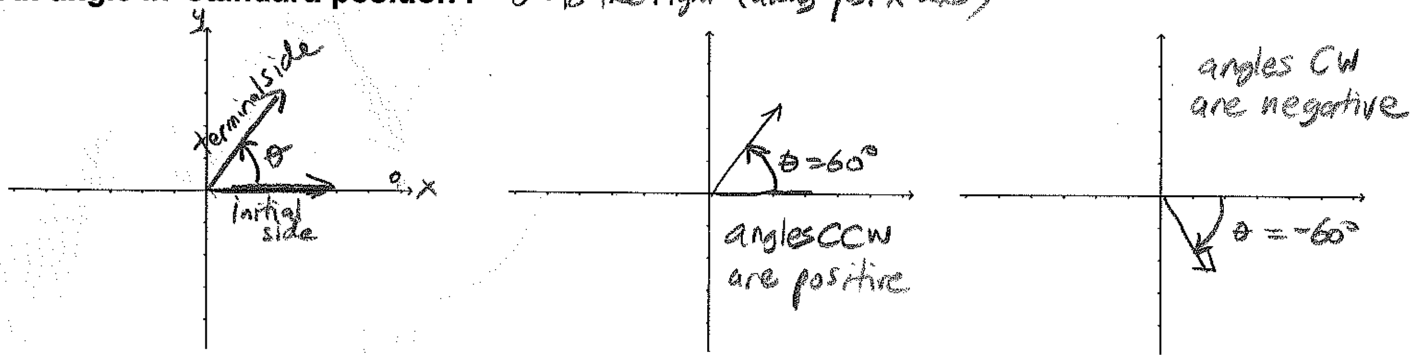
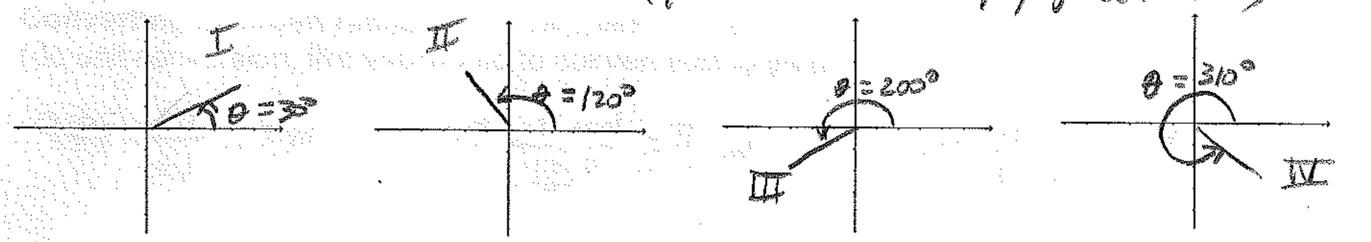
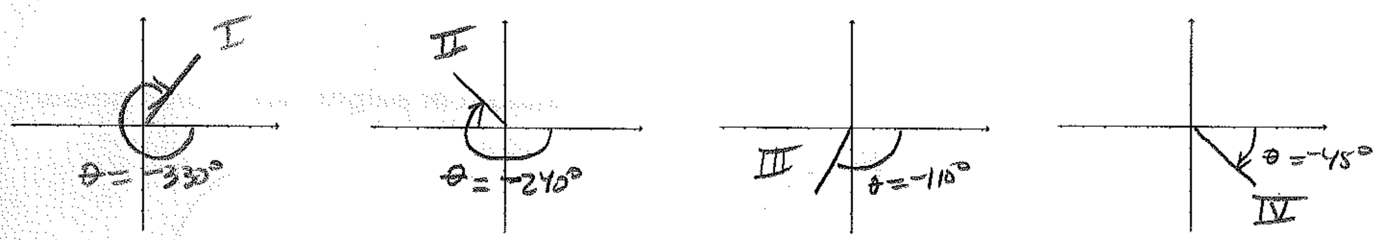
HAlg3-4, 4.1 day 1 Notes – Trig: Radian and Degree measure

An angle in 'standard position': 0° to the right (along pos. x-axis)

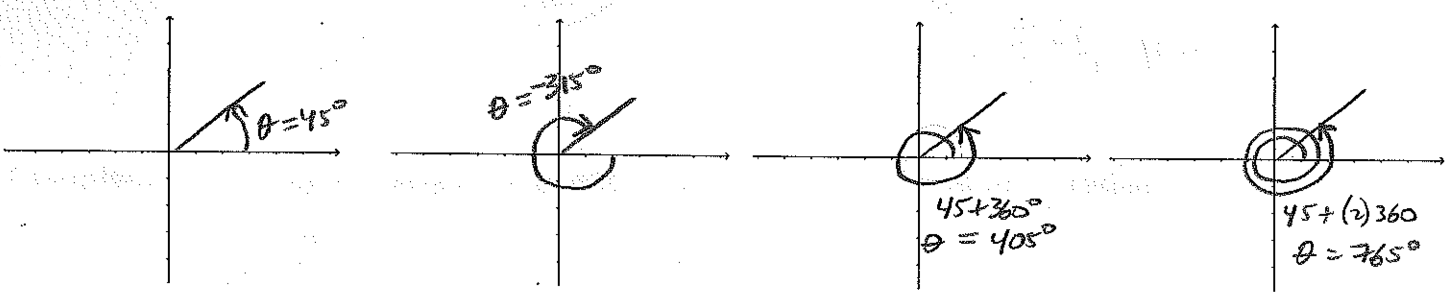


The terminal side can be in any quadrant: (quadrants start on right, go CCW also)

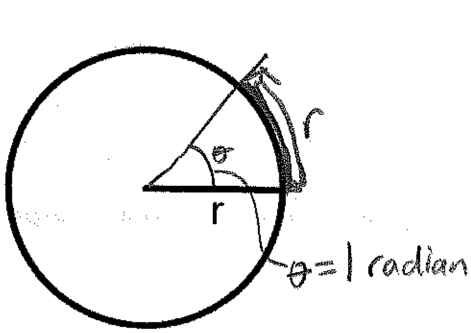




Coterminal angles = angles with the same terminal side. All of the following are coterminal:

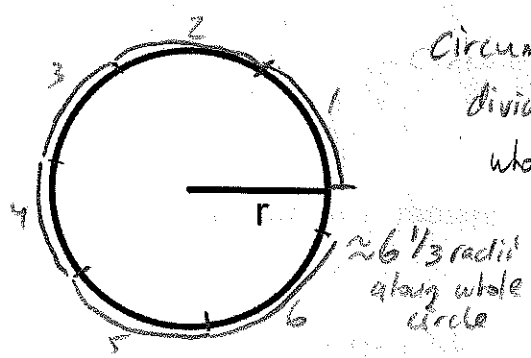


Radians:



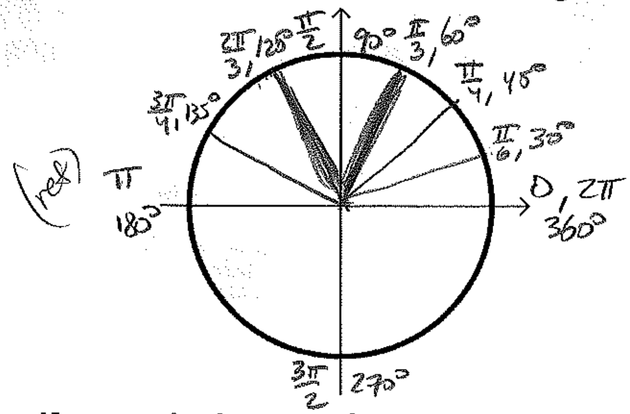
1 radian = measure of the central angle that intercepts arc equal to radius of the circle

How many radians in a whole circle?



