

HAlg3-4, 10.1 Notes – Conic Sections and Parabolas

Conic Sections – curves formed by intersection of a double-napped cone with a plane.

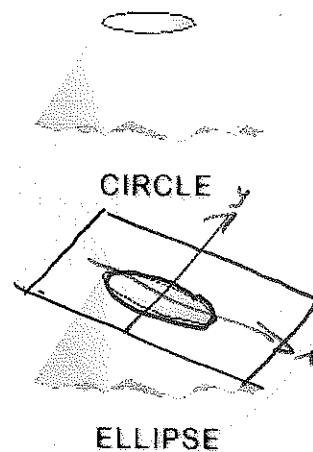
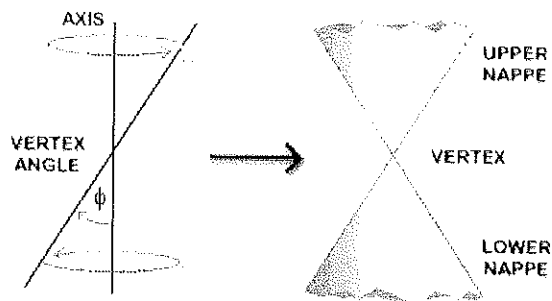
The shape of the intersection curve depends upon the angle the plane makes with the vertical axis of the cone:

Circle - if the plane is perpendicular to the cone axis, the curve is a circle.

Ellipse – if the plane's angle is greater than the vertex angle, but not perpendicular to cone axis, the curve is an ellipse.

Parabola – if the plane's angle matches the vertex angle, the curve is a parabola.

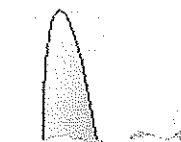
Hyperbola – if the plane's angle is smaller than the vertex angle, the curve cuts through both nappes and is a hyperbola.



ELLIPSE



PARABOLA

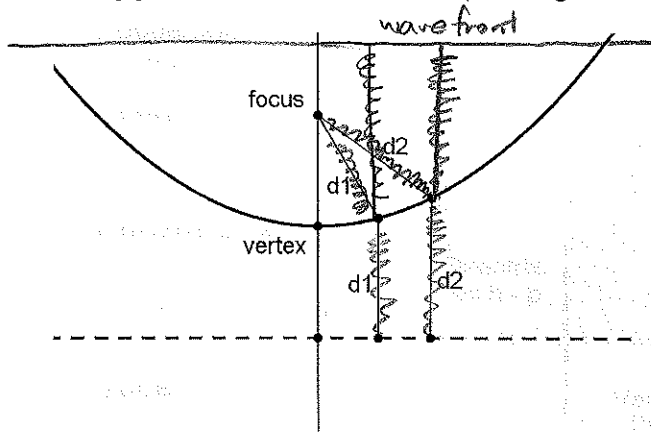


HYPERBOLA

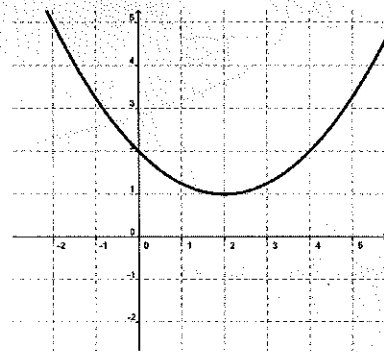
Conic Sections are defined by the general equation: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
 ...but each curve has a standard form that makes it easier to sketch.

Parabola = The set of all points (x,y) that are equidistant from a fixed line (called the directrix) and a fixed point (called the focus).

Applications – reflectors (flashlight, antenna dishes)



Let's look at an example: $y = \frac{1}{4}x^2 - x + 2$



How could we graph this by hand? Easier to graph if we change the equation into the standard form equation for a parabola:

$$y = \frac{1}{4}x^2 - x + 2$$

$$\frac{1}{4}x^2 - x = y - 2 \quad (\text{x terms on left, everything else on rt})$$

$$x^2 - 4x = 4y - 8 \quad (\text{make } x^2 \text{ coefficient 1})$$

$$x^2 - 4x + \underline{4} = 4y - 8 + \underline{4} \quad (\text{complete the square})$$

$$(x-2)^2 = 4y - 4$$

$$(x-2)^2 = 4(y-1) \quad (\text{factor right side})$$

vertex: $(2, 1)$

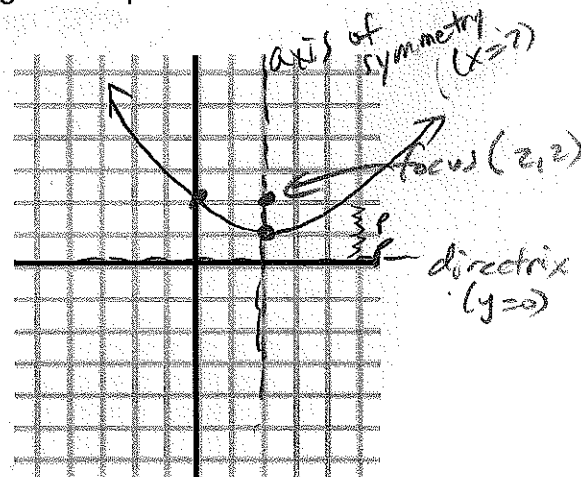
To find another point find an intercept

$$y\text{-int } (x=0) \quad (0-2)^2 = 4(y-1)$$

$$(-2)^2 = 4(y-1)$$

$$4 = 4(y-1)$$

$$1 = y-1 \quad y=2 \quad (0, 2)$$



$$(x-2)^2 = 4(y-1)$$

↑
this number is $4p$

Standard form equation of a parabola:

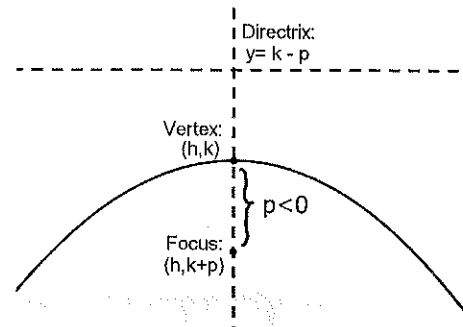
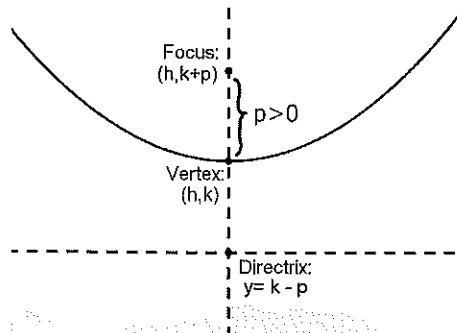
Vertical axis:

$$(x-h)^2 = 4p(y-k)$$

Vertex: (h, k)

Focus: $(h, k+p)$

Directrix: $y = k - p$



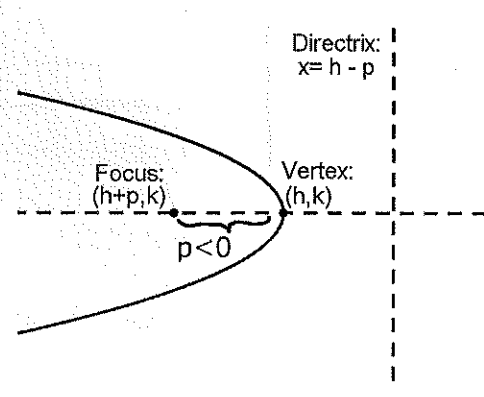
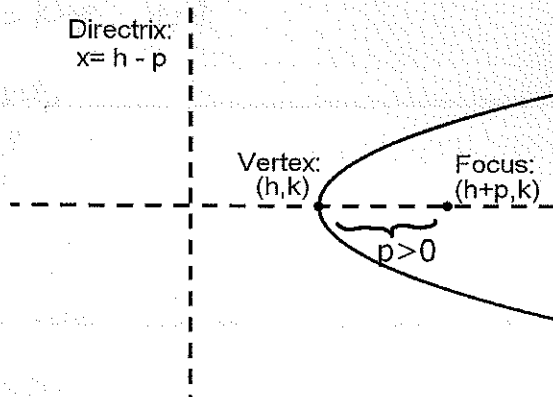
Horizontal axis:

$$(y-k)^2 = 4p(x-h)$$

Vertex: (h, k)

Focus: $(h+p, k)$

Directrix: $x = h - p$



For all parabolas: p = distance between focus and vertex (and vertex to directrix)

To sketch, plug in $x=0$ or $y=0$ to find an intercept.
(Curve is symmetric about its axis)

(parabolas don't have asymptotes)

Example: Find the vertex, focus, directrix and sketch the graph of $(x+1)^2 = 4(y-2)$

$$(x+1)^2 = 4(y-2)$$

$$(x-h)^2 = 4p(y-k)$$

$$h = -1$$

$$k = 2$$

$$4p = 4, p = 1$$

$$\text{vertex: } (h, k) = (-1, 2)$$

$$\text{focus: } (h, k+p) = (-1, 3)$$

$$\text{directrix: } y = k-p, y = 1$$

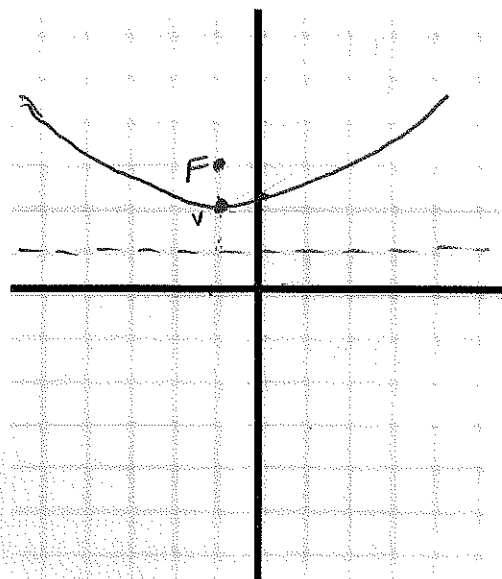
$$(0+1)^2 = 4(y-2)$$

$$1 = 4y - 8$$

$$\begin{array}{r} +8 \\ \hline \end{array} \quad \begin{array}{r} +8 \\ \hline \end{array}$$

