# Geometry, Supplemental – sem2: Matrices

**What is the matrix?**

$$\begin{array}{cc} \text{row1} \rightarrow & \multirow{2}{*}{$\begin{bmatrix} 4 & -\frac{1}{2} & 0 \\ -7 & 1 & 4 \end{bmatrix}$} \\ \text{row2} \rightarrow & \\ & \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{col1} & \text{col2} & \text{col3} \end{array} \end{array}$$

grid of numbers

dimensions : rows x columns (eg. 2 x 3)

**Determining the dimensions of a matrix:**

$$\begin{array}{cc} \text{row1} & \multirow{2}{*}{$\begin{bmatrix} 4 & -\frac{1}{2} & 0 \\ -7 & 1 & 4 \end{bmatrix}$} \\ \text{row2} & \\ & \text{col1 col2 col3} \end{array}$$

2x3

$$\begin{bmatrix} 3 & -7 \\ -1 & 6 \end{bmatrix}$$

2x2

$$\begin{bmatrix} -1 & \frac{3}{7} & 3 \end{bmatrix}$$

1x3

$$\begin{bmatrix} 5 & 4 & 1 \\ -9 & 0 & 5 \\ 7 & -4 & 1 \\ 0 & 3 & 6 \end{bmatrix}$$

4 x 3

**Adding / Subtracting matrices:** ** must have <u>same</u> dimension **

$$\begin{bmatrix} 3 & -7 \\ -1 & 6 \end{bmatrix} + \begin{bmatrix} -1 & \frac{3}{7} & 3 \end{bmatrix}$$

can't add, different dimensions

2x2 , 1x3

$$\begin{bmatrix} 3 & -7 \\ -1 & 6 \end{bmatrix} + \begin{bmatrix} -5 & 9 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 0 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix} - \begin{bmatrix} 9 \\ 1 \\ -6 \end{bmatrix} = \begin{bmatrix} -5 \\ -6 \\ 8 \end{bmatrix}$$

**Multiplying a matrix by a number (scalar multiplication):**

$$10\begin{bmatrix} 4 & -\frac{1}{2} & 0 \\ -7 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 40 & -5 & 0 \\ -70 & 10 & 40 \end{bmatrix}$$

$$-1\begin{bmatrix} 3 & -7 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 7 \\ 1 & -6 \end{bmatrix}$$

$$3\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ -9 \\ 3 \end{bmatrix}$$

**Matrices are equal if:** they have same dimensions and all corresponding elements are equal

Solve for x and y:

$$\begin{bmatrix} 3x & 2 & 4 \end{bmatrix} = \begin{bmatrix} 9 & -y & 4 \end{bmatrix}$$

$3x = 9$

$\boxed{x = 3}$

$-y = 2$

$\boxed{y = -2}$

$$\begin{bmatrix} 2y & -1 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 3x & 0 \end{bmatrix}$$

$2y = 6$

$\boxed{y = 3}$

$3x = -6$

$\boxed{x = -2}$

Sem2 supp day 1 – matrices HW

**Determine the dimensions of the matrix.**

**1.** $\begin{bmatrix} 3 & 5 & -7 \\ 1 & 2 & 9 \\ -2 & 6 & 1 \\ 4 & -3 & 5 \end{bmatrix}$ $\boxed{4\times 3}$

**2.** $\begin{bmatrix} 4 & 9 \\ -5 & 1 \\ 2 & -6 \end{bmatrix}$ $\boxed{3\times 2}$

**3.** $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $\boxed{2\times 1}$

**4.** $\begin{bmatrix} 1 & 4 & 5 & -2 \\ -6 & 2 & 0 & 3 \\ 3 & 8 & -1 & 4 \end{bmatrix}$ $\boxed{3\times 4}$

**Tell whether the matrices are *equal* or *not equal*.**

**5.** $\begin{bmatrix} 3 & 4 \\ -7 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 7 & -1 \end{bmatrix}$ $\boxed{\neq}$