Circumference = 2πr
divide by radius:
whole circle = 2π radians
≈ (6.28 radians)

Any angle can be expressed in either degrees or radians: ** Explain fraction of π



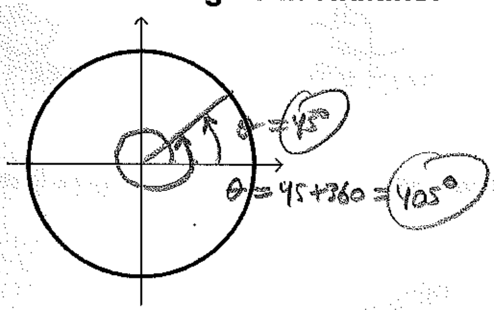
degree	radian	
360°	2π	6.28
180°	π	3.14
90°	$\frac{\pi}{2}$	1.57
45°	$\frac{\pi}{4}$	.785
15°	$15°\left(\frac{\pi}{180}\right) = \frac{\pi}{12} = .262$	

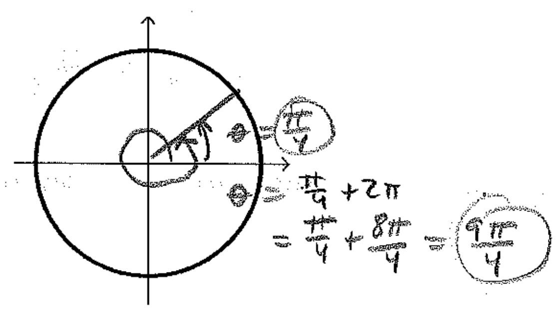
Note: If an angle does not have a degree mark (°) it is in radians.

Converting between radians and degrees: $180° = \pi$ radians
(do unit conversion, like you would to convert feet to yards)

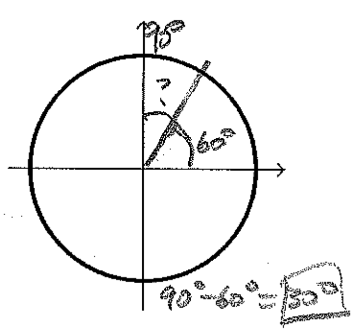
$9\,\cancel{ft}\,\frac{1\,yd}{3\,\cancel{ft}} = 3\,yds$ $\quad 60°\,\frac{\pi\,rad}{180°} = \frac{\pi}{3}\,rad = 1.047$ $\quad \frac{\pi}{4}\,rad\,\frac{180°}{\pi\,rad} = 45°$

Finding coterminal angles in radians:

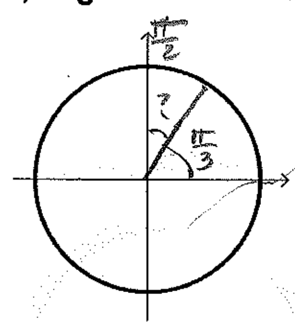




Complementary angles: angles that add to 90°, angles that add to $\frac{\pi}{2}$ radians



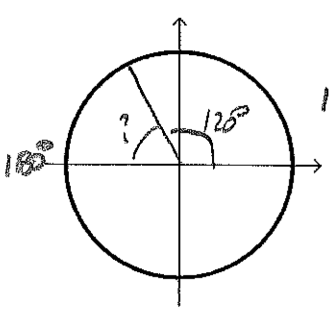
$90° - 60° = \boxed{30°}$



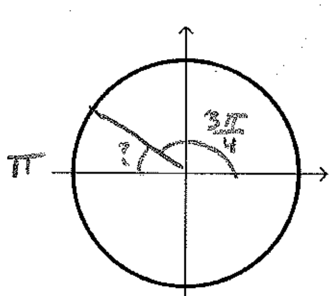
$\frac{\pi}{2} - \frac{\pi}{3}$

$\frac{3\pi}{6} - \frac{2\pi}{6} = \boxed{\frac{\pi}{6}}$

Supplementary angles: angles that add to 180°, angles that add to π radians



$180° - 120° = \boxed{60°}$



$\pi - \frac{3\pi}{4}$

$\frac{4\pi}{4} - \frac{3\pi}{4} = \boxed{\frac{\pi}{4}}$

HAlg3-4, 4.1 day 2 Notes – Trig: DMS and arc length problems

How do we indicate a fraction of a degree?

Could use $41\frac{3}{4}°$, but usually we use Degrees-Minutes-Seconds (DMS):

1 degree = 60 minutes (60')
1 minute = 60 seconds (60")

Converting means multiplying or dividing by 60

Examples: $41\frac{3}{4}° = \boxed{41°45'}$

$\frac{3}{4}° \cdot \frac{60'}{1°} = \frac{180}{4}' = 45'$

$127°15'45" =$

start with smallest part: $45" \cdot \frac{1'}{60"} = \frac{45}{60}' = .75'$

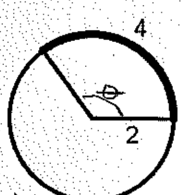
add to next higher part: $15.75' \cdot \frac{1°}{60'} = \frac{15.75}{60}° = .2625°$

$\boxed{127.2625°}$

Calculator feature – degree conversions:

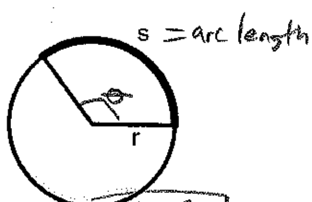
- Decimal degrees to DMS:
  - Enter decimal degrees.
  - 2nd – APPS (angle screen).
  - >DMS, enter twice.
- DMS to decimal degrees: example, convert $127°15'45"$
  - Enter 127, 2nd-APPS(angle), select degree mark, enter
  - Enter 15, 2nd-APPS(angle), select minutes mark, enter
  - Enter 45, Alpha-+key (" mark), enter.

Arc Length Problems:



what is θ if arc is 2 radii?

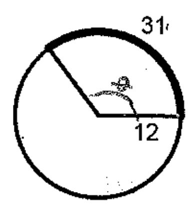
$\theta = 2$ radians



in general: $\boxed{\theta = \frac{s}{r}}$

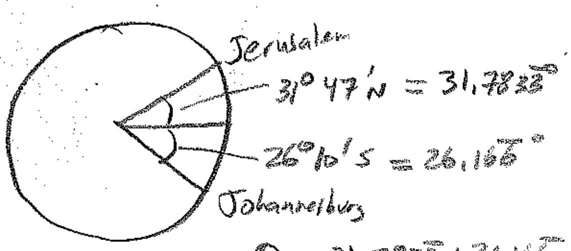
or $\boxed{s = r\theta}$

* $\boxed{\theta \text{ must be in radians}}$



$\theta = \frac{31}{12} = 2.58\overline{3}$ radians

Example: Find the north-south distance between Jerusalem, Israel (with lattitude $31°47'N$) and Johannesberg, South Africa (with lattitude $26°10'S$). The radius of the earth is approximately 4000 miles.



$\theta = 31.78\overline{3} + 26.1\overline{6}$

$\theta = 57.95°$

Convert to radians

$57.95° \cdot \frac{\pi \text{ rad.}}{180°} = 1.01142$ rad.

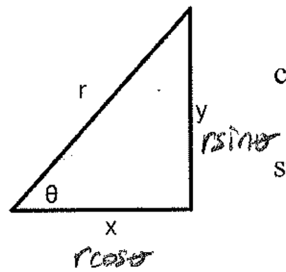
$S = r\theta$

$S = (4000)(1.01142)$

$S \approx \boxed{4045 \text{ mi}}$

HAlg3-4, 4.2 day 1 Notes – Trig: The Unit Circle, Trig Functions

Remember from geometry:



$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$

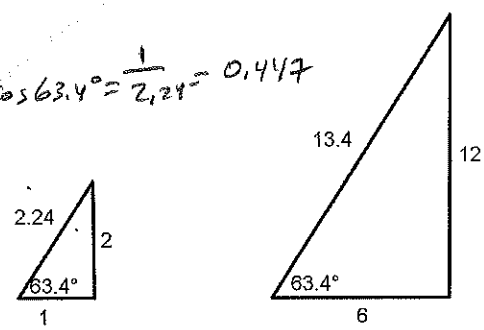
$x = r\cos\theta$

$y = r\sin\theta$

Sine, cosine, and tangent are ratios, so they don't really depend upon the size of the triangle, only on the angle:

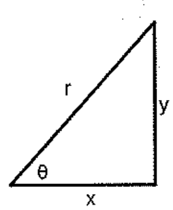
$\cos 63.4° = \frac{1}{2.24} = 0.447$

$\cos 63.4° = \frac{adj}{hyp} = \frac{6}{13.4} = 0.447$



If we think of the angle as the input and the ratio as a number which is the output, sine and cosine can also be thought of, not just as ratios, but also as functions. Here is how the sine and cosine functions are defined:

1) Start with this triangle...

