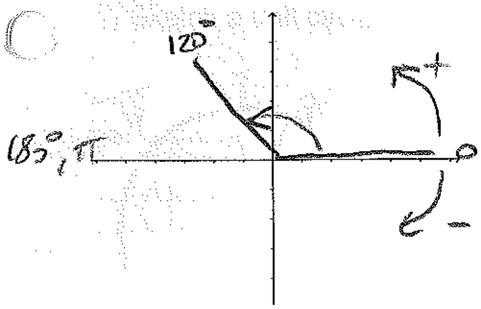
HAlg3-4: Trig topics review

Angles in standard position:



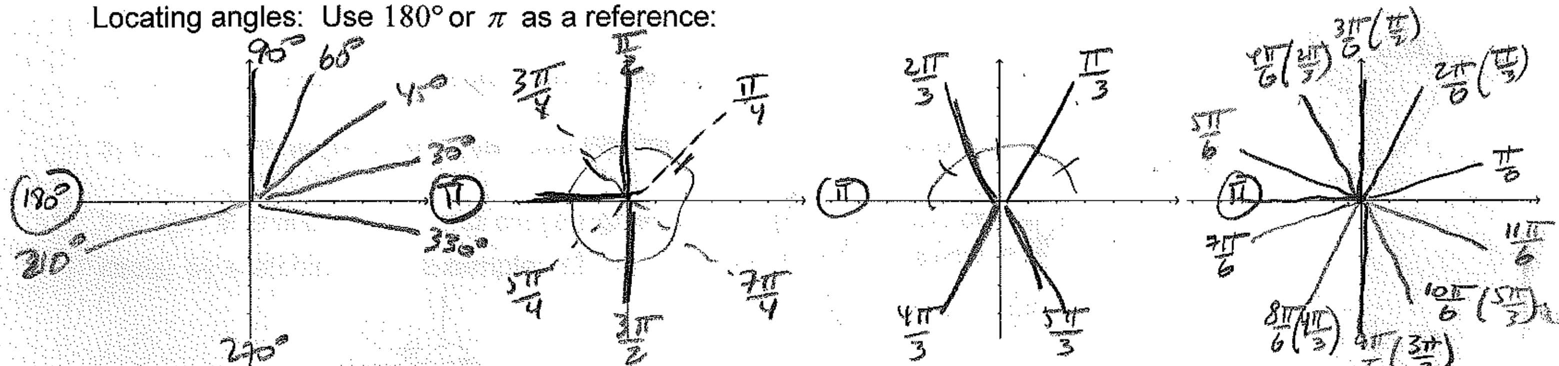
0 reference to right (x-axis), positive CCW, negative CW.

degrees (360 in a circle)

radians (2π in a circle)
1 radian = distance of one radius around the circle.

Converting: $180° = \pi$ 45°; $45° \frac{\pi}{180} = \frac{\pi}{4}$ $\frac{3\pi}{2} \frac{180}{\pi} = 270°$

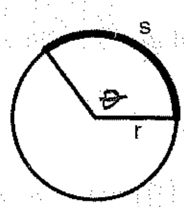
Locating angles: Use $180°$ or π as a reference:



Coterminal angles: add or subtract multiples of 2π *or* $360°$. 60°, 420° coterminal 2π, 4π coterminal

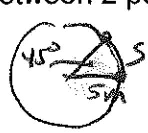
Complementary (add to $90°$ *or* $\frac{\pi}{2}$), Supplementary (add to $180°$ *or* π). comp of 60° = 90 − 60 = 30°
comp of $\frac{\pi}{3} = \frac{\pi}{2} - \frac{\pi}{3} = \frac{2\pi}{3}$

Arc length problems: $s = r\theta$, θ *in radians*



1 radian = distance of one radius around the circle.

What is distance between 2 points on a circle of radius 5m, if angle between points is 45 degrees?

