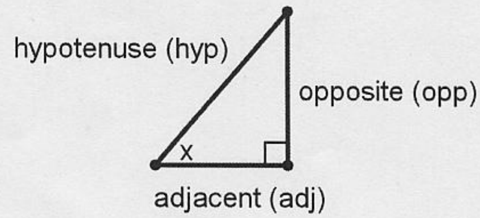
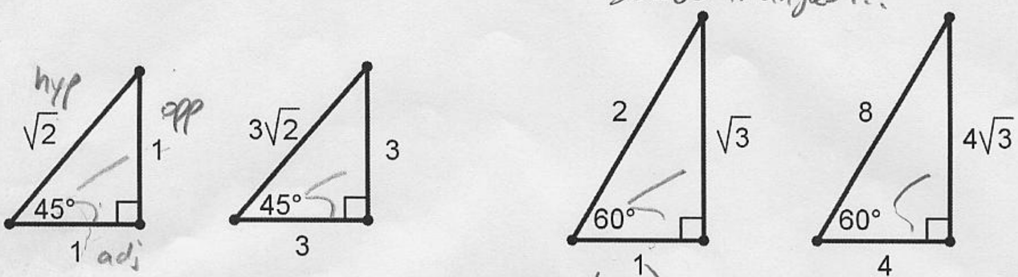
Geometry, 9.9: Intro to Trigonometry (Sine, Cosine, Tangent)

For a right triangle, given one of the non-right angles, each side has a 'name':



We know two special triangle 'patterns':
45-45 triangles...

30–60 triangles...



Ratio of opposite side to hypotenuse: = sine (sin)

$\frac{opp}{hyp} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \frac{opp}{hyp} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \frac{opp}{hyp} = \boxed{\frac{\sqrt{3}}{2}}$ $\quad \frac{opp}{hyp} = \frac{4\sqrt{3}}{8} = \boxed{\frac{\sqrt{3}}{2}}$ } sine

Ratio of adjacent side to hypotenuse: = cosine (cos)

$\frac{adj}{hyp} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \frac{adj}{hyp} = \frac{3}{3\sqrt{2}} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \frac{adj}{hyp} = \boxed{\frac{1}{2}}$ $\quad \frac{adj}{hyp} = \frac{4}{8} = \boxed{\frac{1}{2}}$ } cosine

Ratio of opposite side to adjacent side: = tangent (tan)

$\frac{opp}{adj} = \frac{1}{1} = \boxed{1}$ $\quad \frac{opp}{adj} = \frac{3}{3} = \boxed{1}$ $\quad \frac{opp}{adj} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$ $\quad \frac{opp}{adj} = \frac{4\sqrt{3}}{4} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$ } tangent

The ratio depends on the angle, but not on the size of the triangle. These ratios are called **trigonometric ratios**:

$$\text{sine of } \angle A = \sin A = \frac{opposite}{hypotenuse}$$

$$\text{cosine of } \angle A = \cos A = \frac{adjacent}{hypotenuse}$$

$$\text{tangent of } \angle A = \tan A = \frac{opposite}{adjacent}$$

To help remember:

SOH CAH TOA

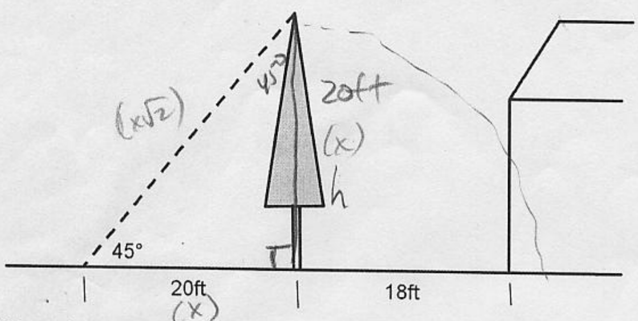
Sine Opposite Hypotenuse

Cosine Adjacent Hypotenuse

Tangent Opposite Adjacent

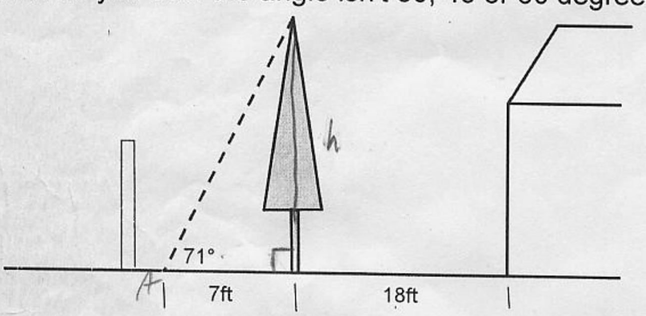
Geometry, 9.10: Solving for missing side using trig ratios

You want to cut down a tree in your yard, but need to know how tall it is – is it safe to cut, or will it hit your house when it falls? You measure off a distance on the ground, and use a protractor to measure the angle to the top of the tree:



$\tan 15° = \frac{h}{20}$

What do you do if the angle isn't 30, 45 or 60 degrees?