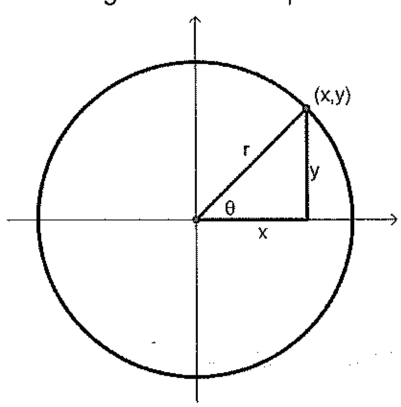


...for which:

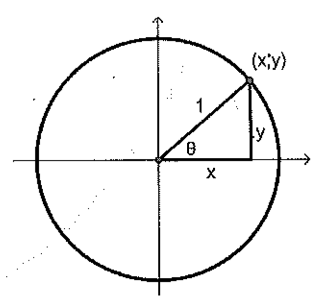
$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$

2) Put this triangle on a circle with the angle in standard position...



3) Reduce the radius to 1 unit...



...and on this 'unit circle'...

$$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

The input to the sine and cosine functions is an angle. If this angle is in standard position on a 'unit circle' (radius 1), we define:

$\cos\theta$ = the x-coordinate of the point on the unit circle at angle θ

$\sin\theta$ = the y-coordinate of the point on the unit circle at angle θ

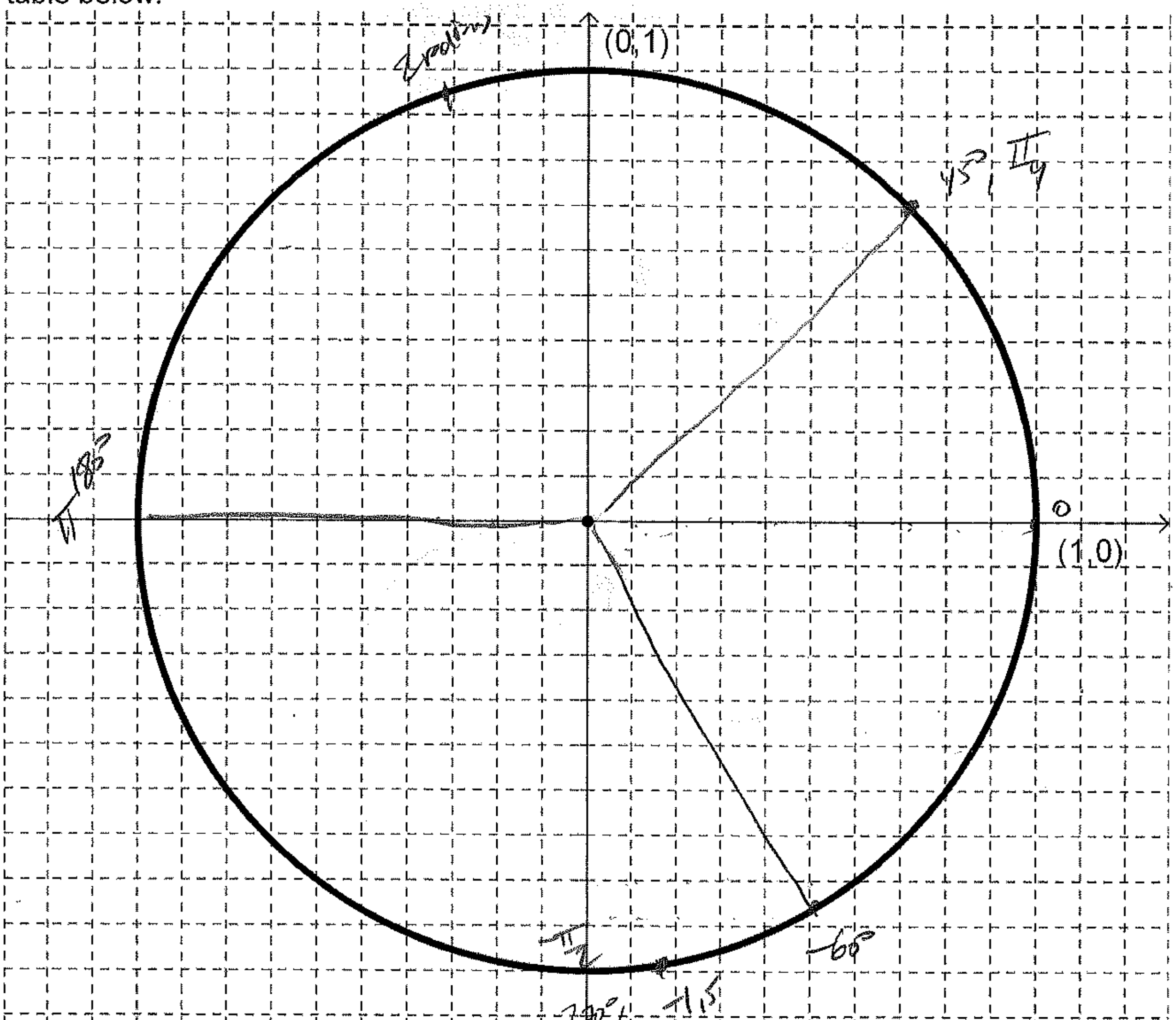
The angle can be any angle…in degrees or radians can be positive or negative.

We define cosine and sine functions to be the x and y coordinates of a point on the unit circle with angle in standard position.

$\theta \rightarrow \boxed{\cos} \rightarrow$ number (ratio)
angle x-coordinate

$\theta \rightarrow \boxed{\sin} \rightarrow$ number (ratio)
angle y-coordinate

Method #1 for finding sine and cosine of an angle #1 – Measuring x or y: Use the grid on this unit circle (r = 1) to estimate the value for cosine and sine of each angle to complete the table below:



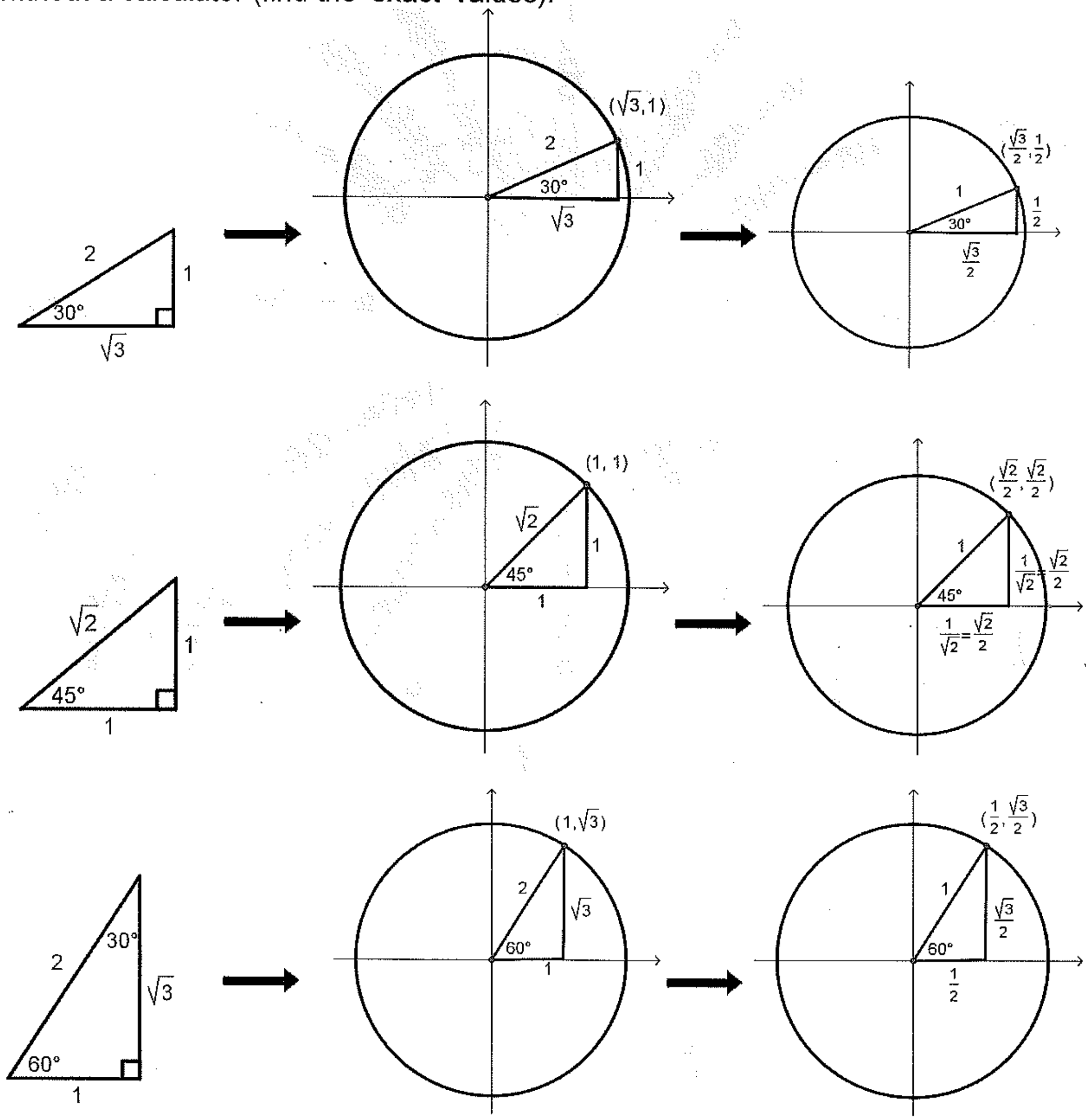
angle θ (deg)	$\cos\theta$	$\sin\theta$
45°	0.71	0.71
180°	-1	0
−60°	0.5	-0.86
630° = 270°	0	-1
−90°	0	-1

630
-360
270

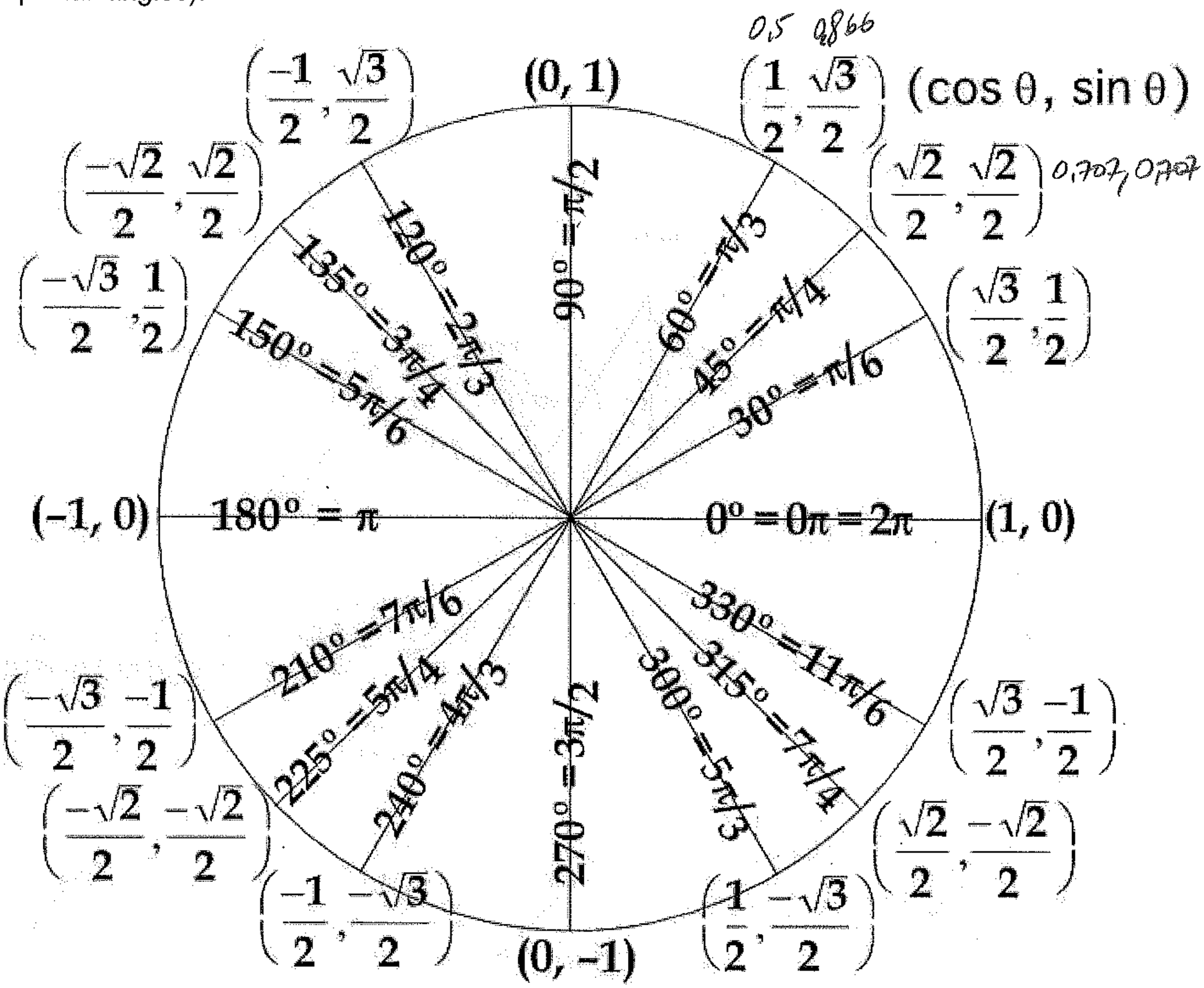
angle θ (radians)	$\cos\theta$	$\sin\theta$
$\frac{\pi}{4}$	0.71	0.71
$-\frac{\pi}{2}$	0	-1
0	1	0
2	-.31	0.94
-1.5	0.19	-0.98

Method #2 for finding sine and cosine of an angle – Calculator: You can also find cosine or sine of an angle with your calculator. Go back and verify a few of these values in the table with your calculator (be sure to set mode to 'degrees' or 'radians' as needed.)

Method #3 for finding sine and cosine of an angle – Unit Circle chart:
For some 'special' values of θ (multiples of 30 and 45 degrees), we can find cosine or sine without a calculator (find the 'exact' values).



The 'unit circle' chart is a list of all the special values. Multiples of 30 and 45 degrees are found by using symmetry of the 30, 45 and 60 degree values. You can only use the chart to find sine and cosine for multiples of 30 and 45 degrees (use a calculator to find any values for 'non-special' angles).



