HAlg3-4, 1.1 Notes – Functions

Relation: pairs of quantities related by some rule of correspondence.

Examples: area of a circle is related to its radius, temperature outside is related to time of day.

2 quantities:

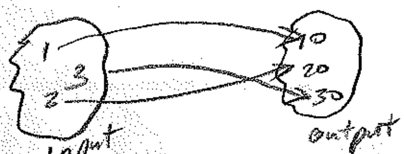
Input = independent variable, x, (x, y)
Set of possible inputs is called the *domain* D

Output = dependent variable, y, (x,y)
Set of possible outputs is called the *range* R

Ways to show relationships:

1) mapping

domain D: {1, 2, 3}

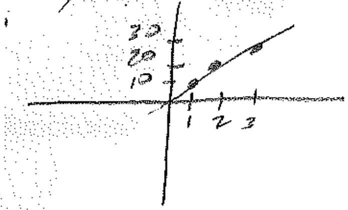


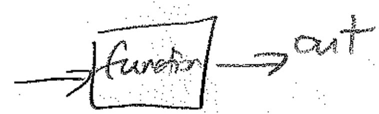
range R: {10, 20, 30}

2) rule/formula: $y = 10x$ D: $(-\infty, \infty)$ R: $(-\infty, \infty)$

3) set of ordered pairs: {(1, 10), (2, 20), (3, 30)} (table)

4) graph:

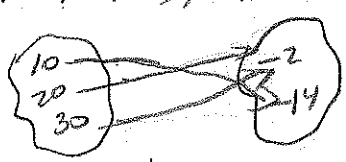


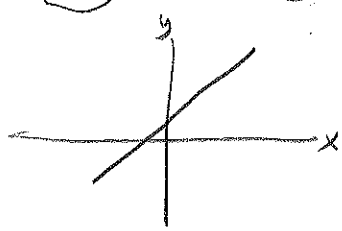


If a relation has *exactly one output for each input* the relation is a *function*.

Examples of functions

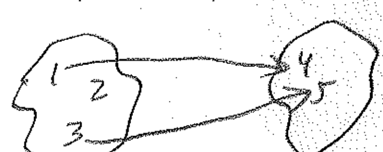
{(1, 10), (2, 30), (3, -10)}



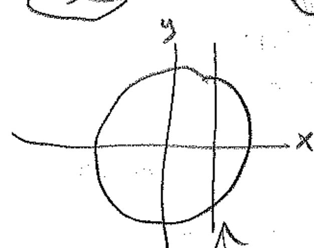


Examples that are not functions, and why

{(1, 10), (2, 30), (1, 40), (3, -20)} input can have only one output



all inputs must have an output



each x can have only one y

vertical line test

If a function can be written as a rule, it can be written in **function notation**:

$f(x)$ = an equation with x

x = input

$f(x)$ = output for a given input x

important
$f(x)$
does not mean
f times x

To evaluate a function for a given input, plug the input into the rule and compute the output:

ex $f(x) = 3x^2+2$

$f(1) = 3(1)^2+2 = 5$

$f(2) = 3(2)^2+2 = 14$ ← include $f(h+2) = 3(h+2)^2+2$

$= 3(h^2+4h+4)+2$

$= 3h^2+12h+12+2$

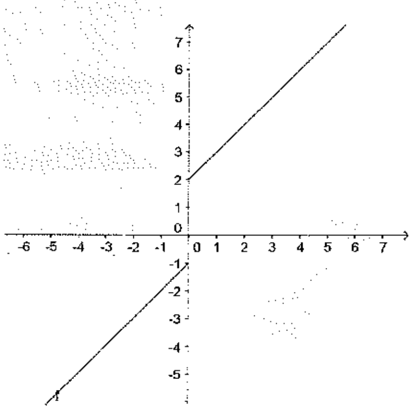
$= 3h^2+12h+14$

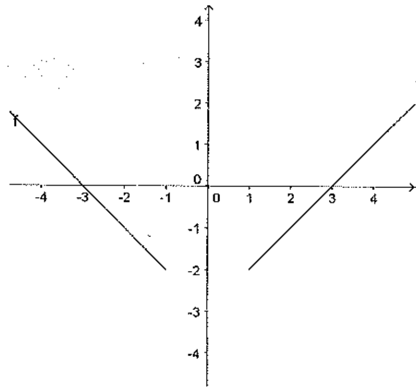
$h(b) = \frac{b}{9-b}$

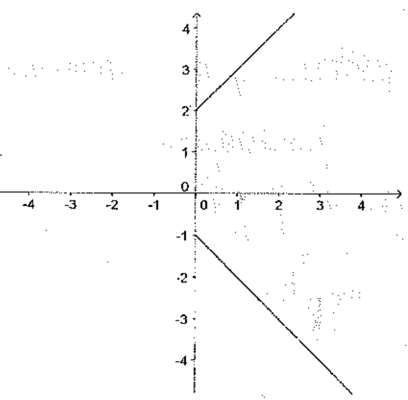
$h(1) = \frac{(1)}{9-(1)} = \frac{1}{8}$

$h(8) = \frac{(8)}{9-(8)} = 8$

Are these functions?







yes

$f(x) = \begin{cases} x-1, & x<0 \\ x+2, & x\geq 0 \end{cases}$

Called a piece-wise linear function

yes

not all x's have to be in the domain

no

x cannot have multiple y (fails vertical line test)

algebraically, solve for y:

$x^2+y^2=4$

$y = \pm\sqrt{4-x^2}$

HAlg3-4, 1.1 (day2) Notes – Functions

Domain of a function = possible or allowable values of the input (x) $f(x) = 2x^2 - 4$

$(-\infty, \infty)$ or $\mathbb{R}$

Determining the domain of a function:

1) Domain may be given by the function definition:

$f: \{(-3,0), (-1,4), (0,2), (2,2), (4,-1)\}$

domain $D: \{-3, -1, 0, 2, 4\}$

2) Domain may be implied (the values for which the rule is defined):

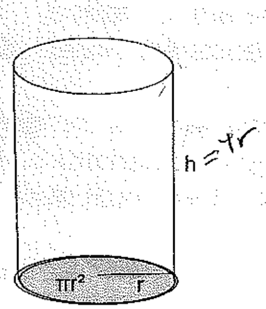
$f(x) = 3x+4$ $D: (-\infty, \infty)$ or all real numbers, $\mathbb{R}$

$f(x) = \frac{x^2}{x-2}$ $D: \{\mathbb{R}, x \neq 2\}$ or $D: (-\infty, 2) \cup (2, \infty)$

$f(x) = \sqrt{x-4}$ $D: \{\mathbb{R}, x \geq 4\}$ or $D: [4, \infty)$

3) Domain may be restricted due by physical or other stated constraints:

Example: A soda can is cylindrical with height 4 times its radius. $h = 4r$



Express the volume of the can as a function of the radius:

$V = \pi r^2 h = \pi r^2 (4r)$

$V(r) = 4\pi r^3$

What is the domain of V(r)?

$D: (0, \infty)$ can't have zero or negative radius

If we add the constraint that the maximum height of the can is 6 inches, what is the domain of V(r)?

