{5}}$

$\frac{1}{6}\sqrt{48}$
$\frac{1}{6}\sqrt{16} \cdot \sqrt{3}$
$\frac{1}{6} \cdot 4 \cdot \sqrt{3}$
$\frac{4}{6}\sqrt{3} = \frac{2}{3}\sqrt{3} = \boxed{\frac{2\sqrt{3}}{3}}$

$\sqrt{5^2 + 12^2}$
$\sqrt{25+144}$
$\sqrt{169}$
$\boxed{13}$

$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$
$= \boxed{2\sqrt{3}}$

$\sqrt{72} + \sqrt{75} - \sqrt{48}$
$\sqrt{9}\sqrt{8} + \sqrt{25}\sqrt{3} - \sqrt{16}\sqrt{3}$
$3\sqrt{8} + 5\sqrt{3} - 4\sqrt{3}$
$3\sqrt{4}\sqrt{2} + \sqrt{3}$
$3 \cdot 2\sqrt{2} + \sqrt{3}$
$\boxed{6\sqrt{2} + \sqrt{3}}$

$\sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{21}}{3}}$

Solving Quadratic Equations:

$x^2 = 18$

$\sqrt{x^2} = \pm\sqrt{18}$

$x = \pm\sqrt{18}$

$x = \pm\sqrt{9}\sqrt{2}$

$\boxed{x = \pm 3\sqrt{2}}$

$x^2 = 12$

$\sqrt{x^2} = \pm\sqrt{12}$

$x = \pm\sqrt{12}$

$x = \pm\sqrt{4}\sqrt{3}$

$\boxed{x = \pm 2\sqrt{3}}$

$x^2 = \left(5\sqrt{3}\right)^2 + \left(\sqrt{5}\right)^2$

$x^2 = 25 \cdot 3 + 5$

$x^2 = 75 + 5$

$x^2 = 80$

$x = \pm\sqrt{80}$

$x = \pm\sqrt{16}\cdot\sqrt{5}$

$\boxed{x = \pm 4\sqrt{5}}$

$\left(\sqrt{5}\right)^2 + \left(\sqrt{11}\right)^2 = x^2$

$5 + 11 = x^2$

$16 = x^2$

$x = \pm\sqrt{16}$

$\boxed{x = \pm 4}$

$x^2 - 36x = 9x$

$x^2 - 45x = 0$

$x(x - 45) = 0$

$x = 0 \qquad x - 45 = 0$

$\boxed{x = 0} \qquad \boxed{x = 45}$

$-x^2 + 5x + 36 = 0$

$x^2 - 5x - 36 = 0$

$(x-9)(x+4) = 0$

$x - 9 = 0 \qquad x + 4 = 0$

$\boxed{x = 9} \qquad \boxed{x = -4}$

*	+
-36	-5
$(-9)(4)$	$-9 + 4$

$x^2 - 2x = 11x$

$x^2 - 13x = 0$

$x(x - 13) = 0$

$x = 0 \qquad x - 13 = 0$

$\boxed{x = 0} \qquad \boxed{x = 13}$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x + 6 = 0 \qquad x - 2 = 0$

$\boxed{x = -6} \qquad \boxed{x = 2}$

*	+
-12	4
$(6)(-2)$	$6 - 2$

Geometry, 9.2: Intro to Circles

Activity – measure a circle's diameter (distance across middle) and circumference (distance around, same as perimeter). Regardless of the size of the circle, the ratio of the circumference to the diameter is:

$$\frac{circumference}{diameter} = 3.14\ldots = \pi \quad (pi) \quad (\text{use } 3.1)$$

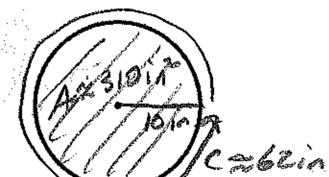
Formulas for circumference and area of a circle: $D \cdot \frac{C}{D} = \pi \cdot D$

$C = \pi D = \pi(2r) = 2\pi r$

circumference, C = $2\pi r$ or πD

area, A = πr^2

Ex: Find the circumference and area of a circle whose radius is 10 in:

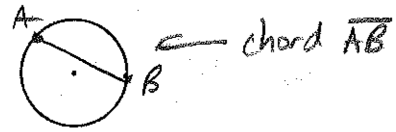


$C = 2\pi r$	$A = \pi r^2$
$C = 2\pi(10in)$	$A = \pi(10in)^2$
$C = 20\pi\, in$	$A = 100\pi\ in^2$
$C \approx 20(3.1)\ in$	$A \approx 100(3.1)\ in^2$
$C \approx 62 in$	$A \approx 310\ in^2$

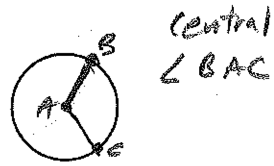
$$\begin{array}{r} 20 \\ \times\, 3.1 \\ \hline 20 \\ 60 \\ \hline 62.0 \end{array}$$

Chords, central angles, and inscribed angles:

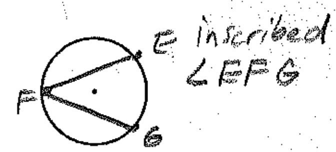
chord = a line segment joining 2 points on a circle:



central angle = an angle formed by 2 radii, vertex is the center:



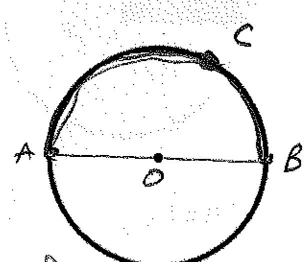
inscribed angle = an angle formed by 2 chords with a common endpoint:



Arcs:

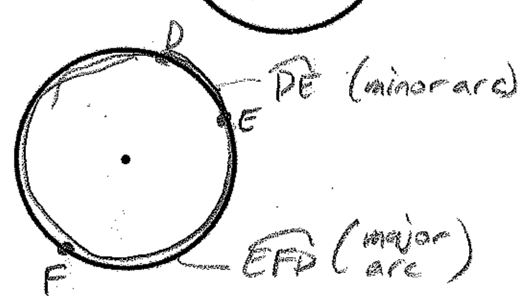
arc = a portion of a circle, consists of 2 endpoints and all the points on the circle between these endpoints:

semicircle: an arc whose endpoints are the endpoints of a diameter. (Named using 3 points) symbol for arc → $\overset{\frown}{ACB}$



minor arc: an arc that is less than a semicircle: (Named using only the 2 endpoints) $\overset{\frown}{DE}$

major arc: an arc that is more than a semicircle: (Named using 3 points) $\overset{\frown}{EFD}$



Finding measure and length of arcs:

Measure of an arc, $m\widehat{AB}$ = the number of degrees it occupies. (degrees)

Length of an arc, length of $\widehat{AB}$ = the length along the circle, a distance (inches, feet, cm, etc).

An arc is always some fraction of the entire circumference of the circle.