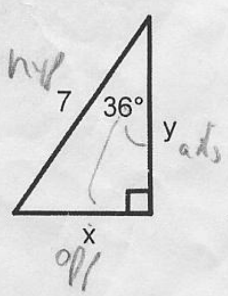
$\tan \angle A = \frac{opp}{adj}$

$\tan 71° = \frac{h}{7}$

$h = 7 \cdot \tan 71°$

$h = 7(2.9042) = 20.3$ ft

Another example:



$\sin 36° = \frac{x}{7}$

$x = 7 \cdot \sin 36°$

$x = 7(.5878)$

$x = 4.1$

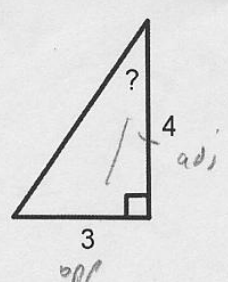
$\cos 36° = \frac{y}{7}$

$y = 7 \cdot \cos 36°$

$y = 7(.8090)$

$y = 5.7$

You can also find an angle, given any two sides:



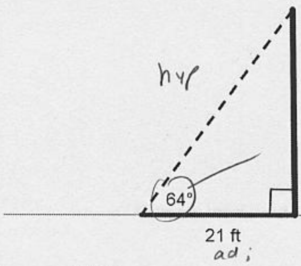
$\tan ? = \frac{3}{4} = 0.76$

? between $\boxed{37°}$ and 38°

Geometry, Notes – Law of Sines

We know how to do right triangle problems like this...

Find the height of the telephone pole:



$\tan A = \frac{opp}{adj}$

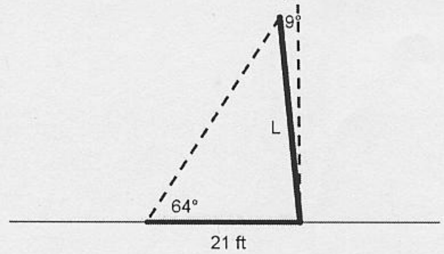
$\tan 64° = \frac{h}{21}$

$2.0503 = \frac{h}{21}$

$h = 21(2.0503)$

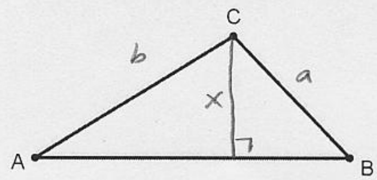
$\boxed{h = 43 ft}$

But what do we do if the pole is not vertical?



Law of Sines

brief proof...

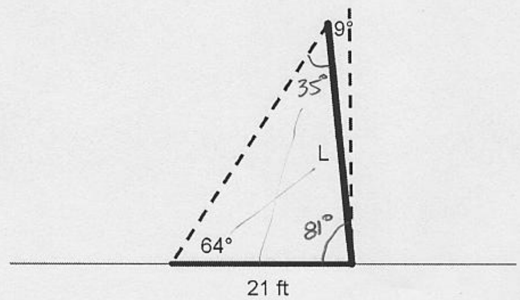


left Δ: $\sin A = \frac{x}{b}$, $x = b\sin A$

right Δ: $\sin B = \frac{x}{a}$, $x = a\sin B$

$\frac{b\sin A}{\sin A \sin B} = \frac{a\sin B}{\sin A \sin B}$ $\quad \frac{b}{\sin B} = \frac{a}{\sin A}$

Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



$\frac{L}{\sin 64°} = \frac{21}{\sin 35°}$

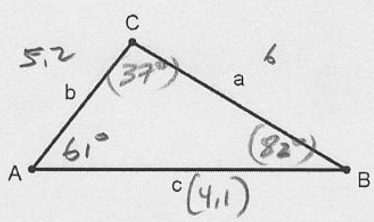
$\frac{L}{.8988} = \frac{21}{.5736}$

$(.5736)L = 21(.8988)$

$L = \frac{21(.8988)}{.5736} = \boxed{32.9 ft}$

Examples:

If a=6, b=5.2, A=61°
find the remaining angle and sides.



