Geometry, Supplemental - Counting Problems: Simple cases, Pascal's Triangle

Example: At an amusement park, you can get a combo meal which includes one main item (pizza or burger), one side (fries or chips) and a drink (diet coke or regular coke). How many different combo meals can you get?

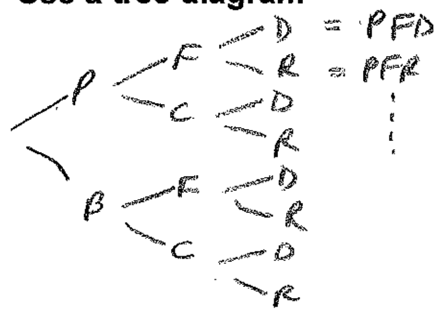
Two strategies for simple counting cases: (pizza = P, Fries = F, Diet = D, etc)

List all possibilities

PFD
PFR
PCD
PCR
BFD
BFR
BCD
BCR

8 combo meals

Use a tree diagram



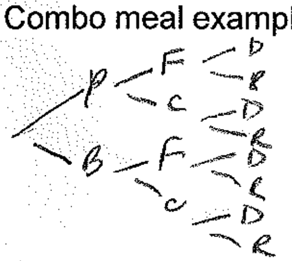
8 combo meals

Example: License plates in a small county consist of 1 letter followed by 3 numbers. How many different license plates can be made?

Sometimes, difficult or impossible to list all possibilities or make a tree diagram.

Use 'boxes' method:

Combo meal example:



$\underset{\text{main item}}{\boxed{2}} \times \underset{\text{side}}{\boxed{2}} \times \underset{\text{drink}}{\boxed{2}} = 8$

License plate example:

$\underset{\text{letter}}{\boxed{26}} \times \underset{\text{num}}{\boxed{10}} \times \underset{\text{num}}{\boxed{10}} \times \underset{\text{num}}{\boxed{10}} = 26{,}000$

Example: John, Cassie, Pat, and Ally line up next to each other. How many different ways can they be lined up?

$\underset{1^{st}\text{ person}}{\boxed{4}} \times \underset{2^{nd}\text{ person}}{\boxed{3}} \times \underset{3^{rd}\text{ person}}{\boxed{2}} \times \underset{4^{th}\text{ person}}{\boxed{1}} = 24$

Pascal's Triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

- 1's on outside
- each number is sum of 2 numbers above it.

Geometry, Supplemental - Counting Problems: Combinations, Permutations

'Choosing' problems can be tricky. There are two different cases: order matters and order doesn't matter. Examples:

On a sports team with 10 players, you need to choose a team captain, a co-captain and an equipment manager. Each person has a different job.

Order matters – 'Permutation'

$\boxed{10} \times \boxed{9} \times \boxed{8} = 720$

Capt. Cocapt. E.m

On a sports team with 10 players, you need to choose 3 players to make a 'leadership team' that work together to share all jobs to lead the team.

Order does not matter – 'Combination'

$\dfrac{\boxed{10} \times \boxed{9} \times \boxed{8}}{\boxed{3} \times \boxed{2} \times \boxed{1}} = \dfrac{720}{6} = 120$

6

Jill	Bob	Jane
Jill	Jane	Bob
Bob	Jill	Jane
Bob	Jane	Jill
Jane	Bob	Jill
Jane	Jill	Bob

when we pick 3 people, those people can be rearranged 6 different ways
= all 6 permutations are just 1 combination.

You can use 'boxes' to solve, but for Combinations (order doesn't matter) you must also divide by boxes for how many ways those you choose can be rearranged.

Or...you can use the Permutation and Combination formulas:

n! = 'n factorial'

Examples: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$3! = 3 \cdot 2 \cdot 1 = 6$

Permutations

$${}_nP_r = \frac{n!}{(n-r)!}$$

(out of 'n' things, choose 'r')
(order matters)

On a sports team with 10 players, you need to choose a team captain, a co-captain and an equipment manager. Each person has a different job.

order matters, permutation:

$${}_{10}P_3 = \frac{n!}{(n-r)!} = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = 10 \cdot 9 \cdot 8 = 720$$

Combinations

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

(out of 'n' things, choose 'r')
(order doesn't matter)

On a sports team with 10 players, you need to choose 3 players to make a 'leadership team' that work together to share all jobs to lead the team.

order doesn't matter, combination:

$${}_{10}C_3 = \frac{n!}{(n-r)!r!} = \frac{10!}{(10-3)!3!} = \frac{10!}{7!3!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot 3 \cdot 2 \cdot 1} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = \frac{720}{6} = 120$$

Permutations, you can just (usually) just use the 'boxes' method. For Combinations, you can find ${}_nC_r$ using Pascal's triangle (count rows and columns starting with zero, not one):

row 0	1
row 1	1 1
row 2	1 2 1
row 3	1 3 3 1
row 4	1 4 6 4 1
row 5	1 5 10 10 5 1
row 6	1 6 15 20 15 6 1

${}_5C_2 = 10$

↑ row ↑ column

$\left(\frac{n!}{(n-r)!\,r!} = \frac{5!}{3!\,2!}\right) = \frac{5\cdot4\cdot\cancel{3\cdot2\cdot1}}{\cancel{3\cdot2\cdot1}\cdot2\cdot1} = \frac{20}{2} = 10$

this 1st

Permutations example (order does matter): Of 20 people in a class, how many ways can you pick a President, Vice-Presidents and a Secretary?

Using formula:

$${}_{20}P_3 = \frac{n!}{(n-r)!} = \frac{20!}{(20-3)!} = \frac{20!}{17!}$$

$$= \frac{20\cdot19\cdot18\cdot\cancel{17\cdot16\cdot15\cdot14\cdot13\cdot12\cdots}}{\cancel{17\cdot16\cdot15\cdot14\cdots}}$$

$$= 20\cdot19\cdot18$$

$$= \boxed{6840}$$

*easiest

Using boxes:

$$\frac{20}{\text{pres}} \times \frac{19}{\text{VP}} \times \frac{18}{\text{sec}} = \boxed{6840}$$

Combinations example: (order does not matter): A pizza shop offers 5 different toppings. How many different 3-item pizzas can be made?

Using formula:

$${}_5C_3 = \frac{n!}{(n-r)!\,r!}$$

$$= \frac{5!}{(5-3)!\,3!}$$

$$= \frac{5!}{2!\,3!}$$

$$= \frac{5\cdot4\cdot\cancel{3\cdot2\cdot1}}{2\cdot1\cdot\cancel{3\cdot2\cdot1}}$$

$$= \frac{5\cdot4}{2} = \frac{20}{2} = \boxed{10}$$

*easiest

Using Pascal's triangle:

${}_5C_3 = \boxed{10}$

↑ row ↑ col

Using boxes:

$$\frac{\text{\# ways to choose 3 out of 5:}}{\text{\# ways these 3 can be rearranged}} \quad \frac{\boxed{5}\times\boxed{4}\times\boxed{3}}{\boxed{3}\times\boxed{2}\times\boxed{1}} = \frac{60}{6} = \boxed{10}$$