Ex: Find:

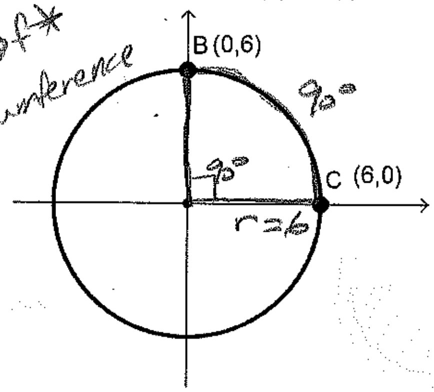
(a) $m\widehat{BC} = \boxed{90°}$

(b) length, l of $\widehat{BC} = \frac{90}{360} C$

$= \frac{1}{4}(2\pi r)$

$= \frac{1}{4}(2\pi 6) = \frac{12\pi}{4} = \boxed{3\pi}$

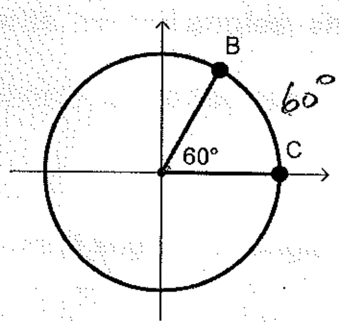
* fraction of * whole circumference



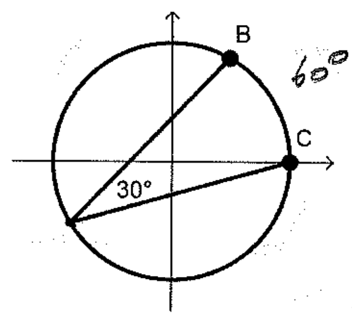
How arcs and angles are related:

An angle can 'intercept' an arc:

If the angle is a central angle, the measure of the arc = the angle:



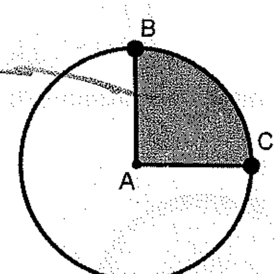
If the angle is an inscribed angle, the measure of the arc = double the angle:



Sectors:

A sector is a region (an area) bounded by 2 radii and an arc of a circle:
(Sectors are named like the central angle that forms them)

Sector BAC



A sector is always some fraction of the entire area of the circle.

Ex: Find the area of sector BAC; if AB = 6 ft.

area of sector BAC $= \frac{120}{360} A$

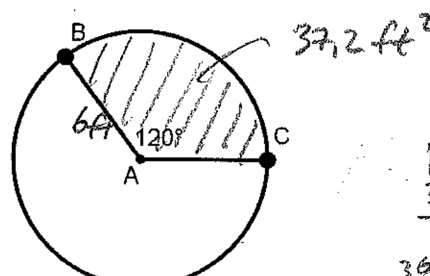
$= \frac{1}{3}(\pi r^2)$

$= \frac{1}{3}(\pi (6\ ft)^2)$

$= \frac{1}{3}\pi\, 36\, ft^2$

$= 12\pi\ ft^2$

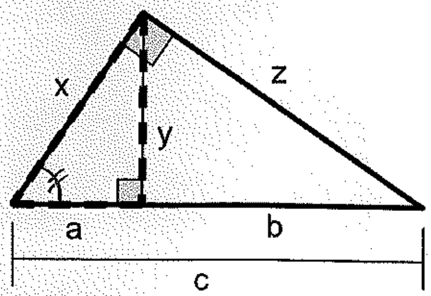
$\approx 12(3.1)\ ft^2 \approx 37.2\ ft^2$

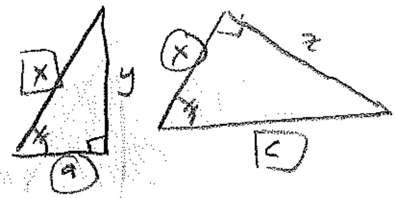


12
× 3.1
12
36
37.2

* fraction of * whole circle area

Geometry, 9.3: Altitude-on-Hypotenuse Theorems



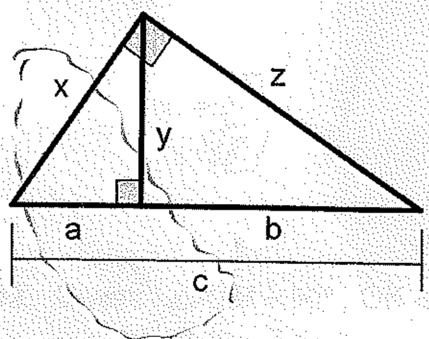


$\frac{a}{x}=\frac{x}{c}$

$x^2=ac$

$x=\sqrt{ac}$

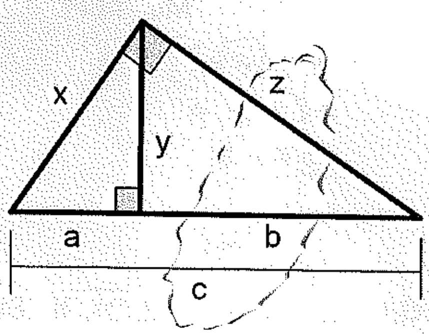
Altitude-on-Hypotenuse Theorems:



$x^2=ac$

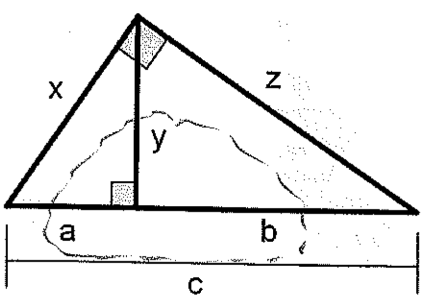
or

$x=\sqrt{ac}$



$z^2=bc$

$z=\sqrt{bc}$

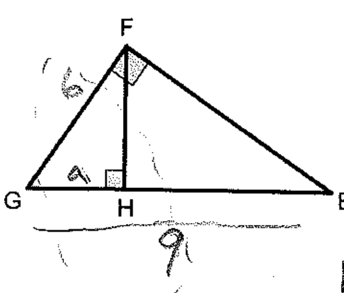


$y^2=ab$

$y=\sqrt{ab}$

Examples:

If GF=6 and EG=9, find HG.



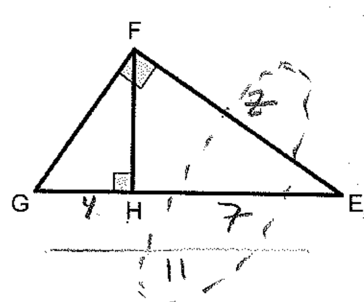
$6^2=9a$

$\frac{36}{9}=\frac{9a}{9}$

$4=a$

$\boxed{HG=4}$

If EH=7 and HG=4, find EF.

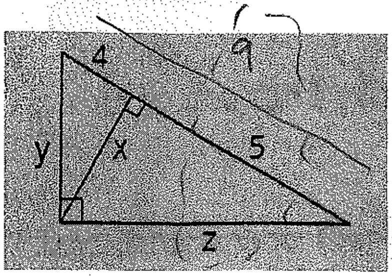


$z^2=7\cdot 11$

$z^2=77$

$\boxed{z=\sqrt{77}}$

ind z+8:



$z^2 = 5 \cdot 9$

$z^2 = 45$

$z = \sqrt{45}$

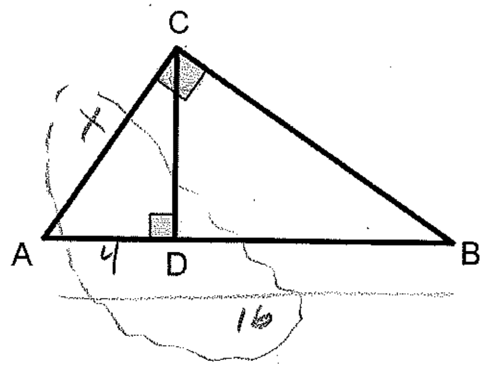
$z = \sqrt{9}\sqrt{5}$

$z = 3\sqrt{5}$

$z+8 = \boxed{3\sqrt{5}+8}$

Practice:

#1. If $\overline{AC} \perp \overline{CB}$, $\overline{CD} \perp \overline{AB}$, AD = 4, and AB = 16, find AC:



$x^2 = 4 \cdot 16$

$x^2 = 64$

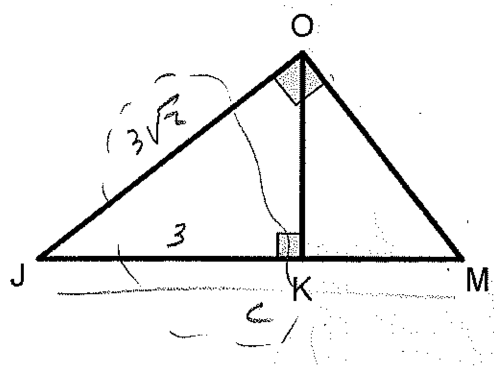
$x = \sqrt{64}$

$x = \boxed{8}$

#2. Given: $\angle JOM = 90°$, OK is an altitude,

$JO = 3\sqrt{2}$, $JK = 3$

Find: JM



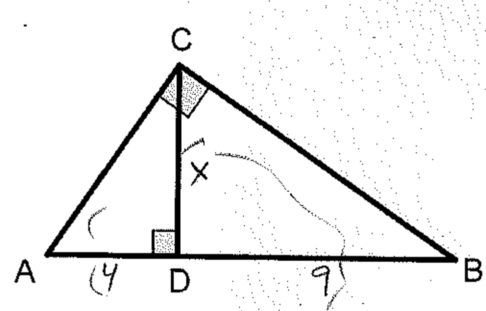
$(3\sqrt{2})^2 = 3c$

$9 \cdot 2 = 3c$

$\frac{18}{3} = \frac{3c}{3}$

$\boxed{6 = c}$

#3. If $\overline{AC} \perp \overline{CB}$, $\overline{CD} \perp \overline{AB}$, AD = 4, and BD = 9, find CD:

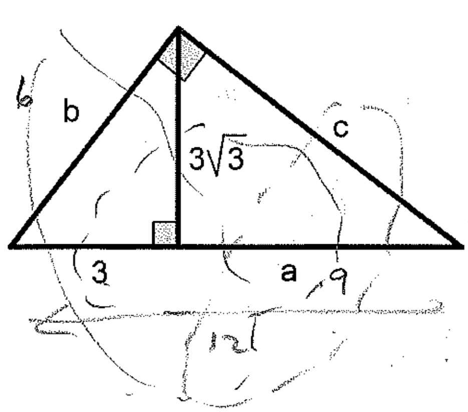


$x^2 = 9 \cdot 4$

$x^2 = 36$

$\boxed{x = 6}$

#4. Find a+b+c:



$(3\sqrt{3})^2 = 3a$

$\frac{9 \cdot 3}{3} = \frac{3a}{3}$

$9 = a$

$b^2 = 12 \cdot 3$

$b^2 = 36$

$b = 6$

$c^2 = 9 \cdot 12$

$c^2 = 108$

$c = \sqrt{108}$

$c = \sqrt{36} \cdot \sqrt{3}$

$c = 6\sqrt{3}$

$\begin{array}{r} 12 \\ \times 9 \\ \hline 108 \end{array}$

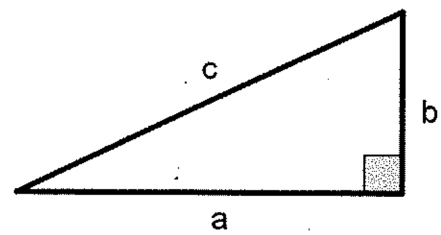
$a+b+c = 9+6+6\sqrt{3}$

$= \boxed{15+6\sqrt{3}}$

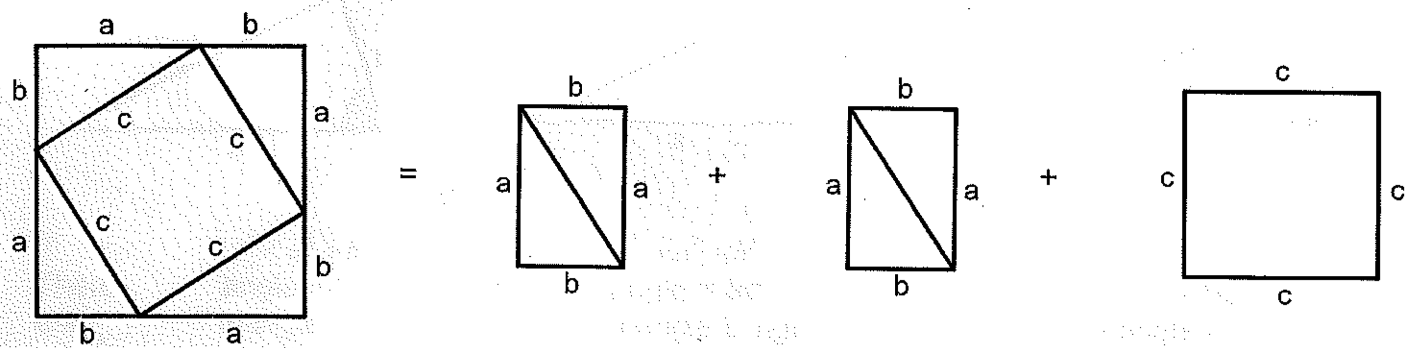
Geometry, 9.4: The Pythagorean Theorem

The most used concept from Geometry...the Pythagorean Theorem is true for <u>right triangles</u>:

$$a^2 + b^2 = c^2$$



There are over 300 proofs of the Pythagorean Theorem. Below is a visual proof based on areas:

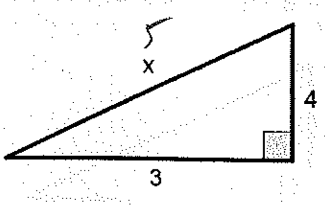


area of $(a+b)(a+b)$ = *area of* ab + *area of* ab + *area of* c^2

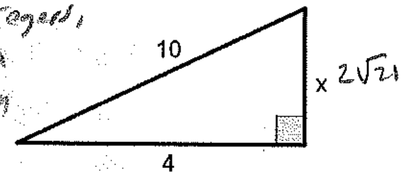
$$(a+b)(a+b) = ab + ab + c^2$$
$$a^2 + ab + ab + b^2 = 2ab + c^2$$
$$a^2 + 2ab + b^2 = 2ab + c^2$$
$$a^2 + \cancel{2ab} + b^2 = \cancel{2ab} + c^2$$
$$a^2 + b^2 = c^2$$

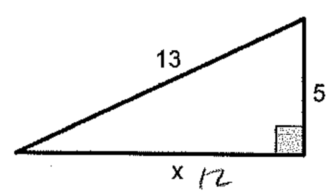
Finding the missing side of a right triangle:

Given any two sides of a right triangle, you can find the missing side. Examples:



when all sides are integers, called a 'Pythagorean Triple'





$3^2 + 4^2 = x^2$

$9 + 16 = x^2$

$25 = x^2$

$x = \sqrt{25}$

$\boxed{x = 5}$

$4^2 + x^2 = 10^2$

$16 + x^2 = 100$

$-16 \quad\quad -16$

$x^2 = 84$

$x = \sqrt{84}$

$x = \sqrt{4}\sqrt{21}$

$\boxed{x = 2\sqrt{21}}$

$x^2 + 5^2 = 13^2$

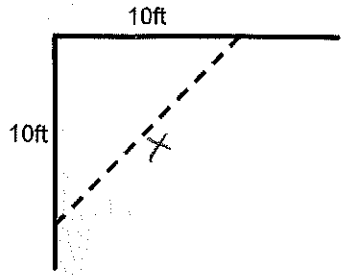
$x^2 + 25 = 169$

$x^2 = 144$

$x = \sqrt{144}$

$\boxed{x = 12}$

In the back corner of your yard, you want to section off a triangular planter area using a line of bricks. If each brick is 1ft long, how many bricks are needed if the brick wall is 3 bricks high?