Examples...Find:

$\sin 120° = \frac{\sqrt{3}}{2}$

$\sin \frac{5\pi}{3} = \frac{-\sqrt{3}}{2}$

$\cos(-60°) = \frac{1}{2}$

$\cos \frac{\pi}{3} = \frac{1}{2}$

$\sin 240° = \frac{-\sqrt{3}}{2}$

$\sin \frac{\pi}{2} = 1$

HAlg3-4, 4.2 day 2 Notes – Trig: Trig function properties, calculator eval

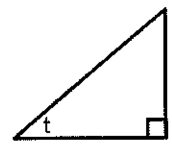
Other trigonometry functions:

From sin(t) and cos(t) we can define 4 additional trigonometric functions, for a total of 6 trig functions (these are general functions...the independent variable could be anything):

sine: $\sin(t) = \frac{1}{\csc t}$ cosecant: $\csc(t) = \frac{1}{\sin t}$

cosine: $\cos(t) = \frac{1}{\sec t}$ secant: $\sec(t) = \frac{1}{\cos t}$

tangent: $\tan(t) = \frac{\sin t}{\cos t} = \frac{1}{\cot t}$ cotangent: $\cot(t) = \frac{1}{\tan t}$

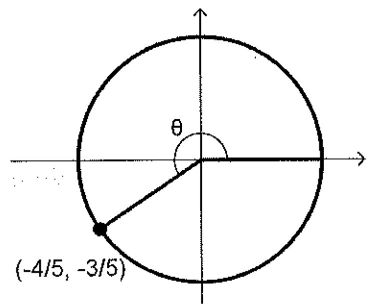


Find all 6 trig functions:

$\sin\theta = -\frac{3}{5}$ $\csc\theta = \frac{1}{\sin\theta} = \frac{-5}{3}$

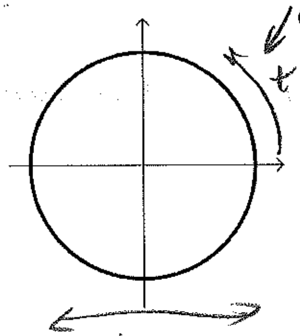
$\cos\theta = -\frac{4}{5}$ $\sec\theta = \frac{1}{\cos\theta} = \frac{-5}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-3/5}{-4/5} = \frac{3}{4}$ $\cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$



Domain and range of sine and cosine functions:

can put any angle in for t



t

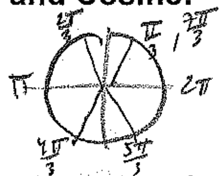
sint varies from -1 to 1

cost varies from -1 to 1

D: $(-\infty, \infty)$ for both sine and cosine functions
R: $[-1, 1]$

Periodicity of Sine and Cosine:

What is $\cos\frac{7\pi}{3}$?



$\frac{7\pi}{3}$ coterminal with $\frac{7\pi}{3} - 2\pi = \frac{\pi}{3}$

$\cos\left(\frac{7\pi}{3}\right) = \cos\left(\frac{7\pi}{3} - 2\pi\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

The values for both sine and cosine repeat every 2π.

A function that repeats this way is called a **periodic function:**

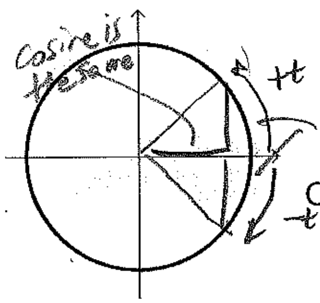
$$f(t+c) = f(t) \qquad \sin(t+2\pi) = \sin(t)$$
$$\cos(t+2\pi) = \cos(t)$$

The **period** of the sine and cosine functions is 2π.

Even/Odd functions: remember, definition of even and odd functions:

Even function if $f(-x) = f(x)$

Odd function if $f(-x) = -f(x)$



$\cos(-t) = \cos(t)$ so cosine is an even function

sine switches sign $\sin(-t) = -\sin(t)$ so sine is an odd function

Other trig functions are found by using sin and cos:

$\tan(-t) = -\tan(t)$ (odd) $\qquad \cot(-t) = -\cot(t)$ (odd)

$\csc(-t) = -\csc(t)$ (odd) $\qquad \sec(-t) = \sec(t)$ (even)

Evaluating with a calculator: use identities and buttons for sin, cos, tan

$\csc\frac{\pi}{8} = \frac{1}{\sin\frac{\pi}{8}} = \frac{1}{.38268} = 2.613 \quad \tan\frac{-3\pi}{4} = \frac{\sin\frac{-3\pi}{4}}{\cos\frac{-3\pi}{4}} = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1 \quad \cos 2.3 = -0.6663$

Make sure your calculator is in correct mode (radians or degrees) – MODE key

note: $\csc\frac{\pi}{8} \neq \sin\frac{8}{\pi}$ * never flip the angle!

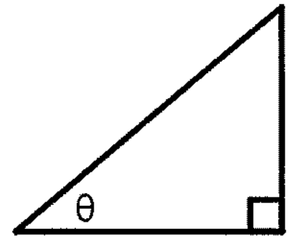
$\csc\frac{\pi}{8} = \frac{1}{\sin\frac{\pi}{8}}$

HAlg3-4, 4.3 day 1 Notes – Trig: Right Triangle Trigonometry

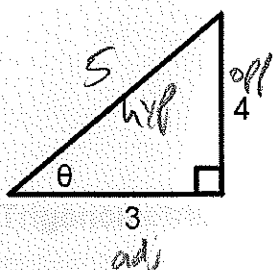
Right triangle definitions of the 6 trig functions:

$\sin\theta = \dfrac{opp}{hyp}$ $\qquad \cos\theta = \dfrac{adj}{hyp}$ $\qquad \tan\theta = \dfrac{opp}{adj}$

$\csc\theta = \dfrac{hyp}{opp}$ $\qquad \sec\theta = \dfrac{hyp}{adj}$ $\qquad \cot\theta = \dfrac{adj}{opp}$



Examples...

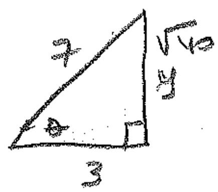


#1. Find all 6 trig functions:

$h^2 = 3^2 + 4^2$
$h^2 = 9 + 16$
$h^2 = 25$
$h = 5$

$\sin\theta = \frac{4}{5}$ $\qquad \csc\theta = \frac{5}{4}$
$\cos\theta = \frac{3}{5}$ $\qquad \sec\theta = \frac{5}{3}$
$\tan\theta = \frac{4}{3}$ $\qquad \cot\theta = \frac{3}{4}$

#2. Sketch the triangle, if $\cos\theta = \dfrac{3}{7} = \dfrac{adj}{hyp}$



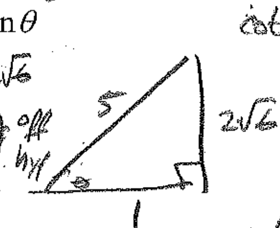
$y^2 + 3^2 = 7^2$
$y^2 + 9 = 49$
$y^2 = 40$
$y = \sqrt{40}$

#3. Given $\sec\theta = 5$ and $\tan\theta = 2\sqrt{6}$, find $\cos\theta, \cot\theta, \cot(90° - \theta), \sin\theta$

$\sec\theta = \frac{1}{\cos\theta} = 5$
$\cos\theta = \boxed{\frac{1}{5}} = \frac{adj}{hyp}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = 2\sqrt{6}$
$\frac{\sin\theta}{1/5} = 2\sqrt{6}$

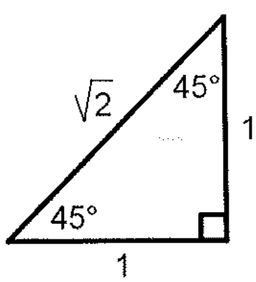
$\sin\theta = \left(\frac{1}{5}\right)2\sqrt{6}$
$\sin\theta = \boxed{\frac{2\sqrt{6}}{5}} \frac{opp}{hyp}$

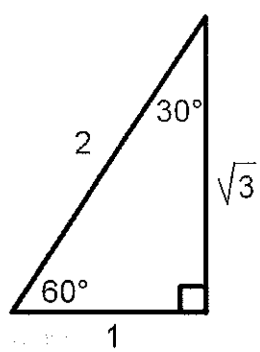


$\cot\theta = \frac{\cos\theta}{\sin\theta}$
$= \frac{1/5}{2\sqrt{6}/5}$
$= \frac{1}{2\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6}}$
$\cot\theta = \boxed{\frac{\sqrt{6}}{12}}$

$\cot(90° - \theta) = \tan\theta$
$= \frac{12}{\sqrt{6}} \frac{\sqrt{6}}{\sqrt{6}}$
$= \frac{12\sqrt{6}}{6} \boxed{2\sqrt{6}}$