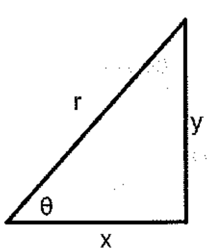


$\theta = 45° \frac{\pi}{180} = \frac{\pi}{4}$

$s = r\theta$
$s = (5m)(\frac{\pi}{4})$
$s = \frac{5}{4}\pi$ m

Two ways to view sine, cosine:

Ratio view



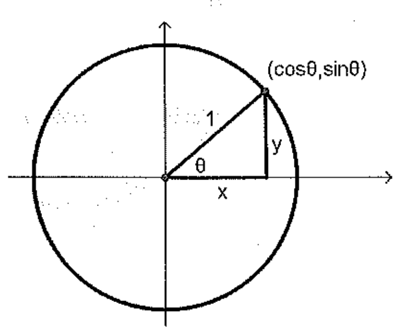
$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$

$$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$$

For angles up to $90°$.
Used for solving real-world problems
(angle of elevation/depression =
angle above/below a horizontal line.)

Unit circle / function view



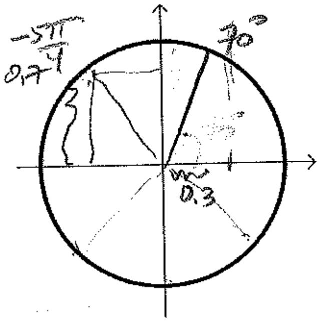
$\cos\theta = x$ *coordinate*
$\sin\theta = y$ *coordinate*
of a point on unit circle

Function definition, input interpreted as an angle,
output is a number (x or y value), input can be any 'angle'
(pos, neg, >360, etc.)

Domain/range of sine and cosine: $D:(-\infty,\infty)$ $R:(-1,1)$

Computing output of a sine or cosine function. Interpret the input as an angle in standard position on a unit circle. Output is x or y value of point on the circle at that angle. 3 ways:

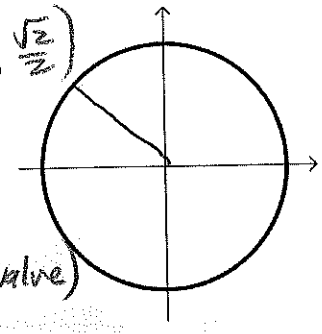
1) Sketch a unit circle and measure the x or y distance:



Find $\cos(70°) \approx 0.3$ $\qquad$ $\sin\left(-\frac{5\pi}{4}\right) \simeq 0.7 = \frac{\sqrt{2}}{2}$

(0.342) $\qquad$ (0.7071)

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

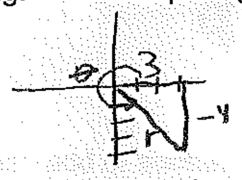


2) Use a calculator (make sure mode is set correctly).

3) For 'special angle values' can use the unit circle to lookup sin or cos: (gives exact value)

You can also find $\sin(\theta)$ and $\cos(\theta)$ given a point (even if it isn't on the unit circle) by sketching: $\quad$ Example: (3, -4) $\qquad$ $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$

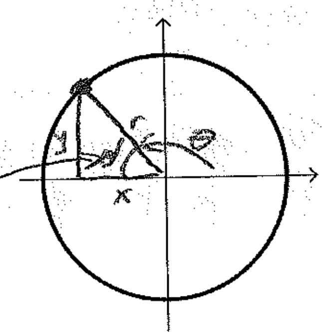
$\cos\theta = \frac{3}{5}$ $\quad$ $\sin = \frac{-4}{5}$



$r=5$

Reference angle: Make a triangle with x, y, and r.
Reference angle is acute angle in triangle at origin:

ref. angle θ'



The other 4 trig functions are defined from sine and cosine:

sine: $\sin(\theta) = \dfrac{1}{\csc(\theta)}$ $\qquad$ cosecant: $\csc(\theta) = \dfrac{1}{\sin(\theta)}$

cosine: $\cos(\theta) = \dfrac{1}{\sec(\theta)}$ $\qquad$ secant: $\sec(\theta) = \dfrac{1}{\cos(\theta)}$

tangent: $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{1}{\cot(\theta)}$ $\qquad$ cotangent: $\cot(\theta) = \dfrac{1}{\tan(\theta)}$