$x^2 = 10^2 + 10^2$

$x^2 = 100 + 100$

$x^2 = 200$

$x = \sqrt{200}$

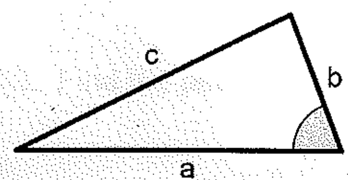
$(x \approx 14.14)$

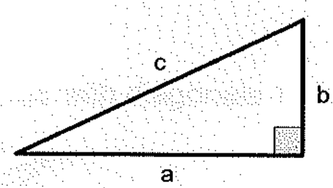
just over 14 bricks needed per layer

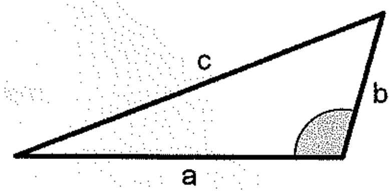
x 3 layers

$3 \times 14 =$ $\boxed{\text{42 bricks needed}}$

Related theorem: You can use the Pythagorean theorem to test whether an angle is a right angle:







If:	$c^2 < a^2 + b^2$	$c^2 = a^2 + b^2$	$c^2 > a^2 + b^2$
Then:	angle < 90°	angle = 90°	angle > 90°
and	triangle is acute	triangle is right	triangle is obtuse

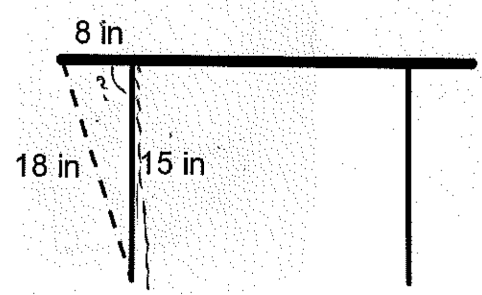
Example: You built a table out of pieces of wood, and measure the lengths shown to check if your table top is exactly perpendicular to your table legs. Do your table legs and table top meet at a right angle?

$18^2 \overset{?}{=} 8^2 + 15^2$

$324 \overset{?}{=} 64 + 225$

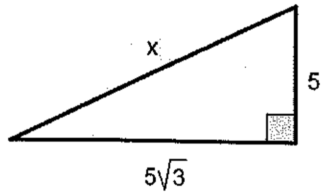
$324 \overset{?}{=} 289$

no angle is >90°



Practice:

#1. Find the missing side:



$x^2 = 5^2 + (5\sqrt{3})^2$

$x^2 = 25 + 25 \cdot 3$

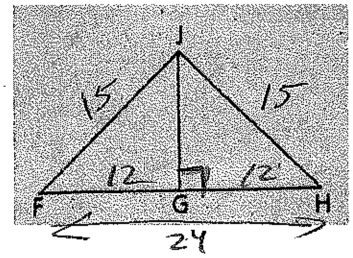
$x^2 = 25 + 75$

$x^2 = 100$

$x = 10$

#2. Given: $\overline{JG}$ is the altitude to base $\overline{FH}$ of isosceles triangle JFH.

$FJ = 15, \quad FH = 24$



Find: JG

by Angle-Bisector theorem bottom divided in half

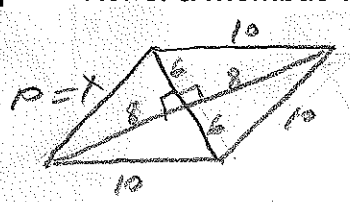
$(JG)^2 + 12^2 = 15^2$

$(JG)^2 + 144 = 225$

$(JG)^2 = 81$

$JG = \boxed{9}$

#3. Find the perimeter of a rhombus with diagonals 12 km and 16 km.



$x^2 = 6^2 + 8^2$

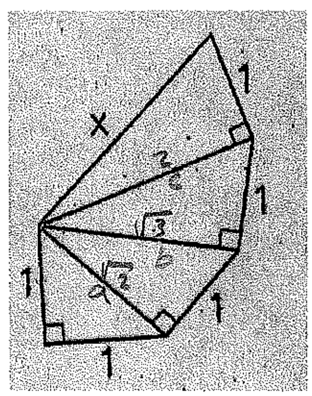
$x^2 = 36 + 64$

$x^2 = 100$

$x = 10$

$P = \boxed{40 \text{ km}}$

#4. Solve for x in the partial spiral to the right:



$a^2 = 1^2 + 1^2$

$a^2 = 2$

$a = \sqrt{2}$

$(\sqrt{2})^2 + 1^2 = b^2$

$2 + 1 = b^2$

$b = \sqrt{3}$

$(\sqrt{3})^2 + 1^2 = c^2$

$3 + 1 = c$

$c = \sqrt{4} = 2$

$2^2 + 1^2 = x^2$

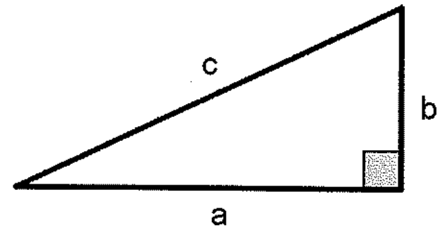
$4 + 1 = x^2$

$\boxed{x = \sqrt{5}}$

Geometry, 9.5: The Distance Formula

We just learned the Pythagorean theorem:

$$c^2 = a^2 + b^2$$



Could we use the Pythagorean theorem to find the length of c if we have just the coordinates of the endpoints of c?

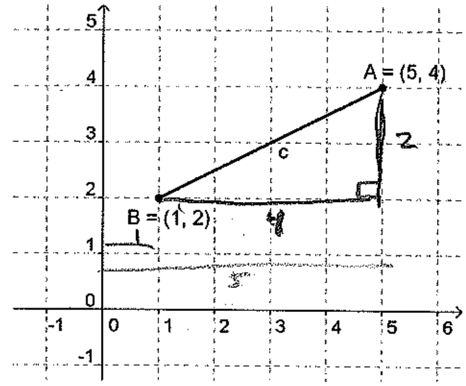
$c^2 = (5-1)^2 + (4-2)^2$

$c = \sqrt{(5-1)^2 + (4-2)^2}$

$c = \sqrt{4^2 + 2^2}$

$c = \sqrt{16+4}$

$c = \sqrt{20} = 2\sqrt{5}$



What if the points were across the x or y axis?

$c^2 = (2-(-1))^2 + (-1-3)^2$

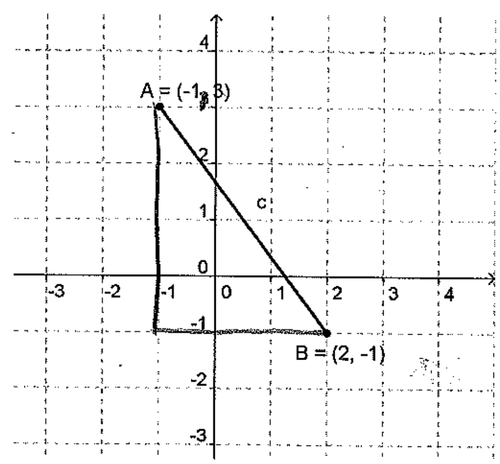
$c = \sqrt{(2-(-1))^2 + (-1-3)^2}$

$c = \sqrt{(2+1)^2 + (-4)^2}$

$c = \sqrt{9+16}$

$c = \sqrt{25}$

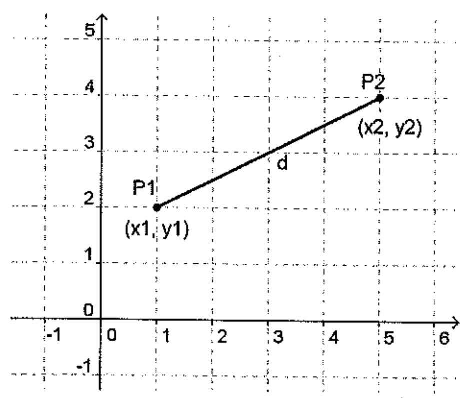
$c = 5$



The Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$



Example: Find the distance between $\overset{x_1\ y_1}{(-2,3)}$ and $\overset{x_2\ y_2}{(4,11)}$

$d = \sqrt{(4--2)^2 + (11-3)^2}$

$d = \sqrt{6^2 + 8^2}$

$d = \sqrt{36+64}$

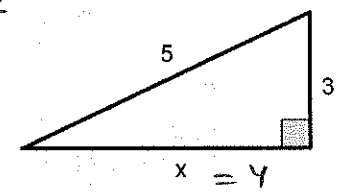
$d = \sqrt{100} = 10$

Geometry, 9.6: Families of Right Triangles

Sometimes, a right triangle has sides that are all whole numbers (integers):