**6.** $\begin{bmatrix} 2 & -1 & 6 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ $\boxed{\neq}$

**7.** $\begin{bmatrix} 1 & 0 \\ 4 & -3 \end{bmatrix}, \begin{bmatrix} \frac{2}{2} & 0 \\ \frac{8}{2} & -\frac{3}{1} \end{bmatrix}$ $\boxed{=}$

**Perform the indicated operation, if possible. If not possible, state the reason.**

**8.** $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 6 & 5 \end{bmatrix}$

**9.** $\begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

**10.** $\begin{bmatrix} 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -4 \end{bmatrix} = \begin{bmatrix} 6 & -4 \end{bmatrix}$

**11.** $\begin{bmatrix} 2 \\ -7 \end{bmatrix} + \begin{bmatrix} -3 & 4 \end{bmatrix}$ dims $\neq$

**12.** $\begin{bmatrix} 0 & 4 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 1 & 3 \end{bmatrix}$

**13.** $\begin{bmatrix} 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} -1 \\ -11 \end{bmatrix}$

**14.** $\begin{bmatrix} 1 & 4 \\ -5 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

**15.** $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 5 \end{bmatrix}$ dims $\neq$

**16.** $\begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -13 & 2 \\ 1 & -7 \end{bmatrix} = \begin{bmatrix} -13 & 2 \\ 1 & -3 \end{bmatrix}$

**Perform the indicated operation.**

**17.** $2\begin{bmatrix} 1 & 6 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ -6 & 4 \end{bmatrix}$

**18.** $-3\begin{bmatrix} 1 & 0 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 9 & -18 \end{bmatrix}$

**19.** $5\begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 10 \\ -25 \end{bmatrix}$

**20.** $-4\begin{bmatrix} -3 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 12 & -24 & -4 \end{bmatrix}$

**21.** $8\begin{bmatrix} 3 \\ 0 \\ -5 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ -40 \end{bmatrix}$

**22.** $-1\begin{bmatrix} 2 & 5 & -3 \\ 6 & -1 & -7 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} -2 & -5 & 3 \\ -6 & 1 & 7 \\ 0 & 0 & -9 \end{bmatrix}$

**Solve the matrix for *x* and *y*.**

**23.** $\begin{bmatrix} x & 3 \\ 5 & y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix}$ $\boxed{x=2}$ $\boxed{y=-4}$

**24.** $\begin{bmatrix} 2x \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 4y \end{bmatrix}$ $2x=10$ $\boxed{x=5}$ $4y=4$ $\boxed{y=1}$

**25.** $\begin{bmatrix} 3x & -21 \end{bmatrix} = \begin{bmatrix} 21 & 7y \end{bmatrix}$ $3x=21$ $\boxed{x=7}$ $7y=-21$ $\boxed{y=-3}$

**26.** ***Endangered and Threatened Species*** The matrices below show the number of endangered and threatened animal and plant species as of June 30, 1996. Use matrix addition to find the total number of endangered and threatened species. *(Source: 1997 Information Please Almanac)*

**ENDANGERED** + **THREATENED** = total

$$\begin{array}{r c} & \begin{array}{cc} \text{U.S.} & \text{Foreign} \end{array} \\ \begin{array}{r} \text{Animal} \\ \text{Plant} \end{array} & \begin{bmatrix} 320 & 521 \\ 431 & 1 \end{bmatrix} \end{array} + \begin{array}{r c} & \begin{array}{cc} \text{U.S.} & \text{Foreign} \end{array} \\ \begin{array}{r} \text{Animal} \\ \text{Plant} \end{array} & \begin{bmatrix} 115 & 41 \\ 94 & 2 \end{bmatrix} \end{array} = \begin{bmatrix} 435 & 562 \\ 525 & 3 \end{bmatrix}$$