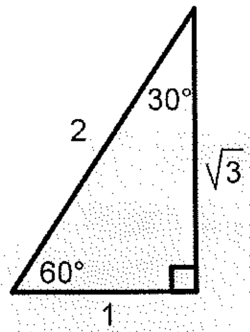
Special Right triangles (memorize these):





#4. Find $\cos 60° = \frac{adj}{hyp} = \frac{1}{2}$

Cofunctions:



Find $\cos 60° = \frac{1}{2}$

Find $\sin 30° = \sin(90° - 60°) = \frac{1}{2}$

$\sin(90° - \theta) = \cos\theta$ $\quad$ $\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$ $\quad$ $\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \csc\theta$ $\quad$ $\csc(90° - \theta) = \sec\theta$

Trigonometric Identities:

Reciprocal identities:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

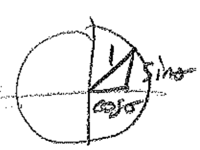
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean identities:

$\sin^2\theta + \cos^2\theta = 1$



$1 + \tan^2\theta = \sec^2\theta$ $\quad \leftarrow \quad \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$ $\quad \leftarrow \quad \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

Examples.... Transform one side of the equation into the other:

#5. $\cos\theta\sec\theta = 1$ (given)

$\cos\theta\left(\frac{1}{\cos\theta}\right) = 1$ $\left(\sec\theta = \frac{1}{\cos\theta}\right)$

$\frac{\cos\theta}{\cos\theta} = 1$ (multiply)

$1 = 1$ ✓ (divide)

#6. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$ (given)

$\sec^2\theta - \sec\theta\tan\theta + \sec\theta\tan\theta - \tan^2\theta = 1$ (FOIL)

$\sec^2\theta - \tan^2\theta = 1$ (terms cancel)

$1 + \tan^2\theta - \tan^2\theta = 1$ $(\sec^2\theta = 1 + \tan^2\theta)$

$1 = 1$ ✓ (terms cancel)

HAlg3-4, 4.3 day 2 Notes – Trig: Real-World Application...Right Triangle Problems

Evaluating Trig functions with a calculator: Use identities if needed to get sin, cos, or tan.

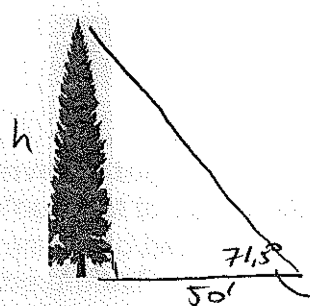
Example: Evaluate sec(5°40'12")

$5°40'12'' \rightarrow 5.67°$

$\sec 5.67° = \frac{1}{\cos 5.67°} \approx 1.0049166$

Real-world application problems

Angle of Elevation:
Example: A surveyor is standing 50 feet from the base of a large tree. The surveyor measures the angle of elevation to the top of the tree as 71.5°. How tall is the tree?

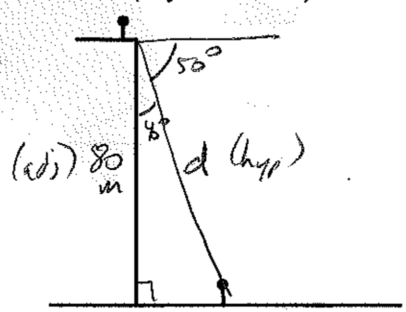


angle of elevation (angle above the horizontal)

$\tan 71.5° = \frac{opp}{adj} = \frac{h}{50}$

$h = 50 \tan 71.5° \approx \boxed{149.4 \text{ ft}}$

Angle of Depression:
Example: You are standing at the edge of the roof of an 80m tall building. Your friend is on the ground below. If you measure the angle of depression to your friend to be 50°, and you can be heard (if you shout) from 100m away, will your friend hear you if you shout?



$\cos 40° = \frac{adj}{hyp} = \frac{80}{d}$

$d = \frac{80}{\cos 40°} = \underline{104.4 \text{ m}}$

no can't hear (>100m)

$\cos 40° = \frac{80}{d}$ swap

because:

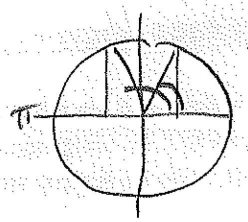
$\frac{\cos 40°}{1} \times \frac{80}{d}$

$\frac{d \cos 40°}{\cos 40°} = \frac{80}{\cos 40°}$

$d = \frac{80}{\cos 40°}$

Given a trig function value, you can find the angle (with unit circle or with calculator)

Example: $\sin\theta = \frac{\sqrt{3}}{2}$, find θ in radians and degrees:



sin is y $= \sqrt{3}/2$

$\theta = 60^\circ$ or 120°

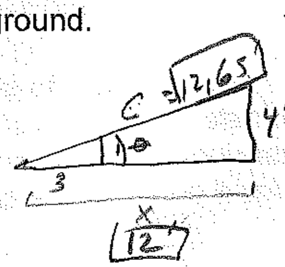
$\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

by calc. $\sin\theta = \frac{\sqrt{3}}{2}$

$\sin^{-1}(\sin\theta) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$ calc. only gives one angle use unit circle to figure out the other

Inverse trig function calculator problems:
You want to build a skateboard ramp that rises 1 foot for every 3 feet of horizontal length. If the ramp is to be 4 feet tall, find the lengths of the other sides, and the angle the ramp makes with the ground.



ratio: $\frac{1}{3} = \frac{4}{x}$

$x = 12$

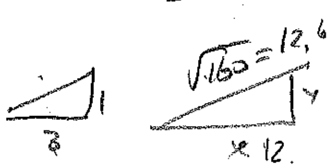
$c^2 = a^2 + b^2$

$c^2 = 12^2 + 4^2$

$c^2 = 144 + 16$

$c^2 = 160$

$c = \sqrt{160} = 12.65$



$\tan\theta = \frac{4}{12} = \frac{1}{3}$

$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{1}{3}\right)$

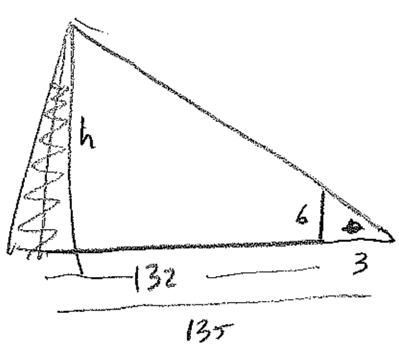
$\theta = \tan^{-1}\left(\frac{1}{3}\right) = \boxed{18.4^\circ}$

You can use 2nd sin (or cos or tan) to find the angle if you know sin (or cos or tan).

Homework problem #63:
A 6 ft person walks from the base of a broadcasting tower directly toward the tip of the shadow case by the tower. When the person is 132 ft from the tower and 3 ft from the tip of the tower's shadow, the person's shadow starts to appear beyond the tower's shadow.

(a) Draw a picture to represent the problem, including the unknown height of the tower.
(b) Write an equation involving the unknown.
(c) What is the height of the tower?