If the sides of a right triangle are integers, they form a **Pythagorean Triple:**

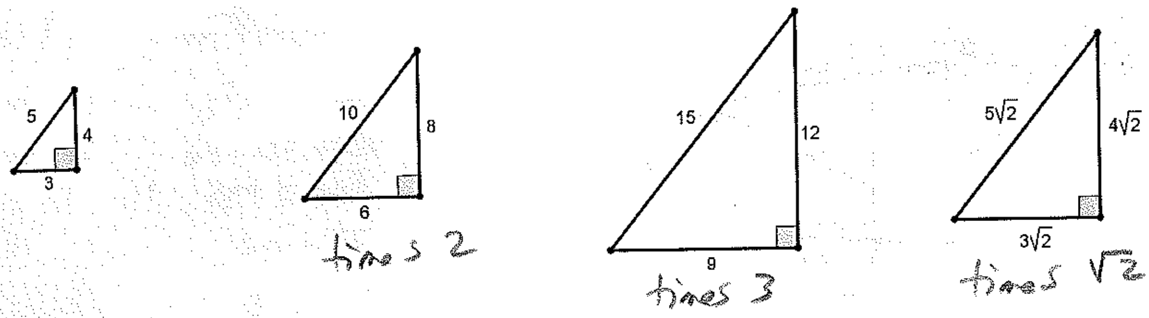
$x^2+3^2=5^2$
$x^2+9=25$
$x^2=16$
$x=4$



Common Pythagorean Triples:

(3, 4, 5) (5, 12, 13) (7, 24, 25) (8, 15, 17)

Families of Triangles:
Triangles that are similar to each other are said to be in the same 'family':



Procedure for using Pythagorean triples to find a missing side:

1) Find a number that divides evenly into the 2 sides given.
2) Divide both sides by that number and write new side lengths in parentheses.
3) Match new sides to a Pythagorean triple and fill in missing side.
4) Make a proportion with two of the sides and an 'x' for the side you need.
5) Solve the proportion for 'x'.

Examples:

Find the missing side:

try it

$x=36$

$\frac{x}{4}=\frac{27}{3}$

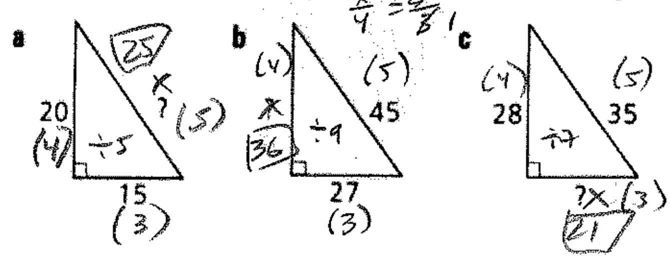
$\frac{x}{3}=\frac{28}{4}=\frac{7}{1}$

$x=21$

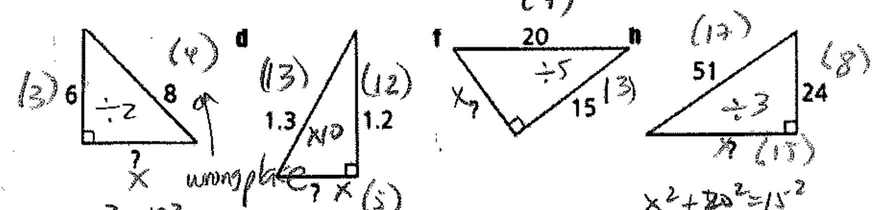
$\frac{20}{4}=\frac{x}{5}$

$4x=100$

$x=25$



$\frac{x}{15}=\frac{243}{81}$ $\boxed{x=45}$

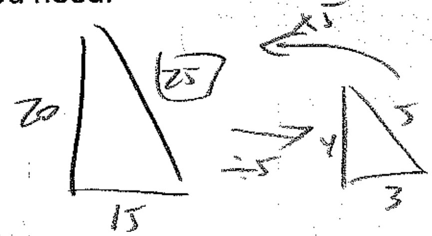


$x^2+6^2=8^2$
$x^2+36=64$
$x^2=28$

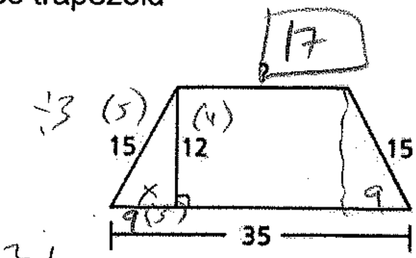
$x=\sqrt{28}$
$\boxed{x=2\sqrt{7}}$

$\frac{x}{5}=\frac{12}{120}=\frac{1}{10}$
$10x=5$
$\frac{10x}{10}=\frac{5}{10}$
$x=\frac{5}{10}=\boxed{.5}$

$x^2+20^2=15^2$
$x^2+225=400$
$x^2=175$
$x=\sqrt{175}=\boxed{5\sqrt{7}}$



Example: Find the length of the upper base of the isosceles trapezoid



$35 - 18 = 17$

Example: Find QD

Compare Pythagorean Theorem!

$9^2+12^2=y^2$

$81+144=y^2$

$225=y^2$

$y=15$

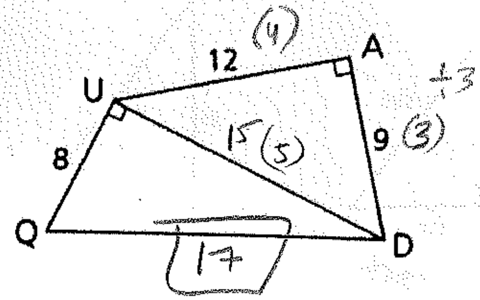
$8^2+15^2=QD^2$

$64+225=QD^2$

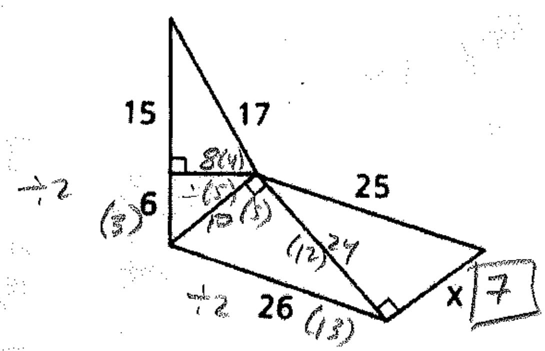
$289=QD^2$

$QD=\sqrt{289}$

$QD=17$



Example: Find x

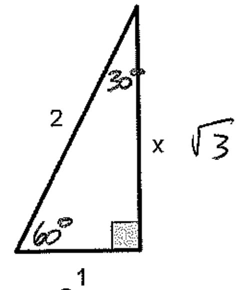


Geometry, 9.7 day 1: Special Right Triangles

Solve for x:

$a^2+b^2=c^2$
$1^2+x^2=2^2$
$1+x^2=4$
$-1 \quad\quad -1$
$x^2=3$

$x=\sqrt{3}$

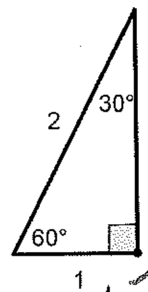


when angles are 30°-60°-90°
~~you get this ratio of sides~~
special because of angles
(Pythagorean triples are special because sides are integers)

This particular combination of sides happens when the angles of the triangle are 30°, 60°, and 90°.