$$9 = 4y$$

$$\frac{9}{4} = y \quad \left(0, \frac{9}{4}\right)$$



Example: Find the vertex, focus, directrix and sketch the graph of $y^2 = -2x$

$$y^2 = -2x$$

$$(y-k)^2 = 4p(x-h)$$

$$(y-k)^2 = 4p(x-h)$$

$$h = 0$$

$$k = 0$$

$$4p = -2, p = -\frac{1}{2}$$

$$\text{vertex } (h, k) = (0, 0)$$

$$\text{focus } (h+p, k) = \left(-\frac{1}{2}, 0\right)$$

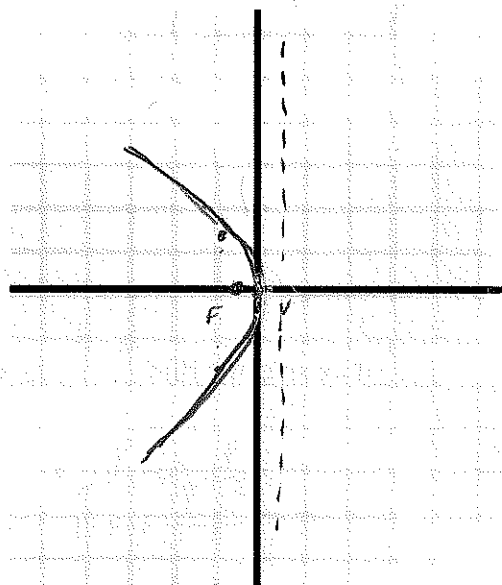
$$\text{directrix: } x = h-p, x = \frac{1}{2}$$

$$\text{plug in } x = -1$$

$$y^2 = -2(-1)$$

$$y^2 = 2$$

$$y = \pm\sqrt{2} \approx \pm 1.414$$



Example: Find the vertex, focus, directrix and sketch the graph of $x^2 - 2x + 8y + 9 = 0$

$$x^2 - 2x = -8y - 9$$

complete the square:

$$(x^2 - 2x + 1) - 1 = -8y - 9$$

$$(x-1)^2 - 1 = -8y - 9$$

$$(x-1)^2 = -8y - 8$$

$$(x-1)^2 = -8(y+1)$$

$$h=1$$

$$k=-1$$

$$4p = -8, p = -2$$

$$\text{vertex: } (h, k) (1, -1)$$

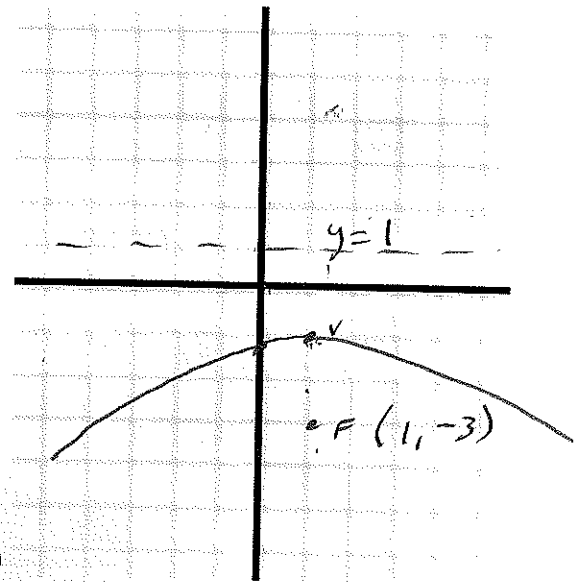
$$\text{focus: } (h, k+p) (1, -3)$$

$$\text{directrix: } y = k - p, y = 1$$

$$(-1)^2 = -8y - 8$$

$$1 = -8y - 8$$

$$9 = -8y \quad y = -\frac{9}{8} \left(0, -\frac{9}{8}\right)$$



Example: Find standard form equation of a parabola with vertex at $(-1, 2)$ and focus at $(-1, 0)$

$$(x-h)^2 = 4p(y-k)$$

$$(x+1)^2 = 4p(y-2)$$

$$(x+1)^2 = 4(-2)(y-2)$$

$$\boxed{(x+1)^2 = -8(y-2)}$$

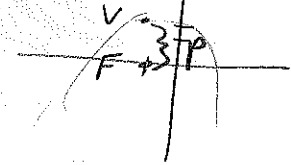
$$k=2$$

$$k+p=0$$

$$2+p=0$$

$$p=-2$$

h, k $h, k+p$



points down, $p < 0$

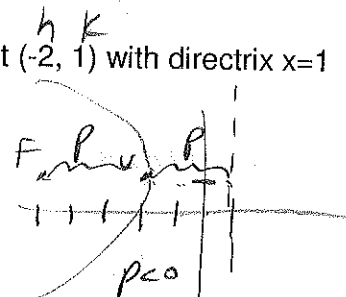
Example: Find standard form equation of a parabola with vertex at $(-2, 1)$ with directrix $x=1$

$$(y-k)^2 = 4p(x-h)$$

$$(y-1)^2 = 4p(x+2)$$

$$(y-1)^2 = 4(-3)(x+2)$$

$$\boxed{(y-1)^2 = -12(x+2)}$$



$p < 0$
 $p = -3$

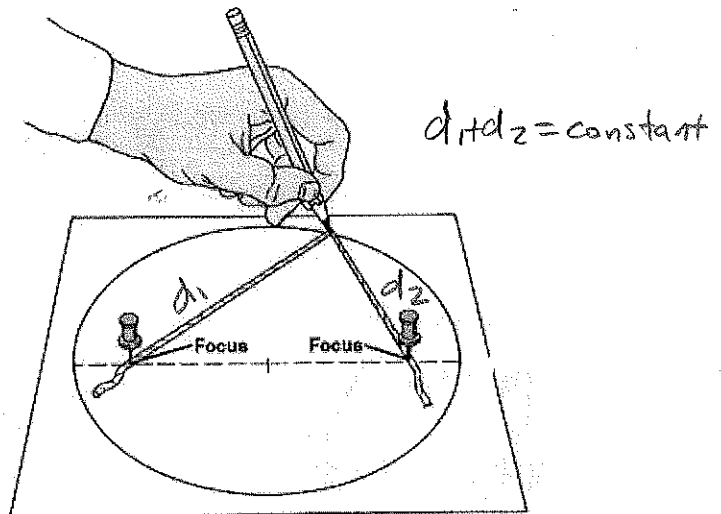
points left, $p < 0$

HA1g3-4, 10.2 Notes - Ellipses

Ellipse = the set of all points (x,y) the sum of whose distances from two fixed points (called foci) is constant.