(a)



2 ways

ratios:

$\frac{6}{3} = \frac{h}{135}$

$3h = 6(135)$

$h = \frac{6(135)}{3}$

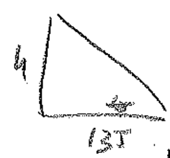
$\boxed{h = 270 \text{ ft}}$

trig

$\tan\theta = \frac{6}{3} = 2$

$\theta = \tan^{-1}(2)$

$\theta = 63.435^\circ$



$\tan\theta = \frac{h}{135}$

$h = 135 \tan 63.435^\circ$

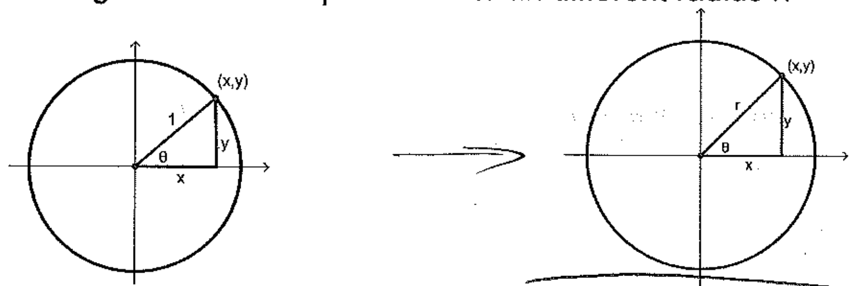
$h = \boxed{270 \text{ ft}}$

HAlg3-4, 4.4 day 1 Notes – Trig: Finding trig functions given a point, constraints

Quick review: cos and sin of an angle are defined as x, y coordinates of a point on a unit circle with angle in standard position:

cosine = the x coordinate
sine = the y coordinate

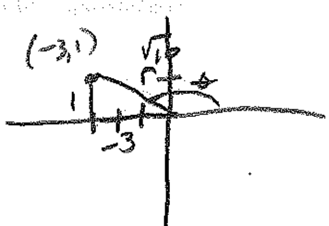
Cosine and sine are also ratios, which means they only depend on the angle, and not on the size of the triangle if we scale up radius 1 to an different radius r:



$\cos\theta = \frac{x}{1}$ $\quad\sin\theta = \frac{y}{1}$ $\longrightarrow$ $\boxed{\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}}$

This means we can find sine or cosine of an angle given the (x,y) coordinates of a point that is not on the unit circle...we just divide the x or y by the radius r.

Ex: Given x, y coordinates of a point (any radius) find 6 trig functions of the angle. (-3,1)



$r = \sqrt{(1)^2+(3)^2}$

$r = \sqrt{10}$

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$

$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{10}/10}{-3\sqrt{10}/10} = -\frac{1}{3}$

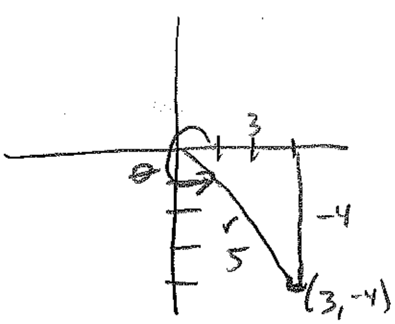
$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{10}}{1} = \sqrt{10}$

$\sec\theta = \frac{1}{\cos\theta} = -\frac{\sqrt{10}}{3}$

$\cot\theta = \frac{1}{\tan\theta} = -3$

1) sketch the point
2) draw in radius, find length w/ Pythagorean Thm,
3) use $\frac{x}{r}, \frac{y}{r}$ to find $\sin\theta, \cos\theta$
4) Find other 4 trig functions from sin, cos

Try it: Given x, y coordinates of a point (any radius) find 6 trig functions of the angle. (3,-4)



$\sin\theta = \frac{y}{r} = \frac{-4}{5}$

$\cos\theta = \frac{x}{r} = \frac{3}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-4/5}{3/5} = \frac{-4}{3}$

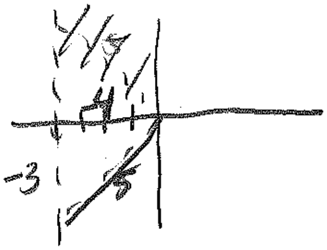
$\csc\theta = \frac{1}{\sin\theta} = \frac{-5}{4}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{-3}{4}$

Ex. Find all 6 trig functions if $\cos\theta = -\frac{4}{5}$, constraint and the angle lies in quadrant III

$= \frac{x}{r}$ so $x = -4$



$\sin\theta = \frac{y}{r} = \frac{-3}{5}$ $\qquad$ $\csc\theta = \frac{1}{\sin\theta} = \frac{-5}{3}$

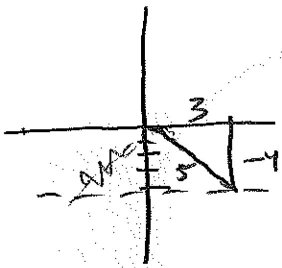
$\cos\theta = \frac{-4}{5}$ $\qquad$ $\sec\theta = \frac{1}{\cos\theta} = \frac{-5}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-3/5}{-4/5} = \frac{3}{4}$ $\qquad$ $\cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$

1) sketch line for x or y given in problem
2) draw in 2 radii
3) reject one from constraint
4) solve for missing side
5) steps in last example

Try it. Find all 6 trig functions if $\sin\theta = -\frac{4}{5}$, constraint and the angle lies in quadrant IV

$y = -4$



$\sin\theta = \frac{y}{r} = \frac{-4}{5}$ $\qquad$ $\csc\theta = \frac{1}{\sin\theta} = \frac{-5}{4}$

$\cos\theta = \frac{x}{r} = \frac{3}{5}$ $\qquad$ $\sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r} = \frac{y}{x} = \frac{-4}{3}$ $\qquad$ $\cot\theta = -\frac{3}{4}$

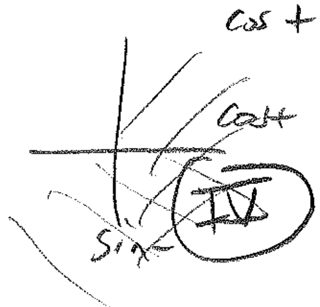
* tan example

The other 4 trig functions are defined from sine and cosine. Signs of the 6 trig functions in each quadrant:

S (students) — Quadrant II: sinθ +, cosθ −, tanθ −, cscθ +, secθ −, cotθ −	Quadrant I: sinθ +, cosθ +, tanθ +, cscθ +, secθ +, cotθ + — A (all).
T (take) — Quadrant III: sinθ −, cosθ −, tanθ +, cscθ −, secθ −, cotθ +	Quadrant IV: sinθ −, cosθ +, tanθ −, cscθ −, secθ +, cotθ − — C (calculus)

Ex: State in which quadrant the angle lies: $\sec\theta > \overset{+}{0}$ and $\cot\theta < 0$

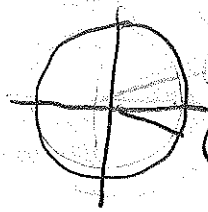
cos + $\qquad$ tan −
(sin −)



Ex: Evaluate sine, cosine and tangent of -750° without using a calculator.

$$\begin{array}{r} \overset{6}{\cancel{7}}50 \\ -360 \\ \hline 390 \\ -360 \\ \hline 30° \end{array}$$

(neg)



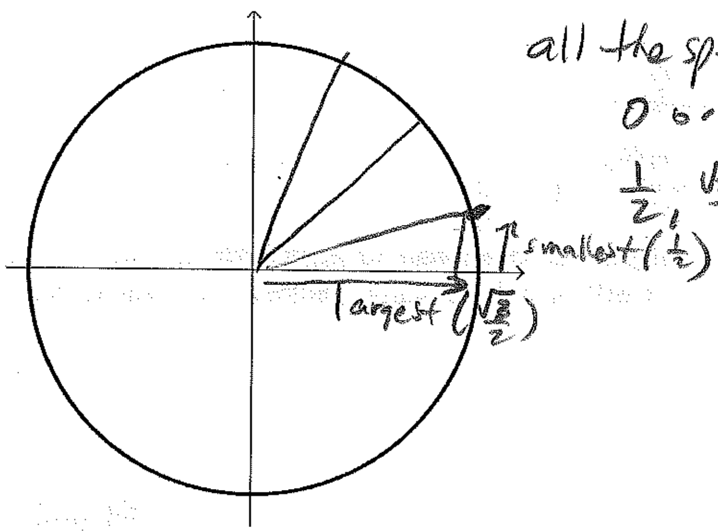
$\sin(-750°) = \frac{-1}{2}$

$\cos(-750°) = \frac{\sqrt{3}}{2}$

$\tan(-750°) = \frac{\sin(-750°)}{\cos(-750°)}$

$= \frac{-1/2}{\sqrt{3}/2} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

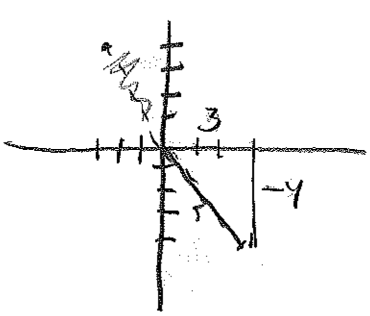
How to 'memorize' the unit circle special values...