Special Triangle: 30-60-90:

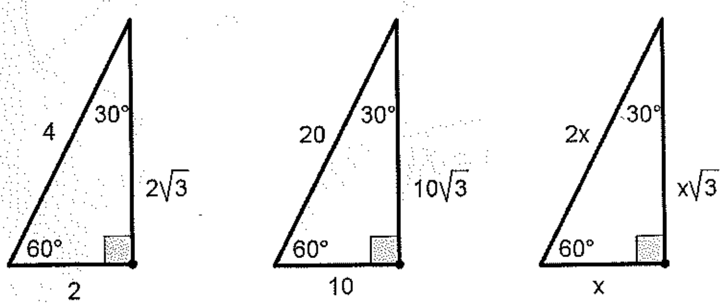
hypotenuse is twice small side



small side

3rd side is $\sqrt{3}$ times small side

Same ratio of sides occurs for all triangles in the 30-60-90 family:



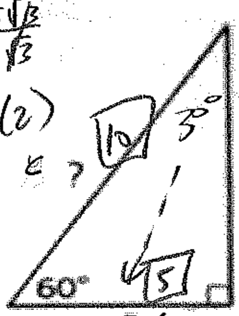
If you know 1 side of a 30-60-90 triangle, you can find the other 2 sides:

Procedure:

1) Add the 'pattern' numbers for the sides in parentheses.
2) Add a variable like x for a side to calculate.
3) Make a proportion with two of the sides.
4) Cross multiply to solve for x.

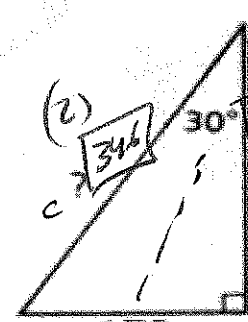
$\frac{c}{2}=\frac{5\sqrt{3}}{\sqrt{3}}$

$c=10$



$\frac{x}{1}=\frac{5\sqrt{3}}{\sqrt{3}}$

$\boxed{x=5}$



$\frac{x}{\sqrt{3}}=\frac{173}{1}$

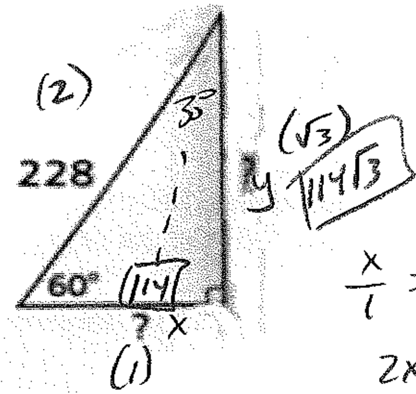
$x=173\sqrt{3}$

$\frac{c}{2}=\frac{173}{1}$

$c=2\cdot 173$

$c=346$

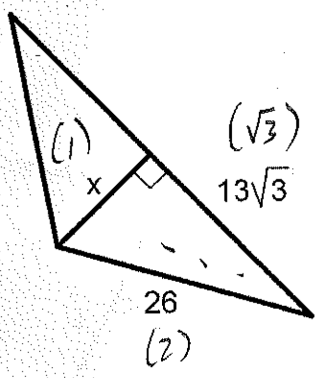
Try it...



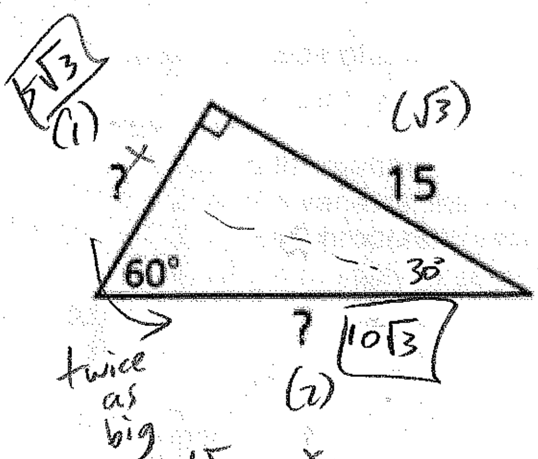
$$\frac{x}{1} = \frac{228}{2}$$
$$2x = 228$$
$$x = 114$$

$$\frac{y}{\sqrt{3}} = \frac{228}{2}$$
$$\frac{y}{\sqrt{3}} = \frac{114}{1}$$
$$y = 114\sqrt{3}$$

More examples:

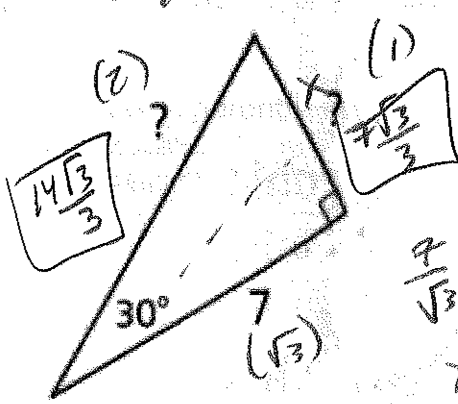


$$\frac{13\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = \frac{x}{1}$$
$$\frac{13}{1} = \frac{x}{1}$$
$$\boxed{x = 13}$$



$$\frac{15}{\sqrt{3}} = \frac{x}{1}$$
$$\frac{x\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = \frac{15}{\sqrt{3}}$$
$$x = \frac{15}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{15}^{5}\sqrt{3}}{\cancel{3}_{1}} = \frac{5\sqrt{3}}{1} = \boxed{5\sqrt{3}}$$

Try it...



$$\frac{7}{\sqrt{3}} = \frac{x}{1}$$
$$x\sqrt{3} = 7$$
$$x = \frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$$

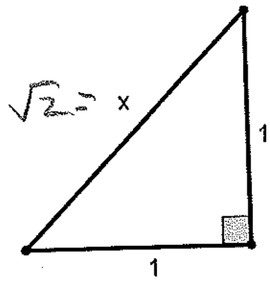
Geometry, 9.7 day 2: Special Right Triangles

Solve for x:

$1^2+1^2=x^2$

$2=x^2$

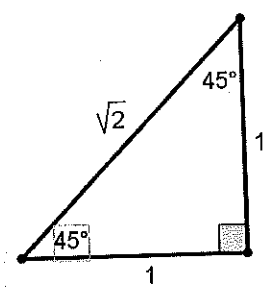
$\sqrt{2}=x$



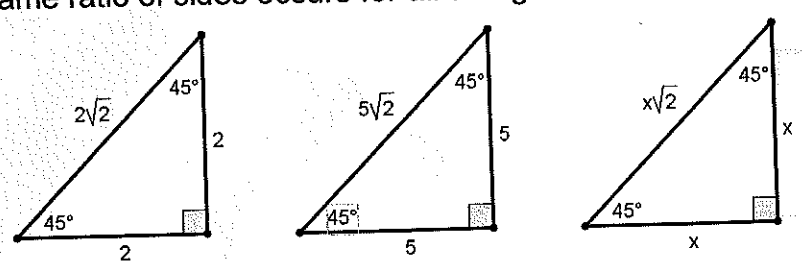
$\sqrt{2}=x$

This particular combination of sides happens when the angles of the triangle are 45°, 45°, and 90°.

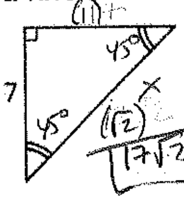
Special Triangle: 45-45-90:



Same ratio of sides occurs for all triangles in the 45-45-90 family:



If you know 1 side of a 45-45-90 triangle, you can find the other 2 sides: (Same procedure as 30°-60°-90° but pattern is different)



$\frac{17}{1}=\frac{x}{\sqrt{2}}$

$x=17\sqrt{2}$

$\boxed{17\sqrt{2}}$

$\sqrt{2}$

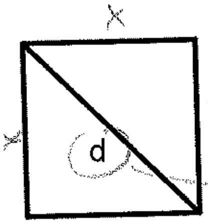
$\frac{38}{\sqrt{2}}=\frac{x}{1}$

$\frac{\sqrt{2}x}{\sqrt{2}}=\frac{38}{\sqrt{2}}$

$x=\frac{38}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{38\sqrt{2}}{2}=\boxed{19\sqrt{2}}$

More examples:

What is the length of the diagonal of a square if the perimeter is 44 inches?