## Geometry, Supplemental – sem2: Sequences and Patterns

**Patterns** Find the next two terms of each sequence:

1. 1, 3, 6, 10, 15, 21, $\underline{28}$, $\underline{36}$ (+2, +3, +4 etc)
2. 3, −12, 48, −192, $\underline{768}$, $\underline{-3072}$ ($\times -4$)
3. 1, 3, 4, 7, 11, 18, $\underline{29}$, $\underline{47}$ (add previous 2 numbers)
4. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, $\underline{\frac{1}{32}}$, $\underline{\frac{1}{64}}$ ($\times \frac{1}{2}$)
5. $\frac{5}{3}, 1, \frac{1}{3}, \frac{-1}{3}, -1$, $\underline{\frac{-5}{3}}$, $\underline{\frac{-7}{3}}$ $\left(-\frac{2}{3}\right)$

   $\frac{5}{3}, \frac{3}{3}, \frac{1}{3}, \frac{-1}{3}, \frac{-3}{3}$

**Sequence Notation** Write the first four terms of each sequence. • start with n=1 • plug in n to get a value of nth term

6. $a_n = \dfrac{n}{n+2}$  $a_1 = \frac{1}{1+2} = \frac{1}{3}$  $a_2 = \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$  $a_3 = \frac{3}{3+2} = \frac{3}{5}$  $a_4 = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$  $\boxed{\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \ldots}$ ($a_1\ a_2\ a_3\ a_4 \ldots$)

   ✱ n = counter
   $a_n$ = value of term

7. $a_n = 4 - 3n$  $a_1 = 4 - 3\cdot1 = 1$  $a_2 = 4-3\cdot2 = -2$  $a_3 = 4-3\cdot3 = -5$  $a_4 = 4-3\cdot4 = -8$  $\boxed{1, -2, -5, -8, \ldots}$

8. $a_n = \dfrac{1}{2n}$  $a_1 = \frac{1}{2\cdot1} = \frac{1}{2}$  $a_2 = \frac{1}{2\cdot2} = \frac{1}{4}$  $a_3 = \frac{1}{2\cdot3} = \frac{1}{6}$  $a_4 = \frac{1}{2\cdot4} = \frac{1}{8}$  $\boxed{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots}$

**Finding the rule for the *n*th term of a sequence** Write a rule for the *n*th term of the given sequence.

1st, find the pattern
✱show: $a_n = a_1 + (n-1)d$   $a_n = a_1(r)^{n-1}$

9. 5, 7, 9, 11, 13,……..  $a_n = 5 + 2(n-1) = 5 + 2n - 2 = \boxed{3+2n}$

10. $\frac{1}{4}, \frac{2}{7}, \frac{3}{10}, \frac{4}{13}, \ldots$ do top and bottom separately; $4 + 3(n-1)$, $4 + 3n - 3$  $a_n = \boxed{\dfrac{n}{1+3n}}$

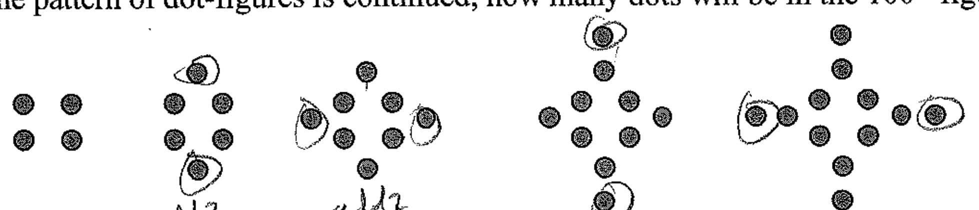
    n=1  2  3  4

11. 3, 0, −3, −6, −9, ……..  $a_n = 3 - 3(n-1) = 3 - 3n + 3$  $a_n = \boxed{6-3n}$

Ex1; 3, 9, 27, …
1, 1(3), 1(3)(3), 1(3)(3)(3)
1, 1·3, 1·3², 1·3³
$\boxed{a_n = 1\cdot(3)^{(n-1)}}$

If the pattern of dot-figures is continued, how many dots will be in the 100th figure?