$h = 4r$
$6 = 4r$
$\frac{3}{2} = r$

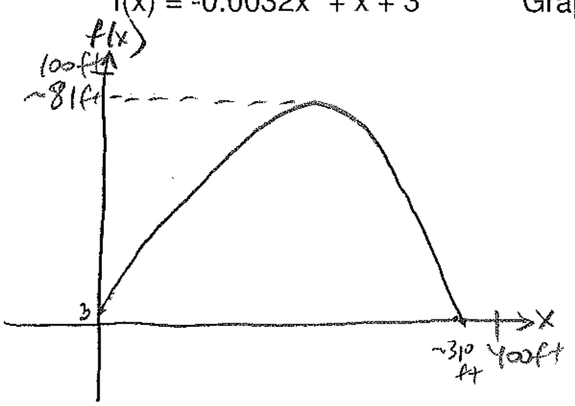
$D: (0, \frac{3}{2}]$

Range of a function = possible values of the output (y)

Plug in all possible input values; the range is the set of all output values that result.

Example: A baseball is hit at a point 3 ft above the ground at a velocity of 100 fps at an angle of 45 deg. The path (height of the ball at distance x) is given by:

$f(x) = -0.0032x^2 + x + 3$ Graph to find the domain and range



$D: [0, 310]$

$R: [0, 81]$

Steps for basic graphing on a TI-83 Plus or equivalent:

1) Enter the equation to graph:
   a. Press the 'y=' key.
   b. On the Y1= line, enter the equation. For the variable x, use the 'X,T,Q,n' key and press 'enter' when done.
2) Set the window:
   a. Press the 'window' key.
   b. Set Xmin, Xmax, Xscl (scale), Ymin, Ymax, Yscl to appropriate values (to graph the baseball example, try:
      i. Xmin = 0
      ii. Xmax=400
      iii. Xscl=10
      iv. Ymin=0
      v. Ymax=100
      vi. Yscl=10
      vii. Xres (usually leave set at 1)
3) Graph:
   a. Press the 'graph' key. The graph should appear.
4) Trace to read points on graph:
   a. Press the 'trace' key. A point should appear along with x and y values at the bottom of the screen.
   b. Move the trace point using the left, right arrow keys.

Another program you can use to graph is called GeoGebra. It is a freeware program similar to Geometer's Sketchpad. You can download and install free at www.geogebra.org (requires an up to date Java Runtime Environment.)

Difference Quotient: A function that is used frequently in Calculus.

$f(1) = -2(1)+4$
$f(23) = -2(23)+4$
$f(x) = -2(x)+4$
$f(x+c) = -2(x+c)+4$

$$\text{Difference Quotient} = \frac{f(x+h)-f(x)}{h}$$

Example: Find the difference quotient and simplify:

$$f(x) = -2x+4 \qquad \frac{f(x+c)-f(x)}{c}, c \neq 0$$

$$\frac{[-2(x+c)+4] - [-2x+4]}{c} = \frac{\cancel{-2x} - 2c \cancel{+4} \cancel{+2x} \cancel{-4}}{c} = \frac{-2c}{c} = \boxed{-2}$$

Example: Find the difference quotient and simplify:

$$g(x) = x^2+1 \quad \frac{g(x+h)-g(x)}{h}, h \neq 0$$

$$\frac{[(x+h)^2+1] - [x^2+1]}{h} = \frac{x^2+2xh+h^2+1-x^2-1}{h} = \frac{2xh+h^2}{h} = \boxed{2x+h}$$

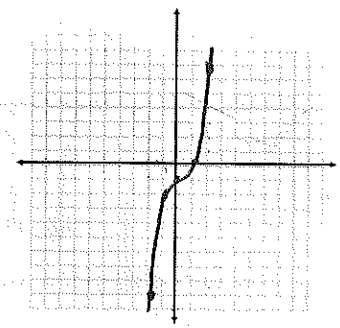
HAlg3-4, 1.2 (~~day1~~) Notes – Graphs of Functions

A graph of a function is a 2-D plot with inputs, x, on x-axis and outputs, f(x), on y-axis.

Point plotting method: make a table and select some input values, compute corresponding output values and plot (x,y) pairs.

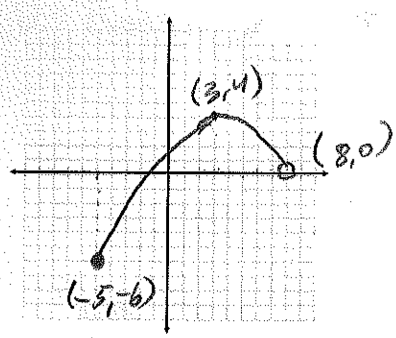
Example: graph $f(x) = x^3 - 1$:

x	f(x)
-2	-9
-1	-2
0	-1
1	0
2	7



Domain and range of a function: From a graph of a function, you can determine the domain and range. Remember, sometimes domain (x) is restricted by problem (no negative lengths) or by function equation (no even roots of negative numbers, no zero denominators.)

Example:



D: $[-5, 8)$

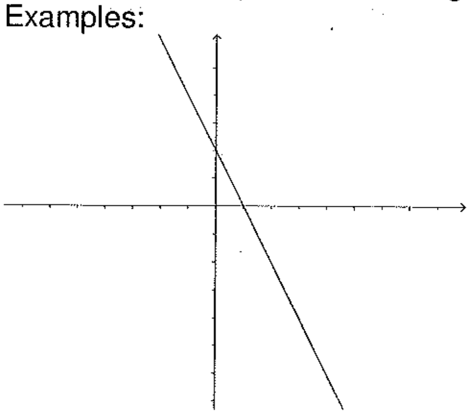
R: $[-6, 4]$

Increasing , decreasing and constant functions: *Moving from left to right:*

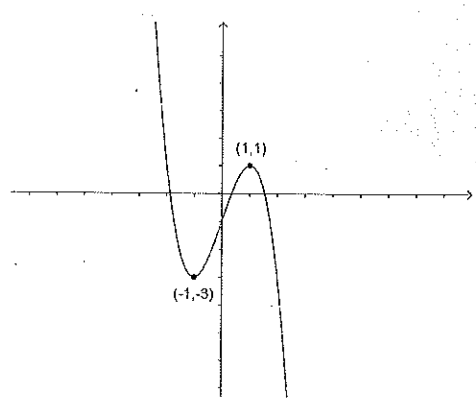
If the graph rises, it is **increasing**
If the graph falls, it is **decreasing**
If the graph does not change, it is **constant**

A graph can be increasing, decreasing or constant for its entire domain, or it can have regions that are different (some increasing, some decreasing, etc.)

Examples:



decreasing x: $(-\infty, \infty)$



decreasing x: $(-\infty, -1) \cup (1, \infty)$

increasing x: $(-1, 1)$

HAlg3-4, 1.2 (~~day 2~~) Notes – Graphs of Functions

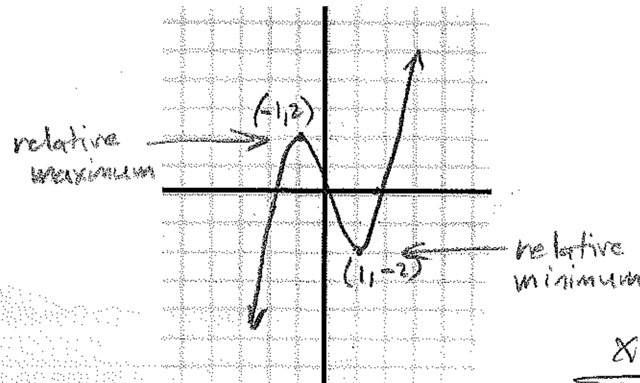
Relative Minimum and Maximum:

Functions may have regions where they are increasing or decreasing.