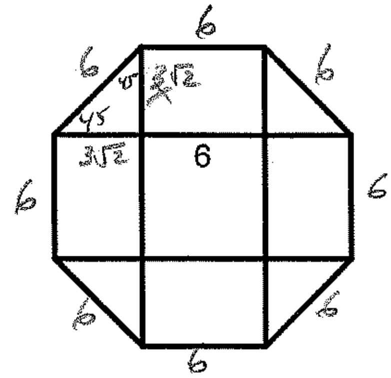


$P=4x=44$

$x=11$

$d=\boxed{11\sqrt{2}\text{ inches}}$

Any regular octagon can be divided into rectangles and triangles.
Here a side of the central square is 6 units long.
Find the perimeter of the octagon.

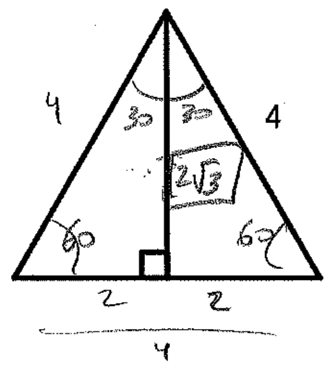


$P = 6 \cdot 8 = \boxed{48}$

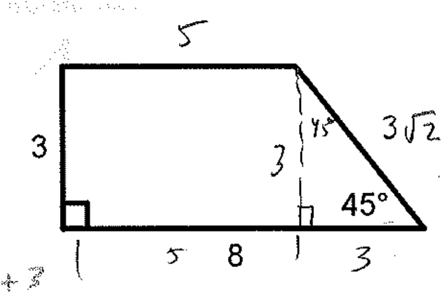
$x\sqrt{2} = 6$

$x = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

What is the length of the altitude of an equilateral triangle with sides 4 units long?



Find the perimeter of the trapezoid.



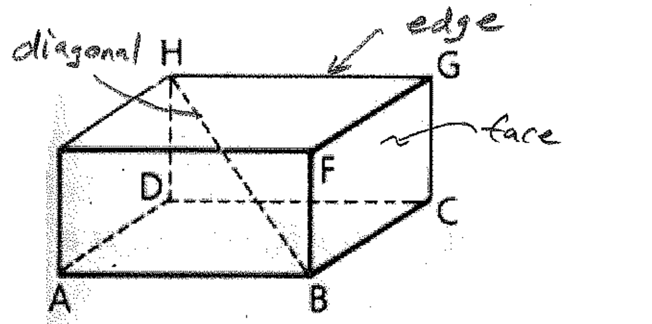
$P = 5 + 3\sqrt{2} + 8 + 3$

$= \boxed{16 + 3\sqrt{2}}$

(start w/ 3D sketching)

Geometry, 9.8: Pythagorean Theorem with 3D shapes

Some 3D shapes...



Rectangular Solid

Rectangular Solid:

Faces: 6 (BCGF, CDHG, etc)

Edges: ~~8~~ 12 ($\overline{GH}$, $\overline{GC}$, $\overline{BC}$, etc)

Diagonals: 4 ($\overline{HB}$, etc)

Note: Rectangular solid is called a Cube if all edges are congruent.

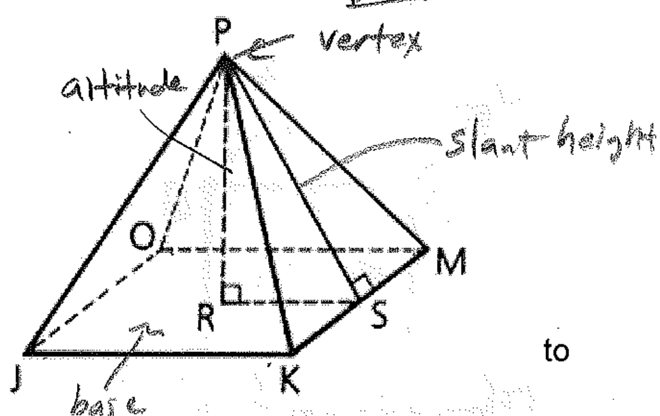
Regular Square Pyramid:

Base: bottom face (JOMK)

Vertex: point at the top (P)

Altitude: height perpendicular to the base ($\overline{PR}$)

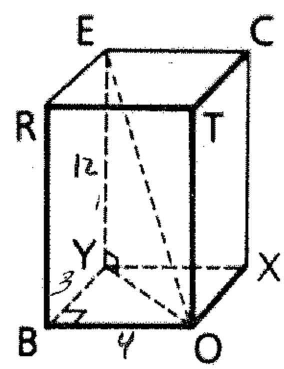
Slant height: length along a face, perpendicular to base ($\overline{PS}$, etc.)



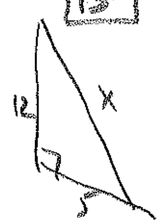
Regular Square Pyramid

Examples:

#1. Given rectangular solid shown, BY=3, OB=4, EY=12.
Find YO (a diagonal face of BOXY) $\boxed{5}$
and EO (a diagonal of the solid) $\boxed{13}$

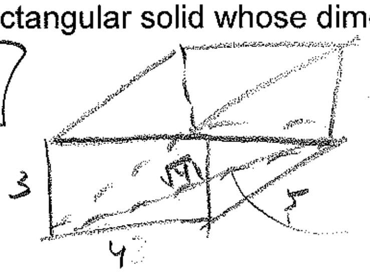


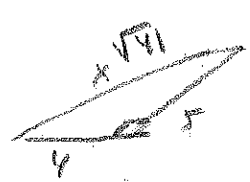
$x^2 = 5^2 + 12^2$
$x^2 = 25 + 144$
$x^2 = 169$
$x = \sqrt{169}$
$x = 13$



#2. Find the diagonal of a rectangular solid whose dimensions are 3, 4, and 5.

$x^2 = 3^2 + (\sqrt{41})^2$ $\boxed{\sqrt{50}}$
$x^2 = 9 + 41 = 50$

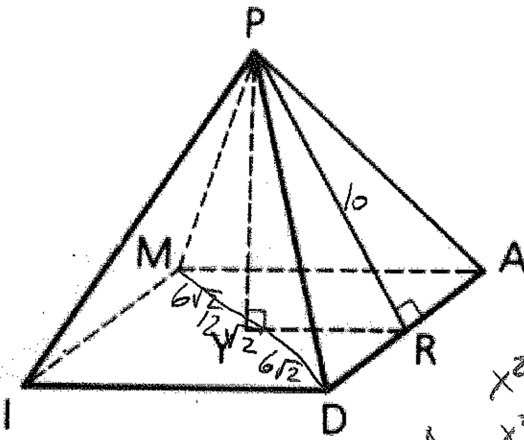




$x^2 = 4^2 + 5^2$
$x^2 = 16 + 25$
$x^2 = 41$

#3. PADIM is a regular square pyramid. Slant height PR measures 10, and the base diagonals measure $12\sqrt{2}$ ~~12~~

a) Find ID $=12$
b) Find the altitude of the pyramid $=8$
c) Find RD $=6$
d) Find PD $=\sqrt{136} = 2\sqrt{34}$



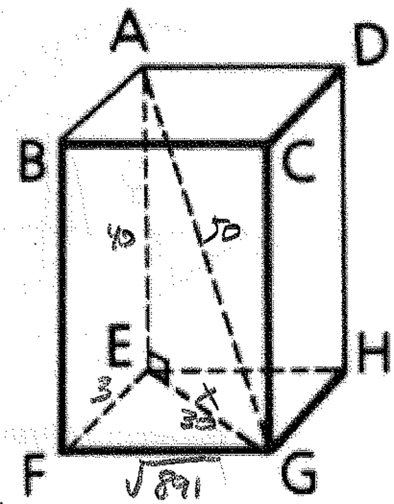
$b^2+b^2=(12\sqrt{2})^2$
$2b^2=144\cdot 2$
$b^2=144$
$b=12$

$h^2+6^2=10^2$
$h^2+36=100$
$h^2=64$
$h=8$

$x^2=10^2+6^2$
$x^2=100+36$
$x=\sqrt{136}$
$\sqrt{4}\ \sqrt{34}$
$\boxed{x=2\sqrt{34}}$

#4. ABCDEFGH is a rectangular solid

If diagonal $\overline{AG}$ measures 50, edge $\overline{AE}$ measures 40, and edge $\overline{EF}$ measures 3, how long is edge $\overline{FG}$?