real-life:

- orbits of planets, comets
- circles in perspective
- tanker truck cross section (lower center of gravity)



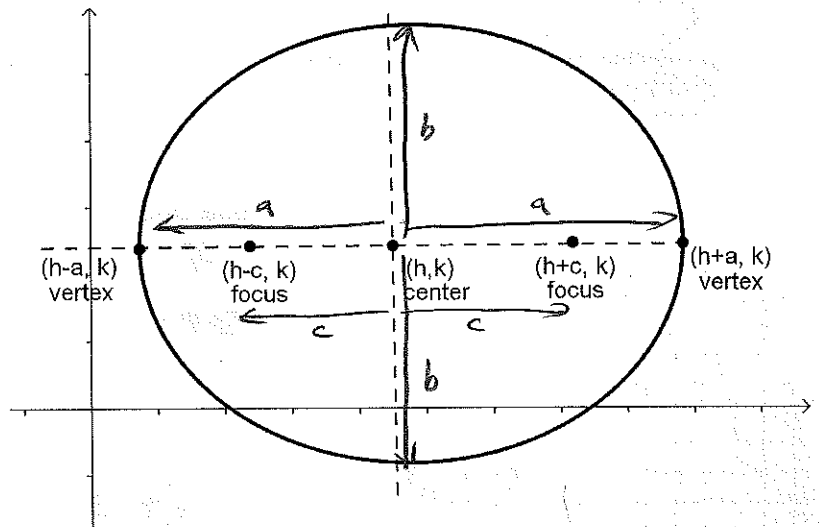
Standard form equation of an ellipse

Horizontal major axis

(bigger number under x^2 term)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

ex: $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$

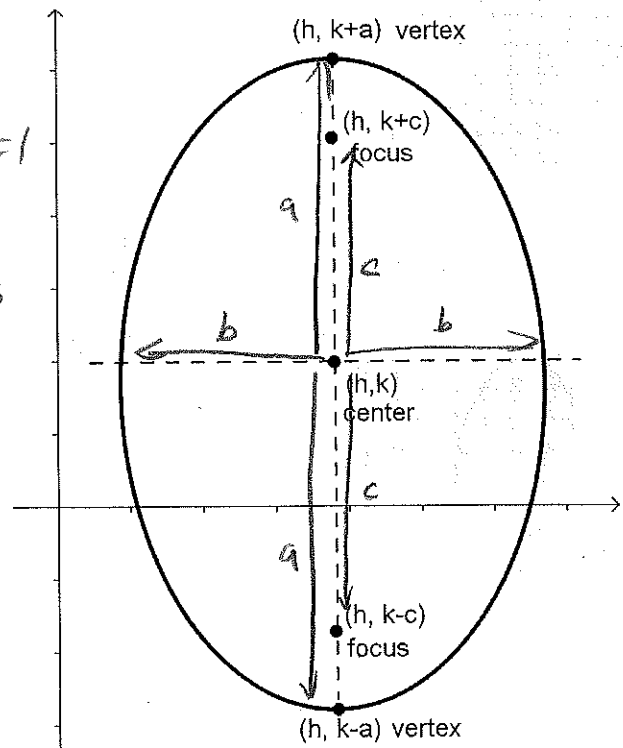


Vertical major axis

(bigger number under y^2 term)

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

ex: $\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$



For all ellipses: a is always bigger than b

(h, k) = center

a = distance from center to a vertex on major (longer) axis

b = distance from center to a point on minor (shorter) axis

c = distance from center to a focus

$$c^2 = a^2 - b^2$$

Since $c^2 = a^2 - b^2$

Eccentricity: $e = \frac{c}{a}$ 'oval-ness' $0 < e < 1$

$e = 0$ when $c = 0$ when $a = b$
($a = b$ ellipse is a circle)

Examples...

#1 Find the center, vertices, foci and eccentricity and sketch: $\frac{(x+3)^2}{16} + \frac{(y-5)^2}{25} = 1$

bigger # under y , vertical major axis

$a = \sqrt{25} = 5$

$b = \sqrt{16} = 4$

$c^2 = a^2 - b^2 = 25 - 16 = 9$

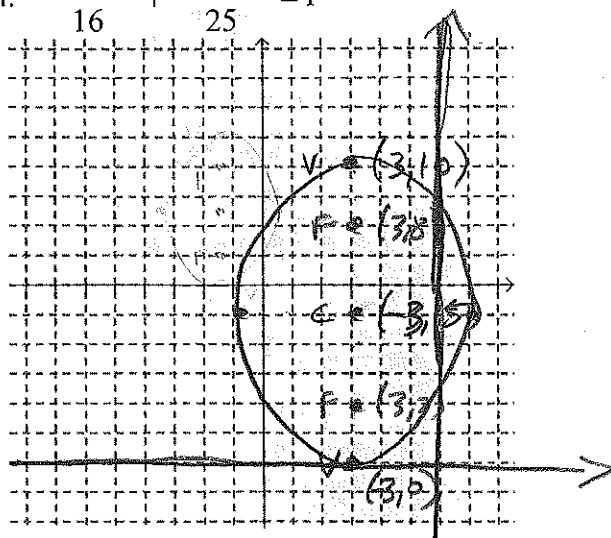
$c = 3$

Center: $(h, k) = (-3, 5)$

Vertices: $(h, k \pm a) = (-3, 10), (-3, 0)$

Foci: $(h, k \pm c) = (-3, 8), (-3, 2)$

Eccentricity: $e = \frac{c}{a} = \frac{3}{5}$



#2 Find the center, vertices, foci and eccentricity and sketch: $x^2 + 4y^2 + 6x - 8y + 9 = 0$