~~12.~~ 13.



add2  add2  add2  add2

4  6  8  10  12

$a_n = 4 + 2(n-1) = 4 + 2n - 2 = 2 + 2n$   so $a_{100} = 2 + 2(100) = \boxed{202 \text{ dots}}$

**Geometry**
Sequences and Patterns Homework

Name Key
Date ____________ Period ______

**Patterns**
Find the next two terms of each sequence.

1. 1, 10, 100, 1000, 10,000 , 100,000 (x10)

2. 0, 10, 21, 33, 46, 60, 75 , 91 (+10, +11, +12, +13, +14

3. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}$, $\frac{1}{49}$ , $\frac{1}{64}$ $\frac{1}{+3, +5, +7 \text{ etc}}$
   +3 +5 +7 +9 +11

4. 1, 2, 4, 8, 16, 32 , 64 x2

5. 0, 3, 8, 15, 24, 35, 48 , 63
   +3 +5 +7 +9 +11 +13

6. 5, -10, 20, -40, 80, -160 , 320 x -2

7. 7, 10, 13, 16, 19, 22, 25 , 28
   +3 +3 +3 +3 +3

8. 8, 4, 0, -4, -8, -12 , -16
   -4 -4 -4 -4

**Sequence notation**

Write the first four terms of each sequence.

9. $a_n = \frac{n+4}{n}$
   $a_1 = \frac{1+4}{1} = 5$ $a_2 = \frac{2+4}{2} = \frac{6}{2} = 3$ $a_3 = \frac{3+4}{3} = \frac{7}{3}$ $a_4 = \frac{4+4}{4} = \frac{8}{4} = 2$
   $\boxed{5, 3, \frac{7}{3}, 2}$

10. $a_n = 2n+5$
   $a_1 = 2 \cdot 1 + 5 = 7$ $a_2 = 2 \cdot 2 + 5 = 9$ $a_3 = 2 \cdot 3 + 5 = 11$ $a_4 = 2 \cdot 4 + 5 = 13$
   $\boxed{7, 9, 11, 13}$

11. $a_n = \frac{2}{3} - \frac{1}{3}n$
   $a_1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ $a_2 = \frac{2}{3} - \frac{2}{3} = 0$ $a_3 = \frac{2}{3} - \frac{3}{3} = -\frac{1}{3}$ $a_4 = \frac{2}{3} - \frac{4}{3} = \frac{-2}{3}$
   $\boxed{\frac{1}{3}, 0, -\frac{1}{3}, -\frac{2}{3}}$

**Finding the rule for the *n*th term of a sequence**

Write a rule for the *n*th term of the given sequence.

12. 1, 4, 7, 10, ........ $a_n = 1 + 3(n-1) = 1 + 3n - 3 = \boxed{3n-2}$

+3 +3 +3

13. $\frac{9}{2}, \frac{5}{1}, \frac{11}{2}, \frac{6}{1}, \frac{13}{2}$, ........ $a_n = \frac{9 + 1(n-1)}{2} = \frac{9 + n - 1}{2} = \boxed{\frac{8+n}{2}}$

$\frac{9}{2}, \frac{10}{2}, \frac{11}{2}, \frac{12}{2}, \frac{13}{2}$

14. 2, 4, 8, 16, ........ $2, 2 \cdot 2, 2 \cdot 2 \cdot 2, 2 \cdot 2 \cdot 2 \cdot 2$