$\frac{6}{\sin 61°} = \frac{5.2}{\sin B}$

$\frac{6}{.8746} = \frac{5.2}{\sin B}$

$6\sin B = \frac{5.2}{.8746} = 5.9456$

$\sin B = \frac{5.9456}{6} = 0.9909$

$\boxed{B = 82°}$

$C = 180° - (82° + 61°) = \boxed{37°}$

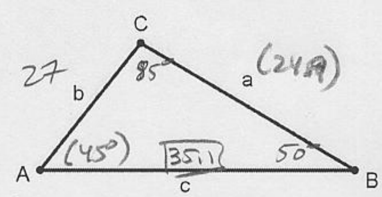
$\frac{c}{\sin 37°} = \frac{6}{\sin 61°}$

$\frac{c}{.6018} = \frac{6}{.8746}$

$.8746c = 6(.6018) = 3.6108$

$c = \frac{3.6108}{.8746} = 4.129$

If C=85°, B=50°, and b=27 ft,
find remaining sides and angles.



$A = 180° - (85° + 50°) = \boxed{45°}$

$\frac{a}{\sin 45°} = \frac{27}{\sin 50°}$

$\frac{a}{.7071} = \frac{27}{.7660}$

$.7660a = 27(.7071)$

$a = \frac{19.0917}{.7660}$

$\boxed{a = 24.9}$

$\frac{c}{\sin 85°} = \frac{27}{\sin 50°}$

$\frac{c}{.9962} = \frac{27}{.7660}$

$.7660c = 27(.9962) = 26.8974$

$c = \frac{26.8974}{.7660} = \boxed{35.1}$

Geometry, Notes – Law of Cosines

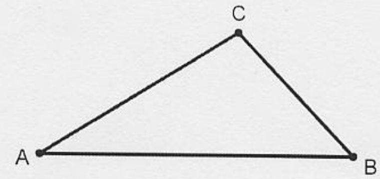
Law of Sines doesn't work if we don't have one known angle across from a known side. Need another method to solve these triangles.

Law of Cosines: 'modified' version of Pythagorean Theorem

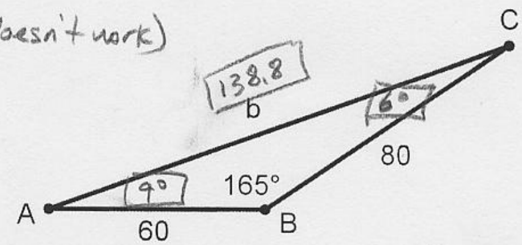
$$a^2 = b^2 + c^2 - 2bc\cos A \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



Examples:
Find remaining sides and angles. (Law of sines doesn't work)



Law of cosines

$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 80^2 + 60^2 - 2(80)(60)\cos 165°$ $(-\cos 15°)$

$b^2 = 10000 - 9600(-.9659)$

$b^2 = 10000 + 9272.64$

$b^2 = 19272.64$

$b = \sqrt{19272.64} = \boxed{138.8}$

Then, use law of sines for angle A:

$\frac{80}{\sin A} = \frac{138.8}{\sin 165°}$ $(\sin 15°)$

$\frac{80}{\sin A} = \frac{138.8}{.2588}$

$(138.8)\sin A = (.2588)80$

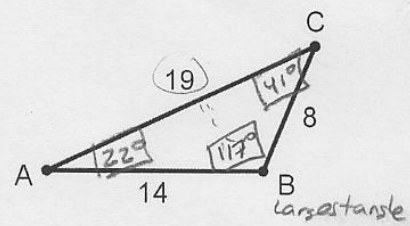
$\sin A = \frac{(.2588)80}{138.8}$

$\sin A = .1492$

$\boxed{A = 9°}$

so $C = 180 - (165 + 9) = \boxed{6°}$

Find the angles. (Law of sines doesn't work)



* NOTE: ALWAYS SOLVE FOR LARGEST ANGLE FIRST

Solve for ∠B 1st:

$b^2 = a^2 + c^2 - 2ac\cos B$

$19^2 = 8^2 + 14^2 - 2(8)(14)\cos B$

$361 = 260 - 224\cos B$

$-260 \quad -260$

$\frac{101}{-224} = \frac{-224\cos B}{-224}$

$\cos B = -.4509$

neg, so supplement of angle for .4509 (63°)

$B = 180° - 63° = \boxed{117°}$

Now Law of sines for ∠A

$\frac{8}{\sin A} = \frac{19}{\sin 117°}$ $(\sin 63°)$

$\frac{8}{\sin A} = \frac{19}{.8910}$

$(19)\sin A = (.8910)8$

$\sin A = \frac{(.8910)8}{19}$

$\sin A = .3752$

$\boxed{A = 22°}$

Then for ∠C:

$C = 180 - (117 + 22)$

$\boxed{C = 41°}$