horiz. major axis $(x^2 + 6x) + (4y^2 - 8y) = -9$

$a = \sqrt{4} = 2$ $(x^2 + 6x + 9) + 4(y^2 - 2y + 1) = -9 + 9 + 4$

$b = 1$

$c^2 = 4 - 1 = 3$ $c = \sqrt{3}$ $\frac{(x+3)^2}{4} + \frac{(y-1)^2}{1} = \frac{4}{4}$

Center: $(h, k) = (-3, 1)$

Vertices: $(h \pm a, k) = (-1, 1), (-5, 1)$

Foci: $(h \pm c, k) = (-3 + \sqrt{3}, 1), (-3 - \sqrt{3}, 1)$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{3}}{2}$

$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{1} = 1$



Try it... Find the center, vertices, foci and eccentricity and sketch: $9x^2 + 4y^2 - 36x - 24y + 36 = 0$

vertical major axis $(9x^2 - 36x) + (4y^2 - 24y) = -36$

$a = 3$

$b = 2$

$c^2 = 9 - 4 = 5$ $c = \sqrt{5}$

Center: $(2, 3)$

Vertices: $(2, 6), (2, 0)$

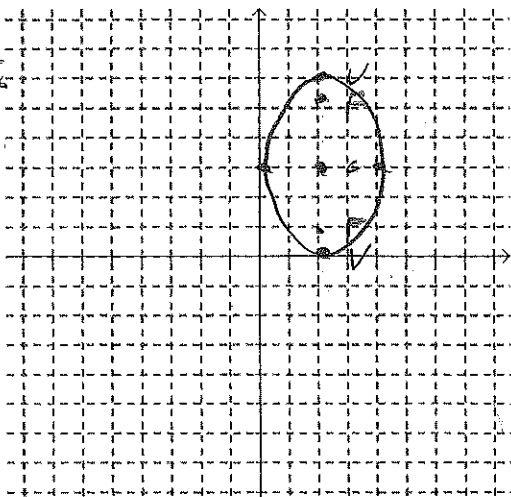
Foci: $(2, 3 + \sqrt{5}), (2, 3 - \sqrt{5})$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

$9(x^2 - 4x + 4) + 4(y^2 - 6y + 9) = -36 + 36 + 36$

$\frac{9(x-2)^2}{36} + \frac{4(y-3)^2}{36} = \frac{36}{36}$

$\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$



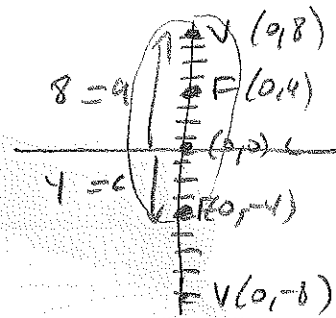
#3 Find the standard form of the equation of the ellipse if:

Vertices: (0,8) and (0,-8)

Foci: (0,4) and (0,-4)

$$\frac{(x-0)^2}{48} + \frac{(y-0)^2}{64} = 1$$

$$\boxed{\frac{x^2}{48} + \frac{y^2}{64} = 1}$$



$$c^2 = a^2 - b^2$$

$$16 = 64 - b^2$$

$$b^2 = 64 - 16 = 48$$

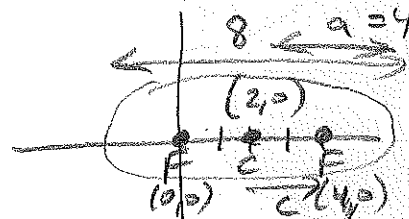
#4 Find the standard form of the equation of the ellipse if:

Foci: (0,0) and (4,0)

Major axis of length 8

$$\frac{(x-2)^2}{16} + \frac{(y-0)^2}{12} = 1$$

$$\boxed{\frac{(x-2)^2}{16} + \frac{y^2}{12} = 1}$$



$$c = 2$$

$$c^2 = a^2 - b^2$$

$$4 = 16 - b^2$$

$$b^2 = 16 - 4$$

$$b^2 = 12$$

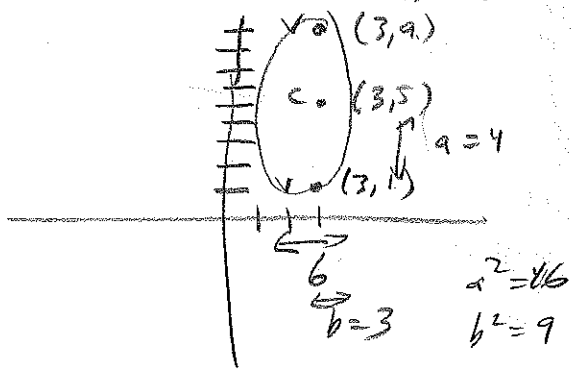
$$a^2 = 16$$

Try it... Find the standard form of the equation of the ellipse if:

Vertices: (3, 1) and (3, 9)

Minor axis of length 6

$$\boxed{\frac{(x-3)^2}{9} + \frac{(y-5)^2}{16} = 1}$$



$$a^2 = 16$$

$$b^2 = 9$$

HA1g3-4, 10.3 Notes – Hyperbola

Hyperbola = the set of all points (x,y) the difference of whose distances from two fixed points (called foci) is a positive constant.