Trig identities: $\quad$ <u>Reciprocal identities:</u> $\quad$ <u>Quotient identities:</u> $\quad$ <u>Pythagorean identities:</u>

$\sin\theta = \dfrac{1}{\csc\theta}$ $\quad$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\quad$ $\tan\theta = \dfrac{1}{\cot\theta}$ $\qquad$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\sin^2\theta + \cos^2\theta = 1$

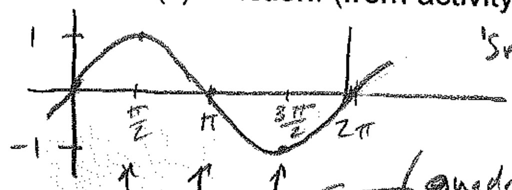
$\csc\theta = \dfrac{1}{\sin\theta}$ $\quad$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\quad$ $\cot\theta = \dfrac{1}{\tan\theta}$ $\qquad$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ $\qquad$ $1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Simplify: $\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \dfrac{1}{\sin\theta} = \boxed{\csc\theta}$

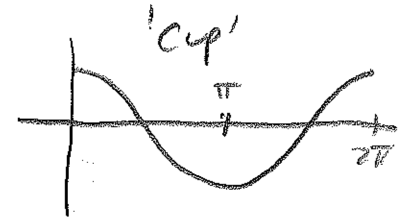
HAlg3-4, 4.5 Notes – Graphs of Sine and Cosine functions

Graph of basic sin(x) function: (from activity)



'Snake'

cos 'cup'



↑ ↑ ↑ ← (quadrature angles)

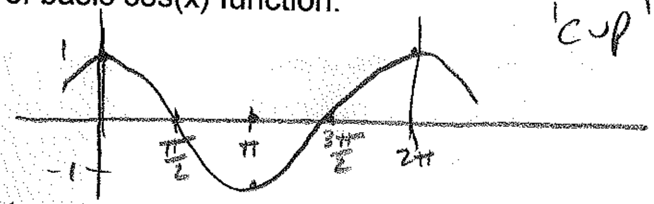
Things to note:

Period: 2π

Domain / Range: D: $(-\infty, \infty)$ R: $[-1, 1]$

Keypoints: start: 0 intercepts: ends and middle
end: 2π max/min: max, 1/4 period, min 3/4 period

Graph of basic cos(x) function: 'cup'



Things to note:

Period: 2π

Domain / Range: D: $(-\infty, \infty)$ R: $[-1, 1]$

Keypoints: start: 0 intercepts: 1/4 period, 3/4 period
end: 0 max/min: max ends, min middle

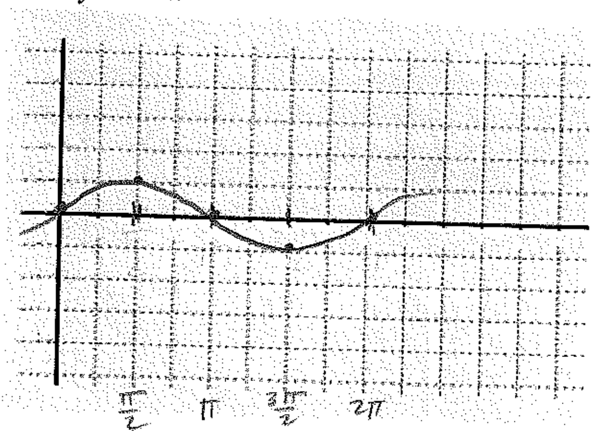
Modifications to basic equation: (sin as an example, cos similar)

$y = d + a\sin(bx - c)$ where a, b, c, d are constants ex: $y = 2 - 3\sin(5x - \frac{\pi}{2})$