all the special values are either
0 or 1 (quadrature angles)
$\frac{1}{2}$, $\frac{\sqrt{2}}{2}$ or $\frac{\sqrt{3}}{2}$ (30°, 45°, 60°)

* 3rd angle example find all 6 if $\tan\theta = \frac{-4}{3} = \frac{\sin}{\cos} = \frac{y}{x}$

$y = 4$, $x = 3$ one is pos, one is neg. Quad IV



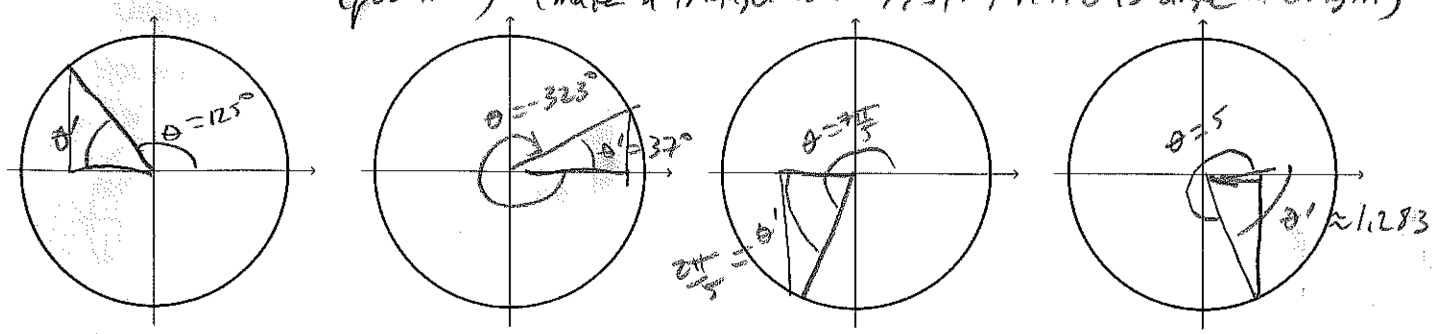
$\sin\theta = \frac{-4}{5}$ $\csc\theta = \frac{-5}{4}$

$\cos\theta = \frac{3}{5}$ $\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{-4}{3}$ $\cot\theta = \frac{-3}{4}$

HAlg3-4, 4.4 day 2 Notes – Trig: Reference angles, solving for angles

Reference angle = The acute angle θ' formed by the terminal side of θ and the horiz (x) axis.
(positive) (make a triangle with x, y, r, ref. θ is angle at origin)



$\theta = 125°$
$\theta' = 180 - 125 = 55°$

$\theta = -323°$
$\theta' = 360 - 323 = 37°$

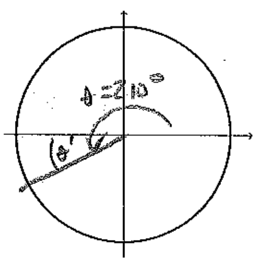
$\theta = \dfrac{7\pi}{5}$
$\theta' = \frac{7\pi}{5} - \pi = \frac{2\pi}{5}$

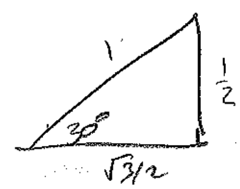
$\theta = 5$
$\theta' = 2\pi - 5 = 1.283..$

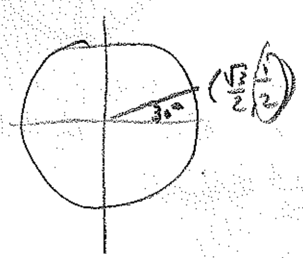
Can use reference angle to evaluate a trig function:

Ex: Find $\sin 210°$

$\theta' = 210 - 180 = 30°$







$\sin 30° = \frac{1}{2}$

quad III $\sin < 0$
so $\sin 210° = -\frac{1}{2}$

Another way to find other trig functions, given one: use identities: (move to end)

Ex: If θ is in quadrant II and $\sin\theta = \dfrac{1}{3}$, find $\cos\theta$ and $\tan\theta$

Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{1}{9}$

$\cos^2\theta = \frac{8}{9}$

$\cos\theta = \frac{\pm\sqrt{8}}{\sqrt{9}} = \frac{\pm\sqrt{8}}{3} = \frac{\pm 2\sqrt{2}}{3}$

quad II, cos neg, $\cos\theta = \boxed{-\frac{2\sqrt{2}}{3}}$

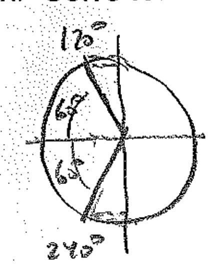
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/3}{-2\sqrt{2}/3} = \frac{-1}{2\sqrt{2}}$

$= \frac{-\sqrt{2}}{2 \cdot 2} = \boxed{-\frac{\sqrt{2}}{4}}$

Solving for the angle, given a trig function value:

Using unit circle:

Ex: Solve for θ: $\cos\theta = -\frac{1}{2}$ $(0 \le \theta \le 360°)$



$\theta = 120°$ or $240°$

(cosines share same x value)

Using calculator: (deg mode)

$\cos\theta = -\frac{1}{2}$

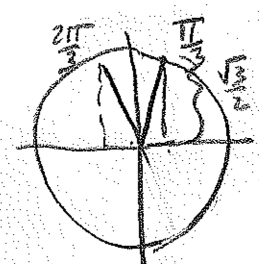
$\theta = \cos^{-1}(-\frac{1}{2}) = 120°$

(use U.C. to find other angle)

$\theta = 180 + 60 = 240°$

Ex: Solve for θ: $\csc\theta = \frac{2\sqrt{3}}{3}$ $(0 \le \theta \le 2\pi)$

$\sin\theta = \frac{3 \cdot \sqrt{3}}{2\sqrt{3} \cdot \sqrt{3}} = \frac{3\sqrt{3}}{2 \cdot 3} = \frac{\sqrt{3}}{2}$



$\boxed{\theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}}$

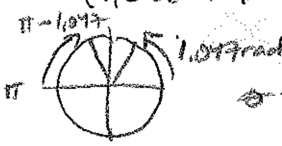
(sines share same y value)

(rad mode)

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \sin^{-1}(\frac{\sqrt{3}}{2}) = 1.047\ldots$ $(= \frac{\pi}{3})$

(need U.C. to find other angle



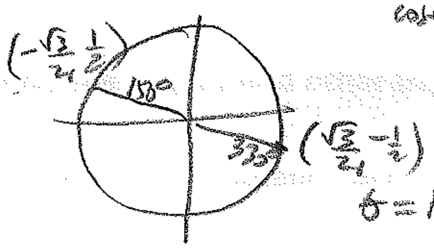
$\theta = \pi - 1.047 = 2.094\ldots$ $(= \frac{2\pi}{3})$

Ex: Solve for θ: $\cot\theta = -\sqrt{3}$ $(0 \le \theta \le 360°)$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-1}{\sqrt{3}}$

$\frac{\sin\theta}{\cos\theta} = \frac{-1/2}{\sqrt{3}/2} = \frac{y/r}{x/r}$

or $= \frac{1/2}{-\sqrt{3}/2}$



$\theta = 150°$ or $330°$

(tangents across circle)

(deg mode)

$\cot\theta = -\sqrt{3}$

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}(\frac{-1}{\sqrt{3}})$

$\theta = -30°$

$= -30 + 360 = \boxed{330°}$

(need UC for other angle)

$\theta = 330 - 180 = \boxed{150°}$

Ex: Solve for θ: $\cos\theta = 0.9848$ $(0 \le \theta \le 360°)$

0.9848 not a special value →

$\cos\theta = 0.9848$

$\theta = \cos^{-1}(0.9848)$

$\theta = \boxed{10° \text{ also } 350°}$