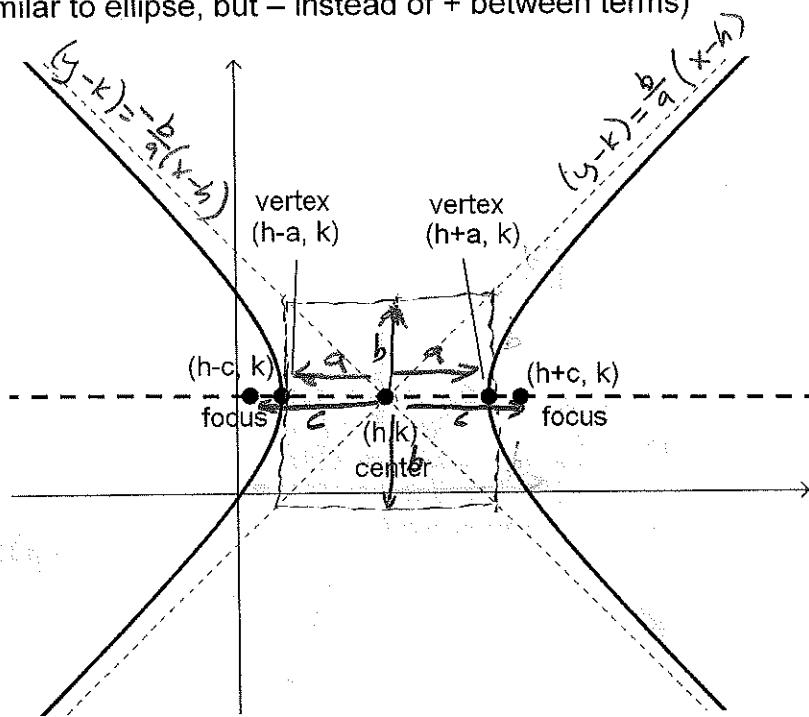
Standard form equation of a hyperbola (similar to ellipse, but $-$ instead of $+$ between terms)

Horizontal transverse axis (x^2 term is first)

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

asymptotes at:

$$(y-k) = \pm \frac{b}{a}(x-h)$$

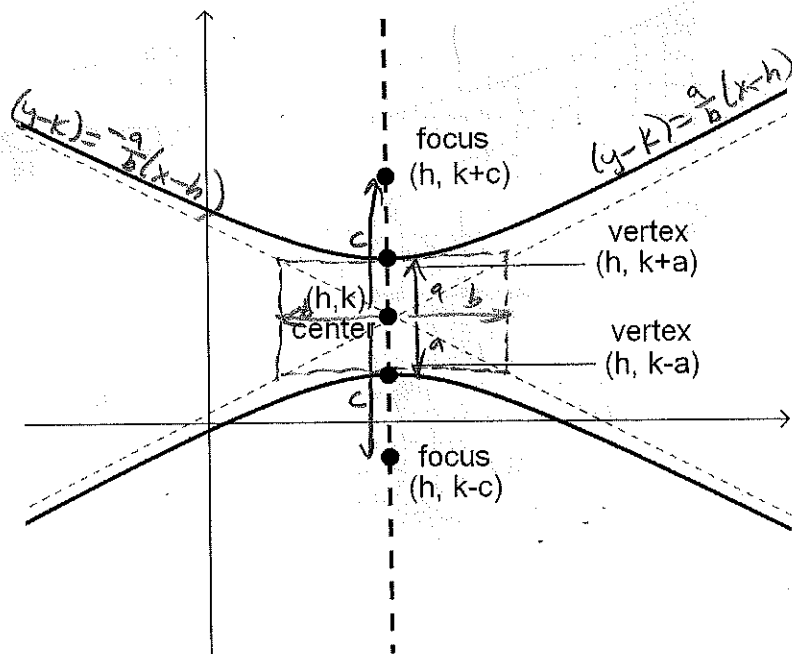


Vertical transverse axis (y^2 term is first)

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

asymptotes at:

$$(y-k) = \pm \frac{a}{b}(x-h)$$



For all hyperbola:

(h,k) = center

a = distance from center to a vertex

c = distance from center to a focus

$$c^2 = a^2 + b^2 \quad \text{therefore } c > a$$

Eccentricity: $e = \frac{c}{a}$ like ellipse, but now $e > 1$ (higher hyperbola eccentricity = 'flatter' curves)

#1 Find the center, vertices, foci, asymptotes, eccentricity and sketch: $\frac{x^2}{9} - \frac{y^2}{25} = 1$

$$\frac{x^2}{9} - \frac{y^2}{25} = 1 \quad \begin{array}{l} \text{X term } +, \\ \text{horiz. transverse axis} \end{array}$$

$$\begin{array}{l} a^2 = 9, a = 3 \\ b^2 = 25, b = 5 \\ c^2 = a^2 + b^2 \\ c^2 = 9 + 25 = 34 \\ c = \sqrt{34} \\ \approx 5.8 \end{array}$$