Relative Minimum = a point where function changes from decreasing to increasing.

Relative Maximum = a point where function changes from increasing to decreasing.

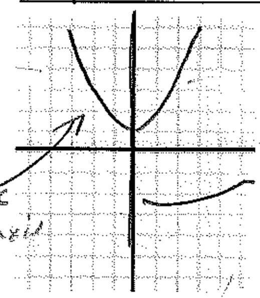
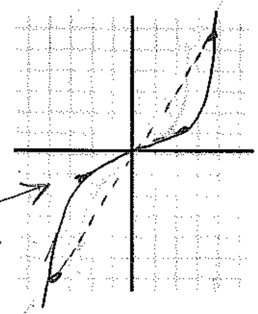
Example: $f(x) = x^3 - 3x$



day 2

Even / Odd Functions:

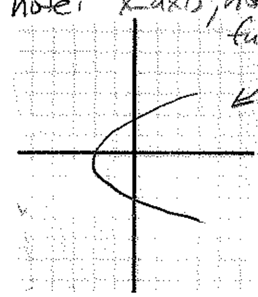
x	x^2
-2	4
-1	1
0	0
1	1
2	4

	Algebraic definition	Graphical meaning
Even function	$f(-x) = f(x)$	
Odd function	$f(-x) = -f(x)$	

axis of symmetry $x=0$

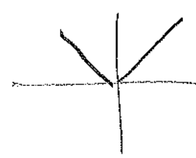
x	x^3
-2	-8
-1	-1
0	0
1	1
2	8

note: symmetric about x-axis, not a function



Examples: Determine if each function is even, odd, or neither.

$f(x) = |x|$ $\quad f(-x) = |-x| = |x| = f(x)$ even

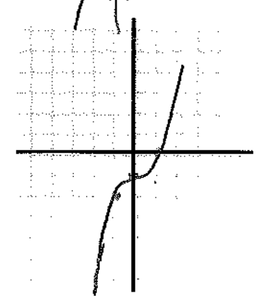


$f(x) = x^3 - x$ $\quad f(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x) = -f(x)$ odd

$f(x) = x^3 - 1$ $\quad f(-x) = (-x)^3 - 1 = -x^3 - 1 = -(x^3 + 1)$

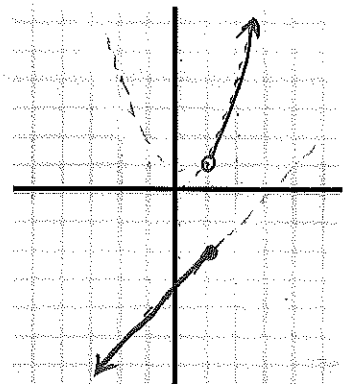
$\neq f(x)$

$\neq -f(x)$ neither



Piece-wise defined functions:

$$f(x)=\begin{cases}x^2, & x>1\\ x-3, & x\le 1\end{cases}$$



Greatest Integer Function: (round down to nearest integer)

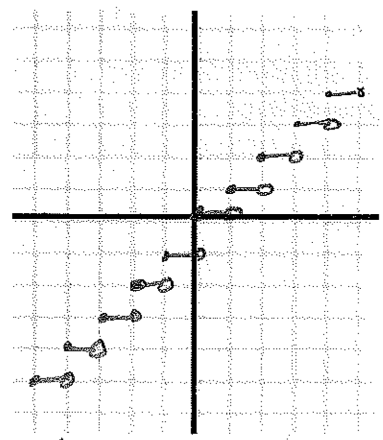
$f(x)=[\![x]\!]$ the greatest integer less than or equal to x

$f(0)=[\![0]\!]=0$

$f(0.5)=[\![0.5]\!]=0$

$f(0.9999)=[\![0.9999]\!]=0$

$f(1)=[\![1]\!]=1$



"step function"

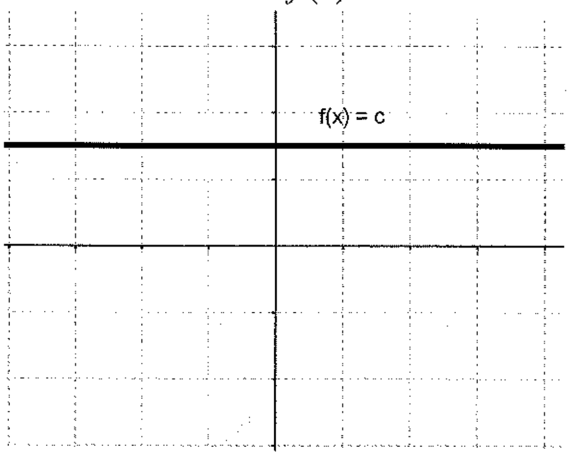
HAlg3-4, 1.3 Notes – Shifting, Reflecting and Stretching Graphs

Sketching functions by hand is made easier when you know:

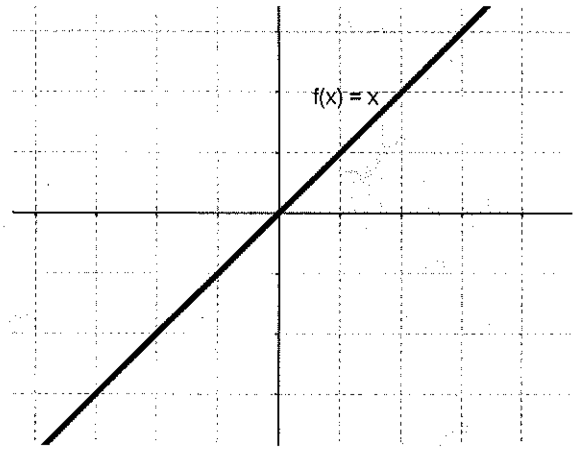
- The shapes of common functions.
- The ways these shapes are transformed by common adjustments to equations.

Basic shapes of common functions:

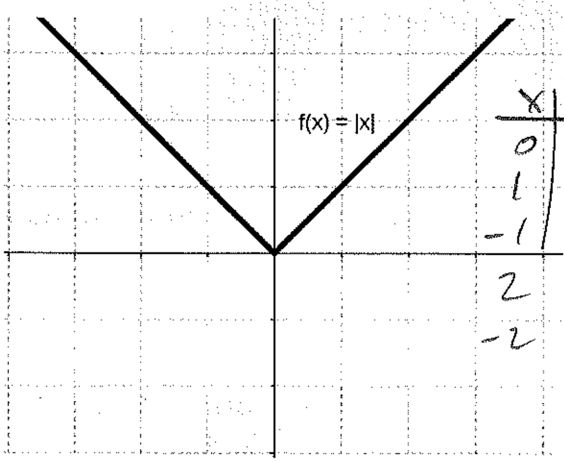
Constant function: $f(x) = c$



Identity function: $f(x) = x$



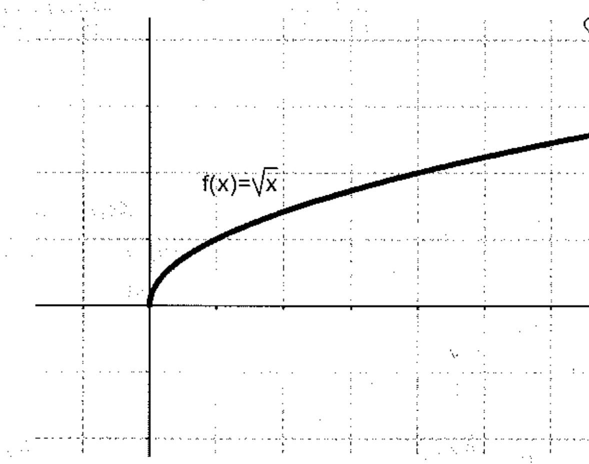
Absolute Value function: $f(x) = |x|$



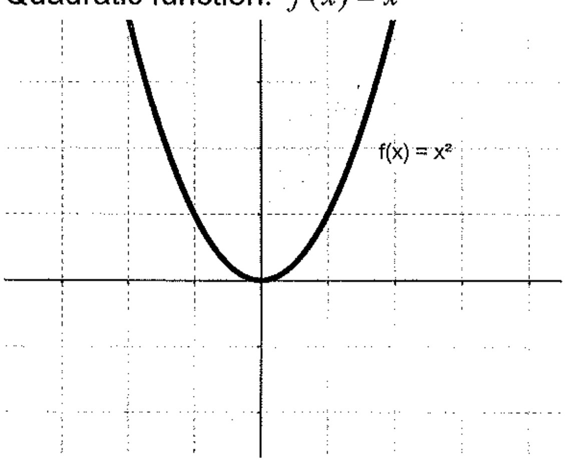
x	$\lvert x \rvert$
0	0
1	1
−1	1
2	2
−2	2

Square Root function: $f(x) = \sqrt{x}$

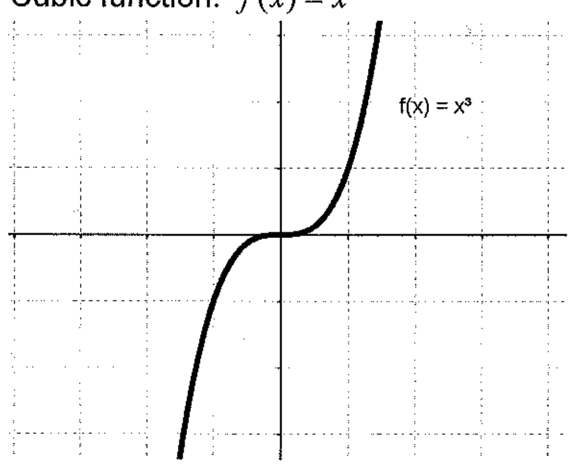
x	$\sqrt{x}$
0	0
1	1
4	2



Quadratic function: $f(x) = x^2$



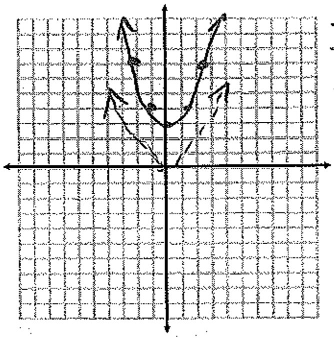
Cubic function: $f(x) = x^3$



Common Transformations:
Rigid Transformations…

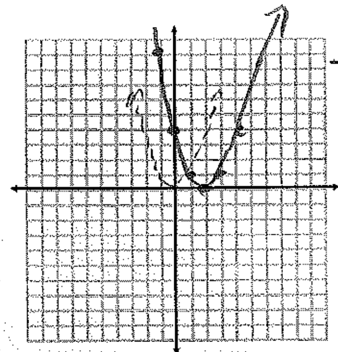
Sketch the functions

$f(x)=x^2+3$



x	x^2+3
-2	7
-1	4
0	3
1	4
2	7

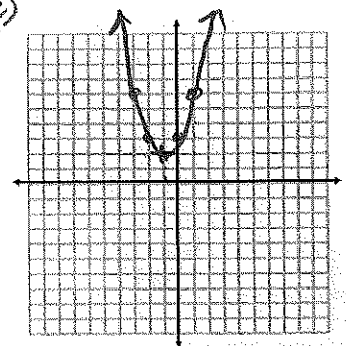
$f(x)=(x-2)^2$



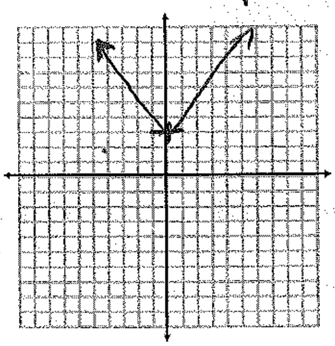
x	$(x-2)^2$
-1	9
0	4
1	1
2	0
3	1
4	4

left 1 up 2

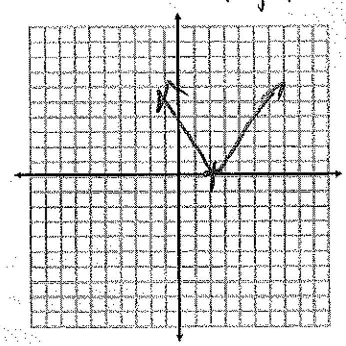
$f(x)=(x+1)^2+2$ $\quad y=x^2$



$f(x)=|x|+3$ up 3

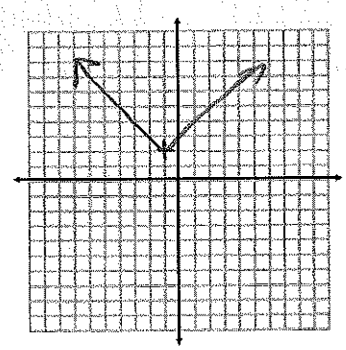


$f(x)=|x-2|$ right 2



left 1

$f(x)=|x+1|+2$ up 2

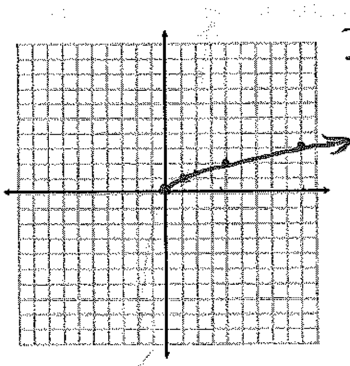


Vertical Shift:
$h(x)=f(x)+c$ (up)
$h(x)=f(x)-c$ (down)

Horizontal Shift:
$h(x)=f(x-c)$ (right)
$h(x)=f(x+c)$ (left)

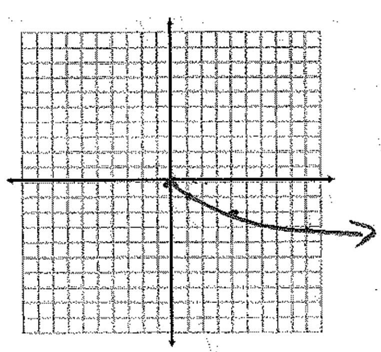
Examples: Sketch the functions

$f(x)=\sqrt{x}$

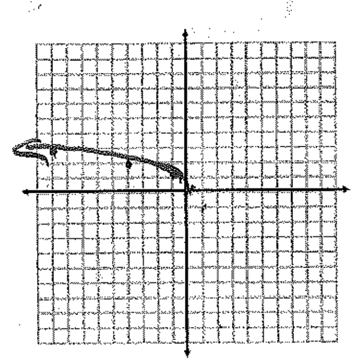


x	$\sqrt{x}$
-1	und
0	0
1	1
2	1.414
4	2
9	3

$f(x)=-\sqrt{x}$



$f(x)=\sqrt{-x}$



Vertical Reflection:
$h(x)=-f(x)$

Horizontal Reflection:
$h(x)=f(-x)$

To sketch:

1) identify the basic shape of the curve
2) identify the shifted origin
3) sketch the basic shape from the shifted origin (plug in points near origin)