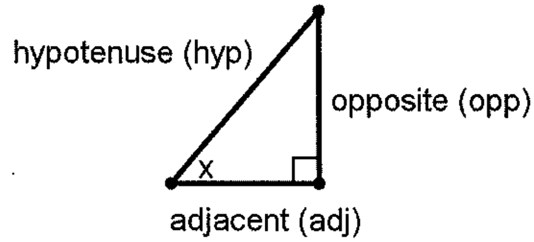


$\sqrt{891}$

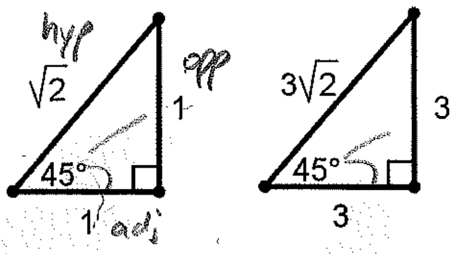
$y^2+3^2=30^2$
$y^2+9=900$
$y^2=891$
$y=\sqrt{891}$

Geometry, 9.9: Intro to Trigonometry (Sine, Cosine, Tangent)

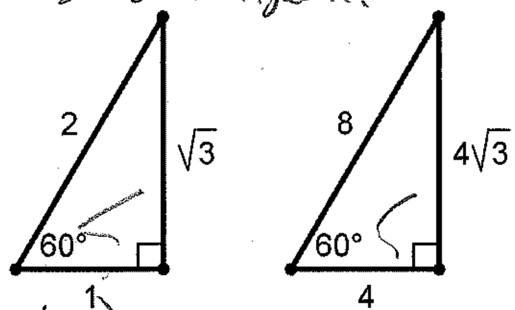
For a right triangle, given one of the non-right angles, each side has a 'name':



We know two special triangle 'patterns':
45-45 triangles...



30-60 triangles...



Ratio of opposite side to hypotenuse: = sine (sin)

$\frac{opp}{hyp} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \frac{opp}{hyp} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \frac{opp}{hyp} = \boxed{\frac{\sqrt{3}}{2}}$ $\quad \frac{opp}{hyp} = \frac{4\sqrt{3}}{8} = \boxed{\frac{\sqrt{3}}{2}}$ } sine

Ratio of adjacent side to hypotenuse: = cosine (cos)

$\frac{adj}{hyp} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \frac{adj}{hyp} = \frac{3}{3\sqrt{2}} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \frac{adj}{hyp} = \boxed{\frac{1}{2}}$ $\quad \frac{adj}{hyp} = \frac{4}{8} = \boxed{\frac{1}{2}}$ } cosine

Ratio of opposite side to adjacent side: = tangent (tan)

$\frac{opp}{adj} = \frac{1}{1} = \boxed{1}$ $\quad \frac{opp}{adj} = \frac{3}{3} = \boxed{1}$ $\quad \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$ $\quad \frac{opp}{adj} = \frac{4\sqrt{3}}{4} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$ } tangent

The ratio depends on the angle, but not on the size of the triangle. These ratios are called **trigonometric ratios**:

$$\text{sine of } \angle A = \sin A = \frac{opposite}{hypotenuse}$$

$$\text{cosine of } \angle A = \cos A = \frac{adjacent}{hypotenuse}$$

$$\text{tangent of } \angle A = \tan A = \frac{opposite}{adjacent}$$

To help remember:

SOH CAH TOA

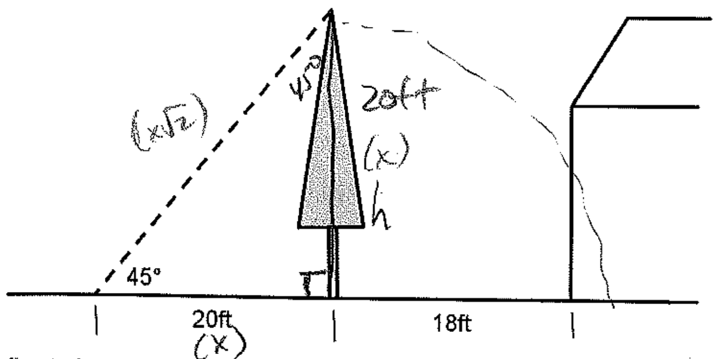
Sine Opposite Hypotenuse
Cosine Adjacent Hypotenuse
Tangent Opposite Adjacent

Here is a useful table of trigonometric ratios to use in a pinch.

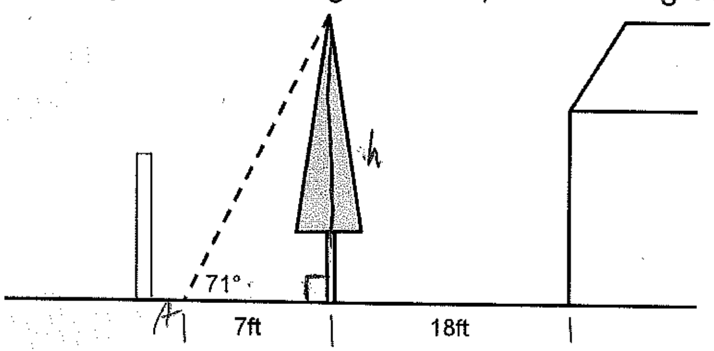
$m°\angle A$	sin A	cos A	tan A	$m°\angle A$	sin A	cos A	tan A
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524	48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699	49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875	50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051	51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228	52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405	53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584	54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763	55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944	56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126	57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309	58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493	59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679	60	0.8660	$0.5\overline{0}$	1.7321
16	0.2756	0.9613	0.2867	61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057	62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249	63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443	64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640	65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839	66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040	67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245	68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452	69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663	70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877	71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095	72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317	73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543	74	0.9613	0.2756	3.4874
30	$0.5\overline{0}$	0.8660	0.5774	75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009	76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249	77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494	78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745	79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002	80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265	81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536	82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813	83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098	84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391	85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693	86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004	87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325	88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657	89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1	90	1	0	Undefined

Geometry, 9.10: Solving for missing side using trig ratios

You want to cut down a tree in your yard, but need to know how tall it is – is it safe to cut, or will it hit your house when it falls? You measure off a distance on the ground, and use a protractor to measure the angle to the top of the tree:



What do you do if the angle isn't 30, 45 or 60 degrees?



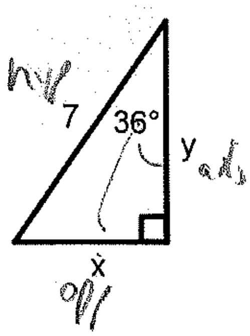
$\tan \angle A = \frac{opp}{adj}$

$\frac{\tan 71°}{1} = \frac{h}{7}$

$h = 7 \cdot \tan 71°$

$h = 7(2.9042) = 20.3$ ft

Another example:



$\sin 36° = \frac{x}{7}$

$x = 7 \cdot \sin 36°$

$x = 7(.5878)$

$x = 4.1$

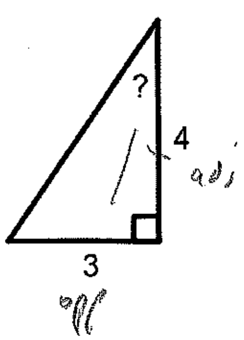
$\cos 36° = \frac{y}{7}$

$y = 7 \cdot \cos 36°$

$y = 7(.8090)$

$y = 5.7$

You can also find an angle, given any two sides:



$\tan ? = \frac{3}{4} = 0.76$

? between $\boxed{37°}$ and $38°$