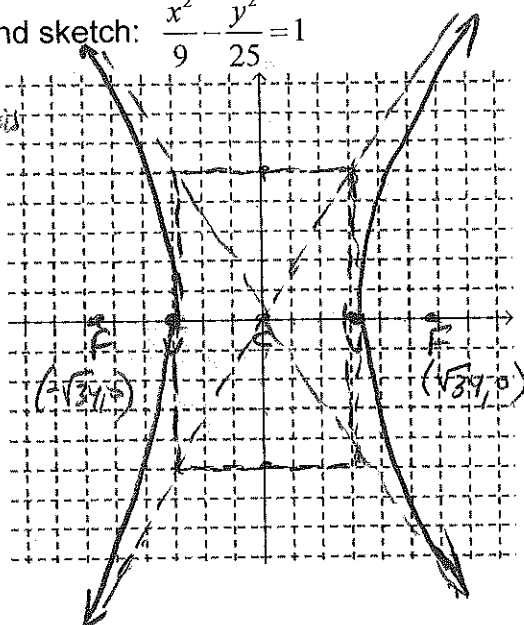
Center: $(h, k) : (0, 0)$

Vertices: $(h \pm a, k) : (3, 0) \text{ \& } (-3, 0)$

Foci: $(h \pm c, k) : (\sqrt{34}, 0) \text{ \& } (-\sqrt{34}, 0)$

Asymptotes: $(y - k) = \pm \frac{b}{a}(x - h) : (y - 0) = \pm \frac{5}{3}(x - 0)$
 $y = \pm \frac{5}{3}x$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{34}}{3}$



#2 Find the center, vertices, foci, asymptotes, eccentricity and sketch:

$$4y^2 - x^2 - 16y - 6x - 29 = 0$$

$$(4y^2 - 16y) + (-x^2 - 6x) = 29$$

$$4(y^2 - 4y + 4) - (x^2 + 6x + 9) = 29$$

$$\frac{4(y-2)^2}{36} - \frac{(x+3)^2}{36} = \frac{36}{36}$$

Center: $(-3, 2)$

Vertices: $(-3, 5) \text{ \& } (-3, -1)$

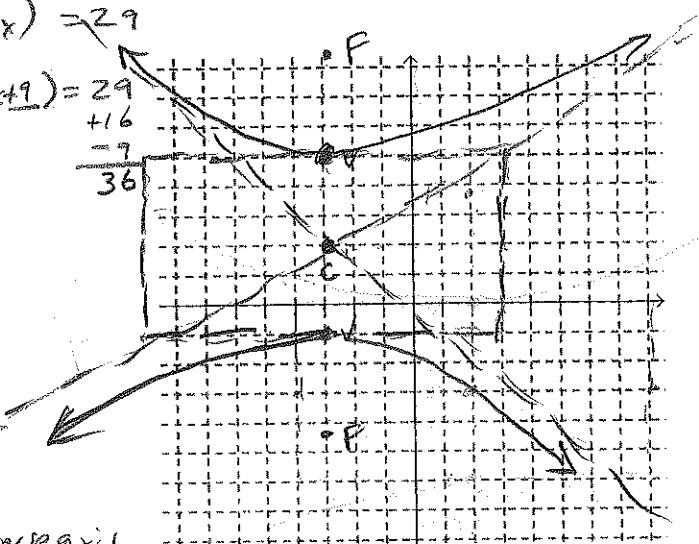
Foci: $(-3, 2 + \sqrt{45}), (-3, 2 - \sqrt{45})$

Asymptotes: $(y - 2) = \pm \frac{1}{2}(x + 3)$

Eccentricity: $e = \frac{c}{a} = \frac{\sqrt{45}}{3}$

$$\begin{array}{l} a = 3 \\ b = 6 \\ c^2 = 9 + 36 = 45 \\ c = \sqrt{45} \end{array}$$

vertical transverse axis



Try it... Find the center, vertices, foci, asymptotes, eccentricity and sketch:

$$x^2 - 9y^2 + 36y - 72 = 0$$

$$x^2 + (-9y^2 + 36y) = 72$$

$$(x-0)^2 - 9(y^2 - 4y + 4) = 72 - 36$$

$$\frac{(x-0)^2}{36} - \frac{9(y-2)^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1$$

horizontal trans. axis

$$a = 6$$

$$b = 2$$

$$c^2 = 40$$

$$c = \sqrt{40} \approx 6.3$$

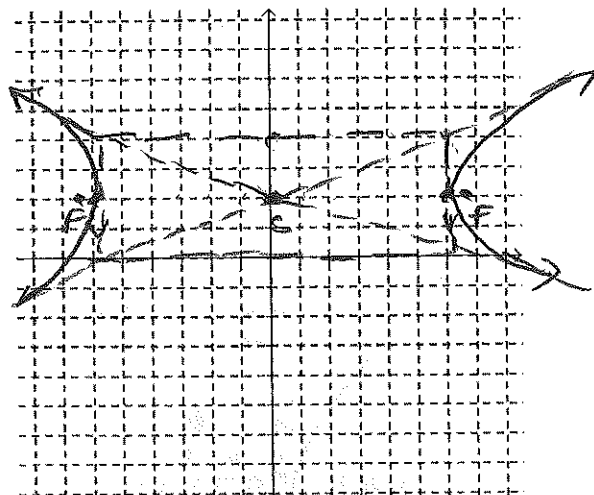
Center: $(0, 2)$

Vertices: $(6, 2)$ & $(-6, 2)$

Foci: $(\sqrt{40}, 2)$ & $(-\sqrt{40}, 2)$

Asymptotes: $(y-2) = \pm \frac{1}{3}x$

$$\begin{aligned} \text{Eccentricity: } e &= \frac{c}{a} = \frac{\sqrt{40}}{6} = \frac{\sqrt{4 \cdot 10}}{6} \\ &= \frac{2\sqrt{10}}{6} = \frac{\sqrt{10}}{3} \end{aligned}$$