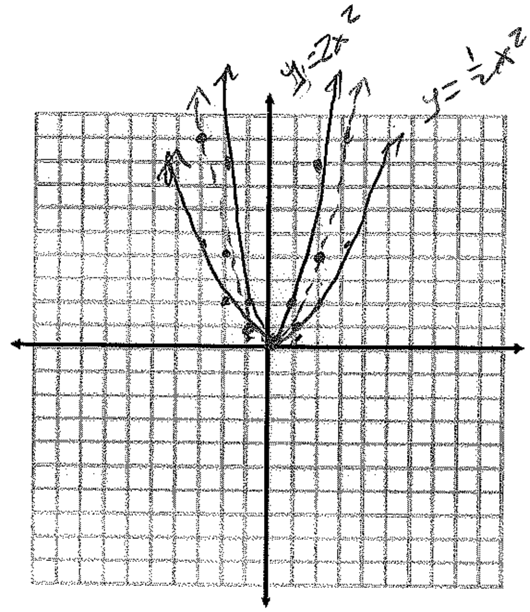
Non-Rigid Transformations...

Vertical Stretch:

$h(x) = cf(x), \quad c > 1$

$f(x) = 2x^2$

$f(x) = \frac{1}{2}x^2$



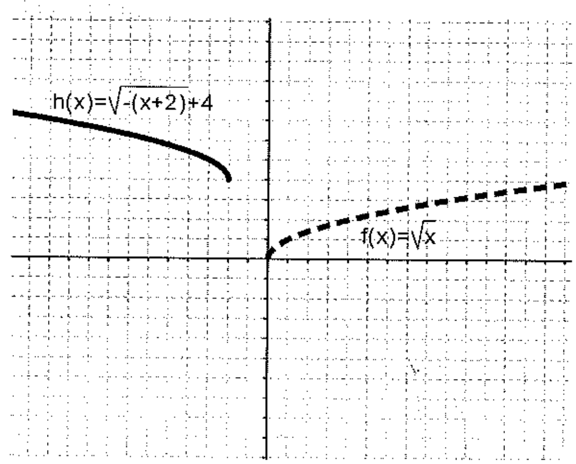
Vertical Shrink:

$h(x) = cf(x), \quad 0 < c < 1$

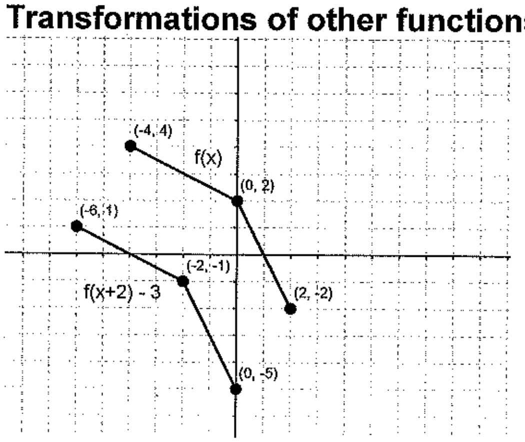
Sequences of Transformations:

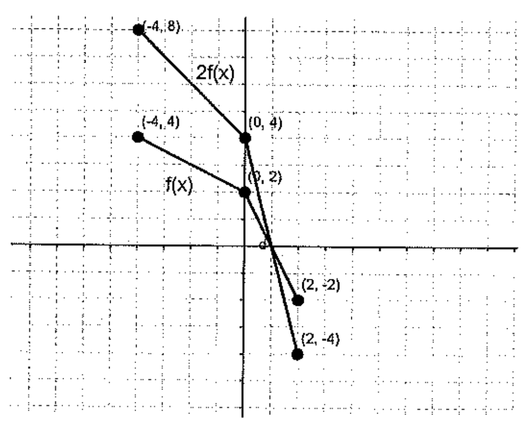
Multiple transformations can be applied. When sketching, start with basic function and apply transformations in order of operations.

Example: Sketch $h(x) = \sqrt{-(x+2)} + 4$



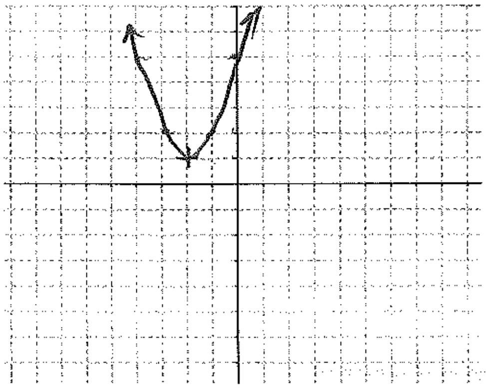
Transformations of other functions:



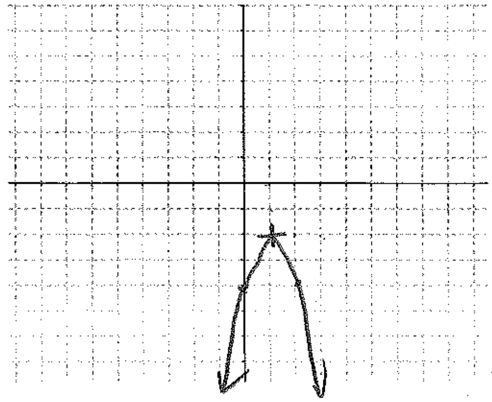


Examples to try….sketch the functions:

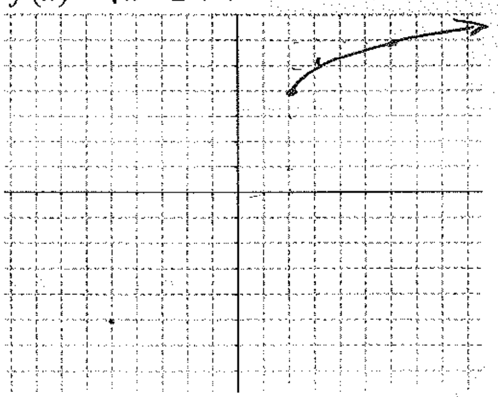
$f(x) = (x+2)^2 + 1$



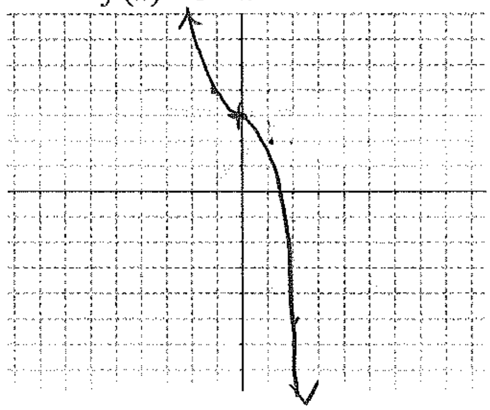
$f(x) = -2(x-1)^2 - 2$



$f(x) = \sqrt{x-2} + 4$



$f(x) = 3 - x^3$



HAlg3-4, 1.4 (day1) Notes – Combinations of Functions

Given two functions, you can provide both with the same input and combine the output values from each function in various ways to create a new function. There are two general ways function outputs are combined: arithmetic combination and composition of functions.