×2 $\boxed{a_n = 2^n}$

15. 4, 9, 16, 25, ........ $\boxed{a_n = (n+1)^2}$

$2^2\ 3^2\ 4^2\ 5^2$

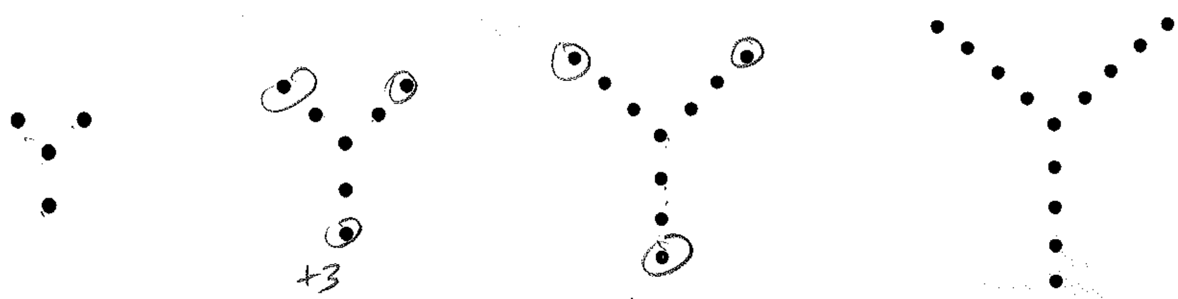
16. Find the value of the *n*th term and the 100th term in each sequence.

| Term | 1 | 2 | 3 | 4 | 5 | 6 | … | *n* | … | 100 |
|---|---|---|---|---|---|---|---|---|---|---|
| Value | 7 | 11 | 15 | 19 | 23 | 27 | … | $3+4n$ | … | 403 |

+4 +4 +4 +4 +4

$a_n = 7 + 4(n-1)$
$= 7 + 4n - 4$
$= 3 + 4n$

17. If the pattern of dot-figures is continued, how many dots will be in the 100th figure?



4    +3 7    +3    +3

$a_n = 4 + 3(n-1)$
$= 4 + 3n - 3$
$= 3n + 1$

$a_{100} = 3(100) + 1$
$= 300 + 1$
$= \boxed{301 \text{ dots}}$

## Geometry, Supplemental – sem2: Recursive Sequences

Can you see a pattern in this sequence?

2, 5, 8, 11, 14, 17 add 3

What about this one?

1, 2, 4, 8, 16, 32 double

How about this one?

1, 1, 2, 3, 5, 8 add previous 2 terms

In patterns like these, the next number is determined by one or more of the numbers before it. This is called a **recursive** pattern because something happens with the first value, and that same something 're-occurs' (recurs) over and over again.

For our rule for generating a sequence, we need a way to write: 'the term before this one' or 'the term two before this one':

$a_n$ is the way we write 'this term – the term we are working on'

$a_{n-1}$ is the way we write 'the term before the term we are working on'

$a_{n-2}$ is the way we write 'the term 2 before the term we are working on'

Examples of finding a recursive pattern:

2, 5, 8, 11, 14, 17 add 3 $a_n = a_{n-1} + 3$

1, 2, 4, 8, 16, 32 double $a_n = 2a_{n-1}$

1, 1, 2, 3, 5, 8 add previous 2 terms

$a_n = a_{n-1} + a_{n-2}$

# Geometry, Supplemental – sem2: Recursive Sequences

A **recursive rule** gives the beginning term or terms of a sequence and then a **recursive equation** that tells how $a_n$ is related to one or more preceding terms. (terms before)

Example: Write the first five terms of the sequence:

$$a_1 = 4, \quad a_n = a_{n-1} + 3$$

$$a_1 = 4$$
$$a_2 = a_{2-1} + 3 = a_1 + 3 = 4+3 = 7$$
$$a_3 = a_{3-1} + 3 = a_2 + 3 = 7+3 = 10$$
$$a_4 = a_{4-1} + 3 = a_3 + 3 = 10+3 = 13$$
$$a_5 = a_{5-1} + 3 = a_4 + 3 = 13+3 = 16$$

Write the first five terms of the sequence.