#4 Find the standard form of the equation of the hyperbola if:

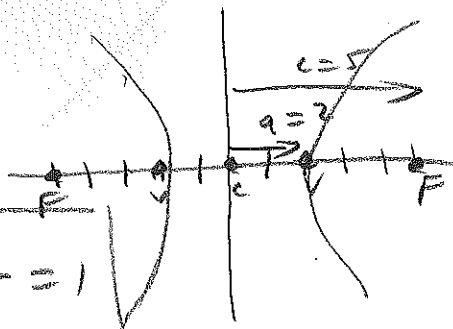
center $(0, 0)$

Vertices: $(2, 0)$ and $(-2, 0)$

Foci: $(5, 0)$ and $(-5, 0)$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 25 &= 4 + b^2 \\ b^2 &= 21 \end{aligned}$$

$$\boxed{\frac{x^2}{4} - \frac{y^2}{21} = 1}$$



Try it... Find the standard form of the equation of the hyperbola if:

Foci: $(2, 5)$ and $(2, -5)$

Vertices: $(2, 3)$ and $(2, -3)$

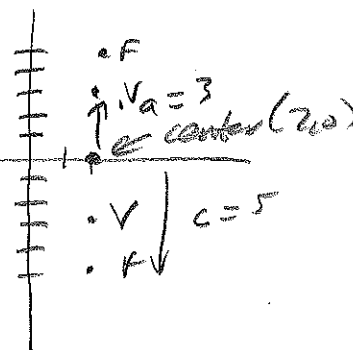
$$c^2 = a^2 + b^2$$

$$25 = 9 + b^2$$

$$b^2 = 16$$

$$\frac{(y-0)^2}{9} - \frac{(x-2)^2}{16} = 1$$

$$\boxed{\frac{y^2}{9} - \frac{(x-2)^2}{16} = 1}$$



HAlg3-4, 10 review day – Comparing conics

Parabola

$$(x-h)^2 = 4p(y-k)$$

$$(y-k)^2 = 4p(x-h)$$

x^2 like $y = x^2$
 y^2 'other one'

p = dist. vertex to focus
and dist. vertex to directrix

Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

a is always bigger,
 a under term of major axis

$$c^2 = a^2 - b^2$$

b = dist. center to point
on minor axis

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

a not always bigger,
 a always under first term
first term is transverse axis

$$c^2 = a^2 + b^2$$

a = dist. center to vertex
 c = dist. center to focus

b = dist. to 'other side of box'

asymptotes from center
through corners of box:

$$(y-k) = \pm \frac{b}{a}(x-h)$$

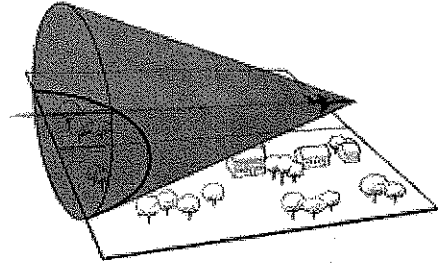
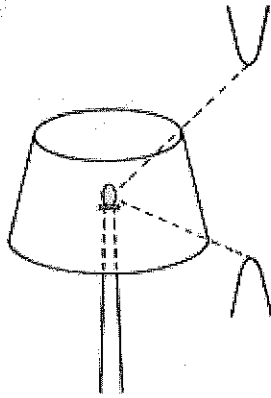
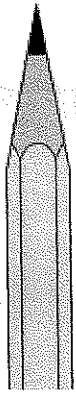
$$(y-k) = \pm \frac{a}{b}(x-h)$$

(look at box to see which)

eccentricity $e = \frac{c}{a}$

HA1g3-4, 10.3 day 2 Notes – Classifying Conic Sections by Equation

Where are hyperbolas found in the real-world? (where planes and cones intersect)



Classifying a conic section from the general equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

not in this course

Look at the A and the C (the x^2 and the y^2 terms)....

$$(4x^2 - y^2) - 4x - 3 = 0 \quad H$$

$$(x^2 + y^2) - 6x + 4y + 9 = 0 \quad C$$

$$(4x^2 + 3y^2) + 8x - 24y + 51 = 0 \quad E$$

$$(x^2 + 4y^2) + 6x + 16y + 21 = 0 \quad E$$

$$(4x^2 - 9x + y - 5 = 0 \quad P$$

$$(4x^2 - y^2) - 4x - 3 = 0 \quad H$$

$$(4x^2 - y^2) + 8x - 6y + 4 = 0 \quad H$$

$$(4y^2 - 2x^2) - 8x - 4y - 15 = 0 \quad H$$

$$(2x^2 + 4y^2) - 4x + 12y = 0 \quad E$$

$$(25x^2 - 10x - 200y - 119 = 0 \quad P$$

$$(2x^2 + 2y^2) - 8x + 12y + 2 = 0 \quad C$$

$$(4x^2 + 4y^2) - 16y + 15 = 0 \quad C$$