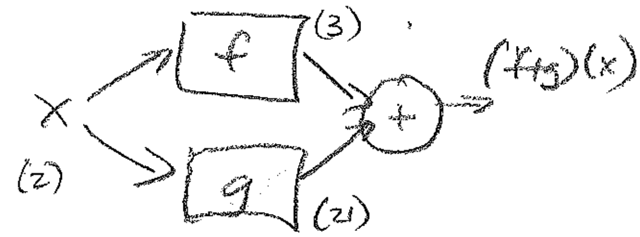
Arithmetic Combination of Functions: Add, subtract, multiply or divide the outputs of the functions. Special notation for these combinations:

Addition: $(f+g)(x)=f(x)+g(x)$

Subtraction: $(f-g)(x)=f(x)-g(x)$

Multiplication: $(fg)(x)=f(x)\bullet g(x)$

Division: $\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)},\quad g(x)\neq 0$



Example: $f(x)=x+1$ $\qquad g(x)=x^2+6x+5$

$(f+g)(x)=f(x)+g(x)=x+1+x^2+6x+5=x^2+7x+6$

There are 2 ways to evaluate this, say, at x=2:

1) Plug 2 into each function first, then add results:

$f(2)=(2)+1=3$

$g(2)=(2)^2+6(2)+5=21$

$(f+g)(2)=f(2)+g(2)$

$=3+21$

$=\boxed{24}$

2) Plug 2 into the generic combination function:

$f(x)=x+1$

$g(x)=x^2+6x+5$

$(f+g)(x)=x^2+7x+6$

$(f+g)(2)=(2)^2+7(2)+6$

$=4+14+6$

$=24$

Note: this is not multiplication...this is function notation with input of x (plug x into equation)

Example: $f(x)=x+1$ $\qquad g(x)=x^2+6x+5$

$\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}=\dfrac{x+1}{x^2+6x+5}=\dfrac{\cancel{(x+1)}\,1}{\cancel{(x+1)}(x+5)}=\boxed{\dfrac{1}{x+5}}$

$\left(\dfrac{f}{g}\right)(2)=\dfrac{f(2)}{g(2)}=\dfrac{(2)+1}{(2)^2+6(2)+5}=\dfrac{3}{21}=\boxed{\dfrac{1}{7}}$

or

$\dfrac{1}{(2)+5}=\boxed{\dfrac{1}{7}}$

Domain of arithmetic combination of functions = Inputs that are in the domain of $\underline{\text{both}}$ functions (intersection)

Example: $f(x)=\sqrt{x}$ $\quad$ $g(x)=\sqrt{4-x^2}$ $\quad$ find $\left(\dfrac{f}{g}\right)(x)$, and its domain.

D: $[0,\infty)$

$\underbrace{4-x^2}_{\geq 0}$

$4-x^2 \geq 0$

$-4 \quad\quad -4$

$-x^2 \geq -4$

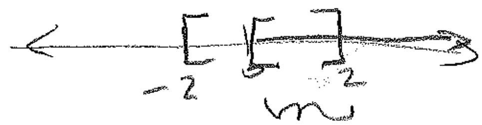
$x^2 \leq 4$

$-4 \leq x^2 \leq 4$

$-2 \leq x \leq 2$

$[-2,2]$

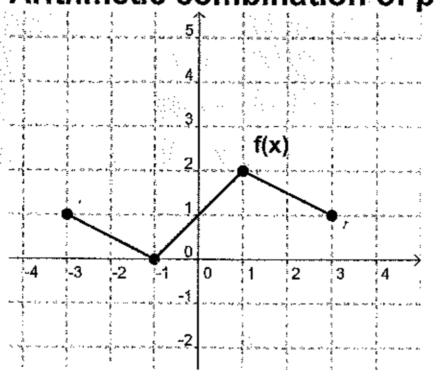
$\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)}=\boxed{\dfrac{\sqrt{x}}{\sqrt{4-x^2}}}$

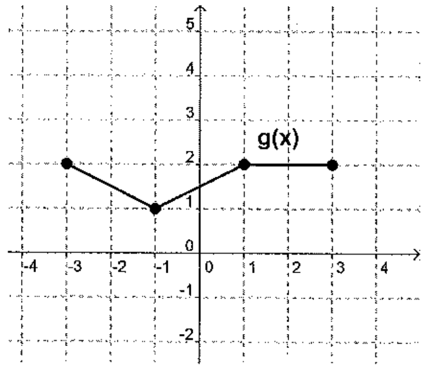


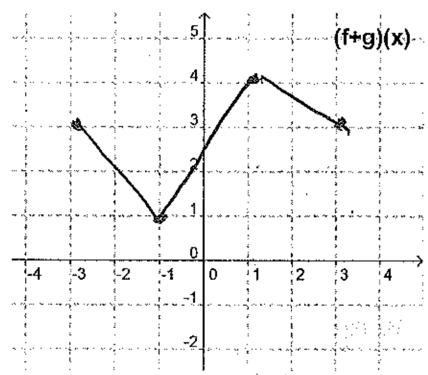
$\boxed{D: [0,2)}$

open because zero in denom.

Arithmetic combination of piece-wise defined functions:







HAlg3-4, 1.4 (day2) Notes – Combinations of Functions

Arithmetic combination of functions…

$$\begin{array}{l} x \nearrow \boxed{f} \to f(x) \searrow \\ \qquad\qquad\qquad\qquad \boxed{(f+g)} \to f(x)+g(x) \\ x \searrow \boxed{g} \to g(x) \nearrow \end{array}$$

Composition of functions…

$$x \to \boxed{g} \to g(x) \to \boxed{f} \to f(g(x))$$ ← composite function

$f(g(x)) = (f \circ g)(x)$ "f of g of x" "composition of f with g"

$h(a(x)) = (h \circ a)(x)$ "h of a of x" "composition of h with a"

Examples: $f(x) = x^2$ $g(x) = x+1$ $(f \circ g)(3) = f(g(3)) = f(4) = (4)^2 = 16$

$g(3) = 3+1 = 4$

$$3 \to \boxed{g(x) = x+1} \to g(3) = 4 \to \boxed{f(x) = x^2} \to f(4) = 16$$

$f(x) = x^2$ $g(x) = x+1$ $(f \circ g)(x) = f(g(x)) = f(x+1) = (x+1)^2 = x^2+2x+2$

$f(x) = x^2$ $g(x) = x+1$ $(g \circ f)(x) = g(f(x)) = g(x^2) = (x^2)+1 = x^2+1$

Note: in general, $(f \circ g)(x) \neq (g \circ f)(x)$

Try it...

$f(x) = x^3$ $g(x) = 2x-1$ $(f \circ g)(2) = f(g(2)) = f(3) = (3)^3 = 27$

$g(2) = 2(2)-1$

$f(x) = \dfrac{1}{x}$ $g(x) = x+1$ $(f \circ g)(x) = f(g(x)) = f(x+1) = \dfrac{1}{x+1}$

Identifying Composite Functions

Example: $h(x) = \dfrac{1}{(x-2)^2} = f(g(x))$ 'outside' 'inside'

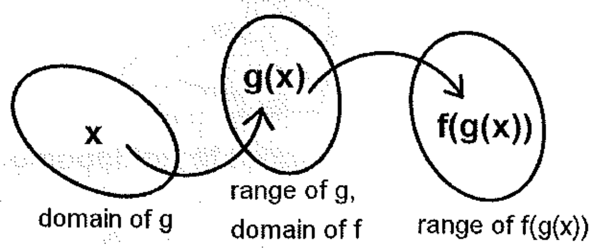
$\dfrac{1}{\boxed{\ }^2}$ outside function $f(x) = \dfrac{1}{x^2}$

$x-2$ inside function $g(x) = x-2$

– or – $\dfrac{1}{\boxed{\ }}$ $f(x) = \dfrac{1}{x}$

$(x-2)^2$ $g(x) = (x-2)^2$

Finding the domain of a composite function



To determine the domain of a composite function, you need to restrict the outputs of g so that they are in the domain of f.