1. $a_1 = 3, \quad a_n = a_{n-1} - 2$

$\boxed{3, 1, -1, -3, -5}$

$a_1 = 3$
$a_2 = a_1 - 2 = 3 - 2 = 1$
$a_3 = a_2 - 2 = 1 - 2 = -1$
$a_4 = a_3 - 2 = -1 - 2 = -3$
$a_5 = a_4 - 2 = -3 - 2 = -5$

2. $a_1 = -3, \quad a_n = -2a_{n-1}$

$\boxed{-3, 6, -12, 24, -48}$

$a_1 = -3$
$a_2 = -2a_1 = -2(-3) = 6$
$a_3 = -2a_2 = -2(6) = -12$
$a_4 = -2a_3 = -2(-12) = 24$
$a_5 = -2a_4 = -2(24) = -48$

3. $a_1 = -2, \quad a_n = 3a_{n-1} + 1$

$a_1 = -2$
$a_2 = 3a_1 + 1 = 3(-2) + 1 = -5$
$a_3 = 3a_2 + 1 = 3(-5) + 1 = -14$
$a_4 = 3a_3 + 1 = 3(-14) + 1 = -41$
$a_5 = 3a_4 + 1 = -122$

$\boxed{-2, -5, -14, -41, -122}$

4. $a_1 = 32, \quad a_n = \frac{1}{2}a_{n-1} + 4$

$a_1 = 32$
$a_2 = \frac{1}{2}a_1 + 4 = \frac{1}{2}(32) + 4 = 20$
$a_3 = \frac{1}{2}a_2 + 4 = \frac{1}{2}(20) + 4 = 14$
$a_4 = \frac{1}{2}a_3 + 4 = \frac{1}{2}(14) + 4 = 11$
$a_5 = \frac{1}{2}a_4 + 4 = \frac{1}{2}(11) + 4 = \frac{11}{2} + \frac{8}{2} = \frac{19}{2}$

$\boxed{32, 20, 14, 11, \frac{19}{2}}$

5. $a_1 = 2, \quad a_n = (a_{n-1})^2$

$a_1 = 2$
$a_2 = (a_1)^2 = (2)^2 = 4$
$a_3 = (a_2)^2 = (4)^2 = 16$
$a_4 = (a_3)^2 = (16)^2 = 256$
$a_5 = (a_4)^2 = (256)^2 = 65536$

$\boxed{2, 4, 16, 256, 65536}$

6. $a_1 = 2, \quad a_2 = 5, \quad a_n = a_{n-1} + a_{n-2}$

$a_1 = 2$
$a_2 = 5$
$a_3 = a_2 + a_1 = 2 + 5 = 7$
$a_4 = a_3 + a_2 = 7 + 5 = 12$
$a_5 = a_4 + a_3 = 12 + 7 = 19$

$\boxed{2, 5, 7, 12, 19}$

**Geometry**
Homework: Recursive Sequences 1st 4 terms

Name Key

1. $a_1 = 3, \quad a_n = a_{n-1} + 5$

   3, 8, 13, 18, ...

2. $a_1 = 15, \quad a_n = a_{n-1} - 5$

   15, 10, 5, 0, ...

3. $a_1 = -2, \quad a_n = 4a_{n-1}$

   −2, −8, −32, −128, ...

   $\begin{array}{r} 32 \\ 4 \\ \hline 128 \end{array}$

4. $a_1 = 1, \quad a_n = 100a_{n-1}$

   1, 100, 10000, 1000000, ...

5. $a_1 = -2, \quad a_n = 3a_{n-1} + 1$

   −2, −5, −14, −41, ...

   $\begin{array}{r} 14 \\ 3 \\ \hline 42 \end{array}$

6. $a_1 = 8, \quad a_n = 2a_{n-1} - 6$

   8, 10, 14, 22, ...

7. $a_1 = -1, \; a_2 = 4,$
   $a_n = a_{n-1} \cdot a_{n-2}$

   −1, 4, −4, −16, 64, ...

8. $a_1 = 2, \; a_n = n^2 - a_{n-1}$

   2, 2, 7, 9, ...

   n=1 n=2 n=3 n=4

   $2^2-2$ $3^2-2$ $4^2-2$