Example: $g(x) = x + 1$ $\quad f(x) = \dfrac{1}{x}$ $\quad (f \circ g)(x) = \dfrac{1}{x+1}$

D: $\mathbb{R}$ $\quad$ R: $\mathbb{R}$ $\quad$ D: $\mathbb{R}$, $x \neq 0$

$x \rightarrow \boxed{g} \xrightarrow{x+1} \boxed{f}$

input $\neq 0$

$x+1 \neq 0$

$x+1 \neq 0$
$-1 \quad -1$
$x \neq -1$

$x \neq -1$

can see this from final equation

Domain of $(f \circ g)$: $\mathbb{R}$, $x \neq -1$

Example: $g(x) = \sqrt{9 - x^2}$ $\quad f(x) = x^2 - 9$ $\quad (f \circ g)(x) = \left(\sqrt{9 - x^2}\right)^2 - 9 = 9 - x^2 - 9 = -x^2$

D: $[-3, 3]$ $\quad$ R: $[0, 3]$ $\quad$ D: $\mathbb{R}$

$x \rightarrow \boxed{g} \xrightarrow{\sqrt{9-x^2}} \boxed{f}$

input can accept all real values

not apparent from final equation

$9 - x^2 \geq 0$

$-x^2 \geq -9$

$x^2 \leq 9$

$-9 \leq x^2 \leq 9$

$-3 \leq x \leq 3$

$[-3, 3]$

Domain of $(f \circ g)$: $[-3, 3]$

(restricted by 1st function)

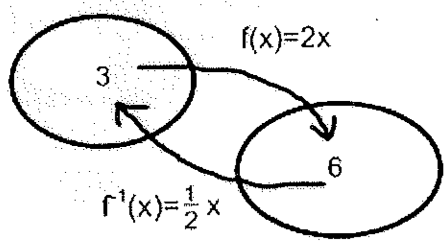
calc check: graph $Y_1 = -x^2$ (zoom out)

$Y_1 = \left(\sqrt{9-x^2}\right)^2 - 9$ (zoom out)

Practical example of composite functions (example #10 in book – read it)

HAlg3-4, 1.5 (day1) Notes – Inverse Functions

Inverse of a function:



Inverse functions 'undo' each other

Notation: inverse of $f(x)$ is $f^{-1}(x)$

Note: $f^{-1}(x) \neq \dfrac{1}{f(x)}$ $\quad 2^{-1} = \frac{1}{2}$, $x^{-1} = \frac{1}{x}$, $(f(x))^{-1} = \frac{1}{f(x)}$, but $f^{-1}(x) =$ inverse of f

Finding an inverse function:

1) Replace f(x) with y
2) Swap x and y
3) Solve for y

Example: $f(x) = x + 6$ find $f^{-1}(x)$

$y = x + 6$
$x = y + 6$
$-6 \quad -6$
$x - 6 = y \qquad f^{-1}(x) = x - 6$

Verifying two functions are inverses:

Put one function in as the input to the other function and simplify. Since they 'undo' each other, the result should be just the input, x: $\quad f(f^{-1}(x)) = x$

Example: verify $f(x) = 2x + 3$ and $f^{-1}(x) = \dfrac{1}{2}x - \dfrac{3}{2}$ are inverses.

$f(f^{-1}(x)) = f\left(\frac{1}{2}x - \frac{3}{2}\right)$
$= 2\left(\frac{1}{2}x - \frac{3}{2}\right) + 3$
$= x - 3 + 3$
$= x$ ✓

Practice problems:

#1 Find the inverse of $f(x) = \dfrac{5}{x-2}$

$y = \frac{5}{x-2}$
$x = \frac{5}{y-2}$
$(y-2)x = 5$
$xy - 2x = 5$
$xy = 5 + 2x$
$y = \frac{5}{x} + 2 \qquad \boxed{f^{-1}(x) = \frac{5}{x} + 2}$

#2. Find the inverse of $f(x) = 2(x-8)$

$y = 2x - 16$
$x = 2y - 16$
$x + 16 = 2y$
$y = \frac{1}{2}x + 8 \qquad \boxed{f^{-1}(x) = \frac{1}{2}x + 8}$

#3. Verify if the following are inverses:

$f(x) = 3x + 12$

$f^{-1}(x) = \dfrac{1}{3}x - 4$

$f(f^{-1}(x)) = 3\left(\frac{1}{3}x - 4\right) + 12$
$= x - 12 + 12$
$= x$ ✓

#4. Verify if the following are inverses:

$f(x) = 2x - 6$

$f^{-1}(x) = 2x + 3$

$f(f^{-1}(x)) = f(2x+3)$
$= 2(2x+3) - 6$
$= 4x + 6 - 6$
$= 4x$ ✗

not inverses

Graphs of inverses:

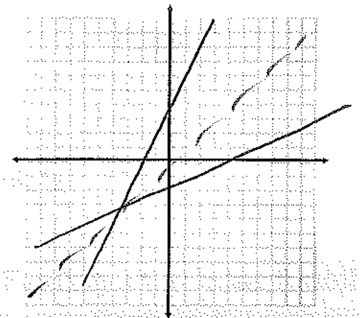
Use a calculator to graph the following functions and their inverses:

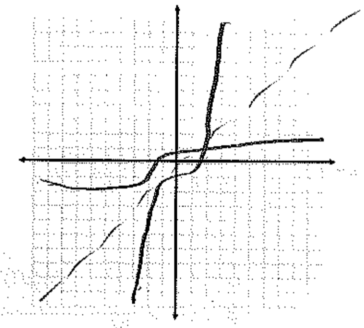
$f(x)=2x+3$

$f^{-1}(x)=\frac{1}{2}x-\frac{3}{2}$

$f(x)=2x^3-1$

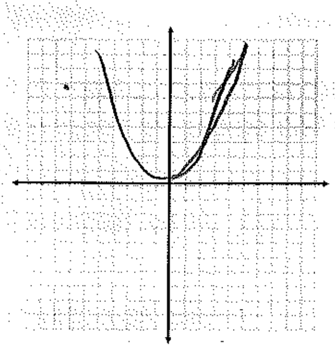
$f^{-1}(x)=\sqrt[3]{\frac{x+1}{2}}=\left(\frac{x+1}{2}\right)^{\left(\frac{1}{3}\right)}$

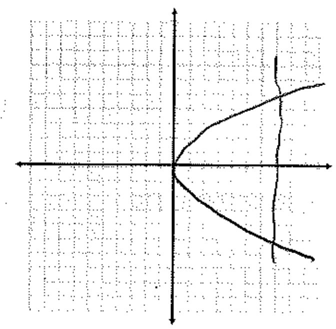




The graphs of functions and their inverses are: reflected over line y=x

Can we find the inverse of $f(x)=x^2$?

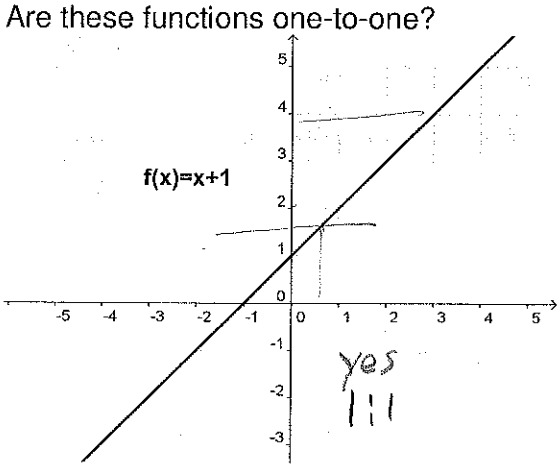




not a function

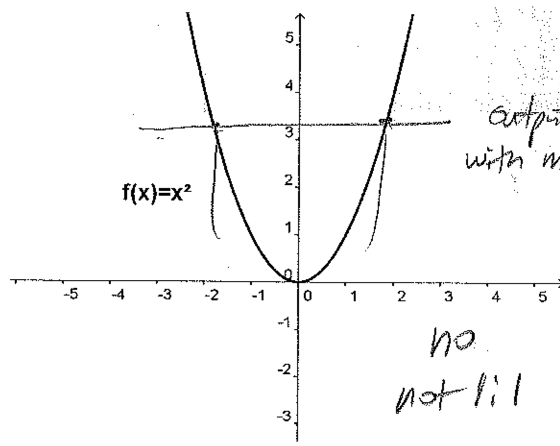
Not all functions have inverses. To have an inverse, a function must be **one-to-one**, which means **every output corresponds to exactly one input.**

Are these functions one-to-one?



yes

1:1



output with multiple inputs

no

not 1:1

To be one-to-one, a function must pass the **horizontal line test.**

HAlg3-4, 1.5 (day2) Notes – Inverse Functions

Existence of an inverse

Not all functions have inverses. To have an inverse, the function must be **one-to-one, having only one input for every output.**

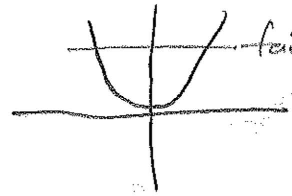
Check for multiple inputs with same output, or use horizontal line test to determine if a function is one-to-one and has an inverse.

Examples:

$f = \{(-1,3),(0,4),(1,5),(2,3),(3,6)\}$

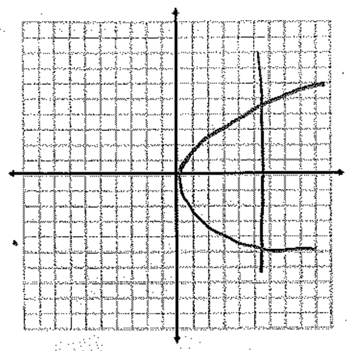
output 3 has two inputs (-1, and 2)
not 1:1 (no inverse)

$f(x) = x^2$



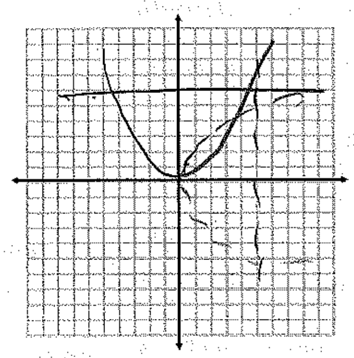
fails horiz. line test
not 1:1
(no inverse)

Horizontal and Vertical line tests



Vertical line test

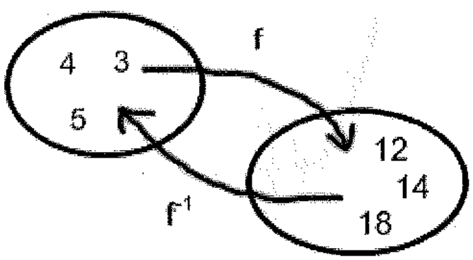
test whether is a function



Horizontal line test

tests whether has an inverse (1:1)
is "invertable"

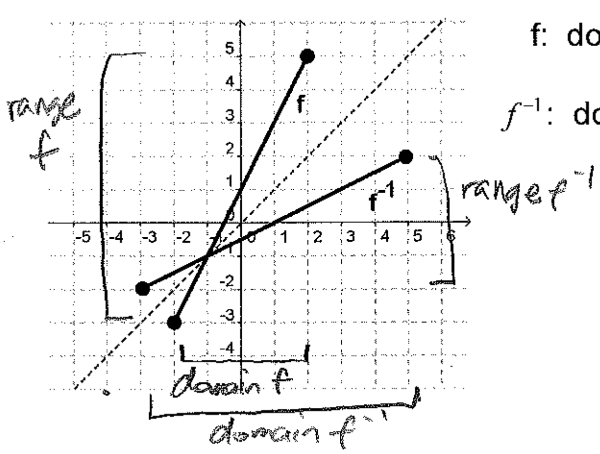
Domain and range of inverse functions



<u>function f:</u>
domain: $\{3,4,5\}$
range: $\{12,14,18\}$

<u>inverse f^{-1}:</u>
domain: $\{12,14,18\}$
range: $\{3,4,5\}$

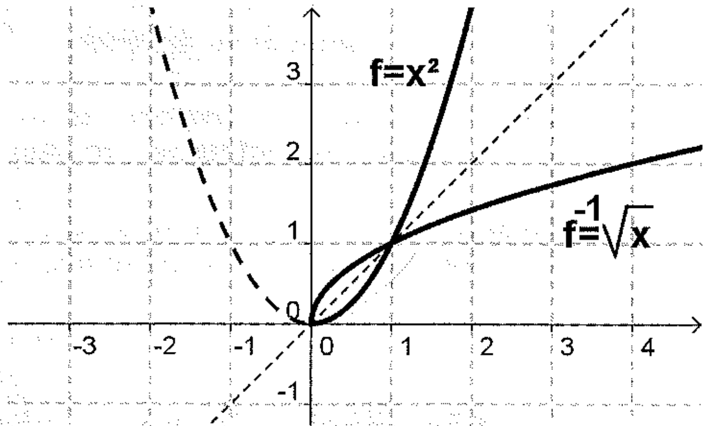
domains and ranges of function and its inverse are swapped.



f: domain: [-2, 2] range: [-3, 5]

f^{-1}: domain [-3, 5], range: [-2, 2]

Sometimes, you can find an inverse for a function that isn't one-to-one by restricting the domain:



inverse of $f(x)$

$y = x^2$

$x = y^2$

$y^2 = x$

$y = \pm\sqrt{x}$

↑ not a function

but if we restrict domain of $f(x)$ to $[0,\infty)$

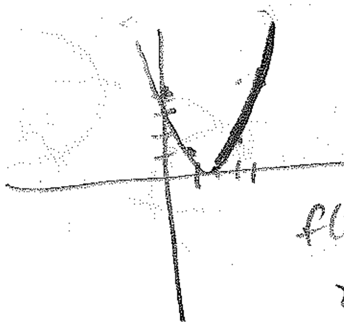
range of $f(x)$ is $[0,\infty)$

$y = \sqrt{x}$ only positive values

Hw example: #69

Delete part of the graph of the function so that the part remaining is 1:1, Find the inverse of the remaining part and domain of the inverse:

$f(x) = (x-2)^2$



$f(x) = (x-2)^2,\ x \geq 2$

$x = (y-2)^2$

$\pm\sqrt{x} = y-2$

$y = 2 \oplus \sqrt{x}$

$f^{-1}(x) = 2+\sqrt{x},\ x \geq 0$

	f	f^{-1}
D:	$[2,\infty)$	$[0,\infty)$
R:	$[0,\infty)$	$[2,\infty)$