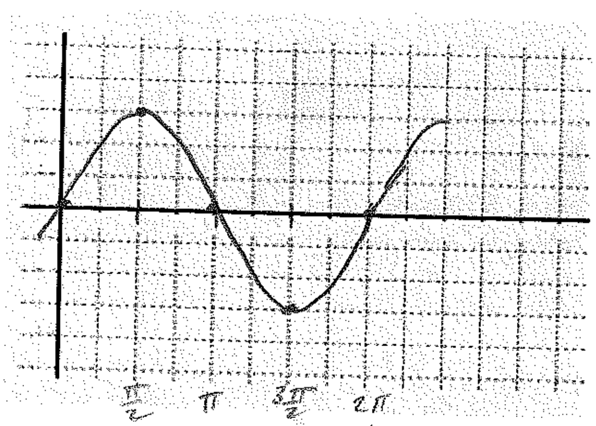
a => affects amplitude (vertical stretch)
b => affects period (horizontal stretch)
c => affects phase shift (horizontal shift)
d => affects vertical shift

Amplitude: Amplitude = $|a|$ (is always positive)

$y = \sin x$

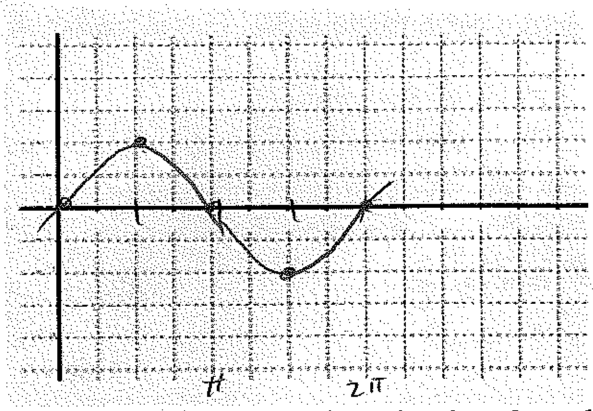


$y = 3\sin x$

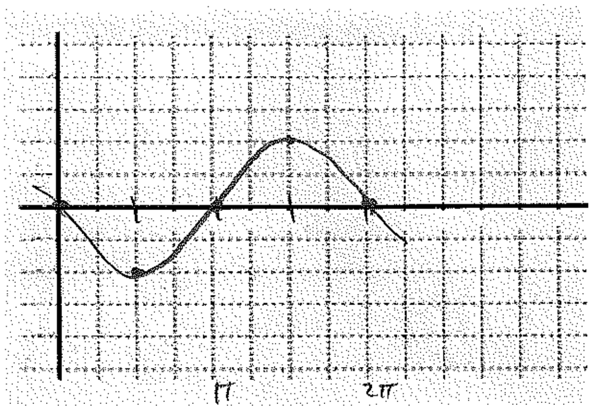


If a is negative, curve is reflected over x-axis (flips vertically):

$y = 2\sin x$



$y = -2\sin x$



Period: period = how long it takes for a full cycle

for basic $y = \sin x$, period = 2π

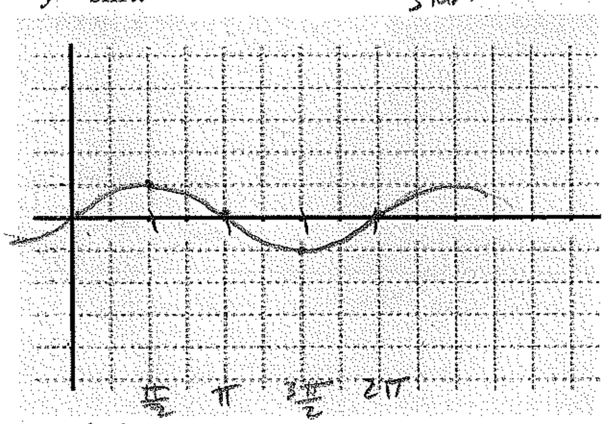
if $y = \sin bx$, how does b affect period? Solve the following inequality for x:

$$\frac{0}{b} \le \frac{bx}{b} \le \frac{2\pi}{b}$$

$$\underset{\text{start}}{0} \le x \le \underset{\text{end}}{\frac{2\pi}{b}}$$

$$\boxed{\text{period} = \frac{2\pi}{b}}$$

$y = \sin x$



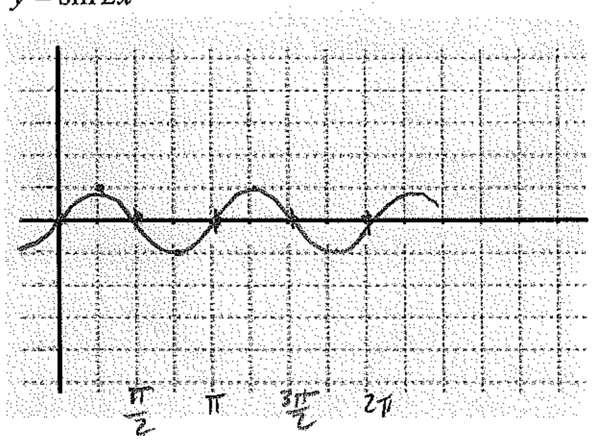
$0 \le x \le 2\pi$

start

period = 2π

end

$y = \sin 2x$



$$\frac{0}{2} \le \frac{2x}{2} \le \frac{2\pi}{2}$$

$0 \le x \le \pi$

start

period = π

end

or $\sin 2x$ ← 2 complete cycles in the original 2π period

Phase Shift: = horizontal shift

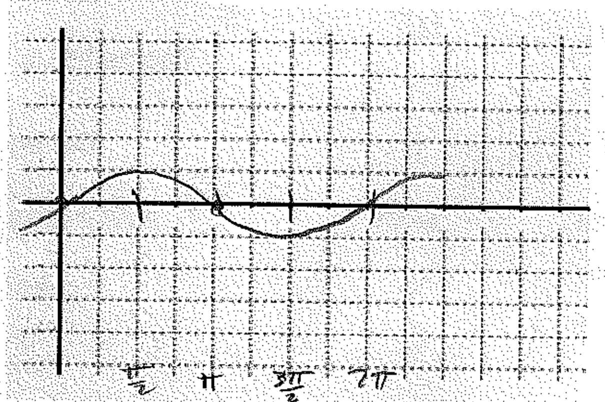
if $y=\sin(bx-c)$, what effect does c have? Solve the following inequality for x:

$$0 \le bx - c \le 2\pi$$
$$+c \quad\quad +c \quad\quad +c$$
$$\frac{c}{b} \le \frac{bx}{b} \le \frac{2\pi}{b} + \frac{c}{b}$$
$$\frac{c}{b} \le x \le \frac{2\pi}{b} + \frac{c}{b}$$

period

$\boxed{\text{phase shift} = \frac{c}{b}}$

$y=\sin x$



$0 \le x \le 2\pi$

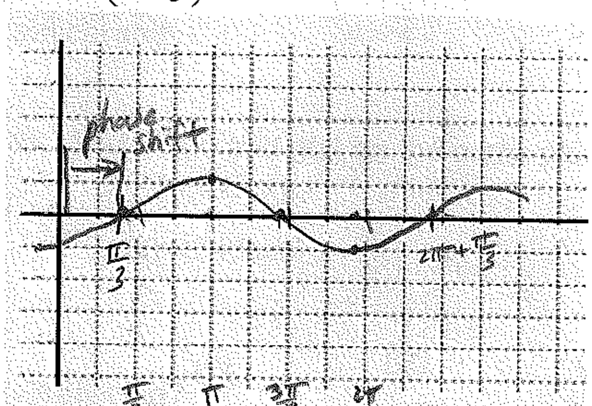
start

end

period $= 2\pi$

phase shift = none

$y=\sin\left(x-\frac{\pi}{3}\right)$



$$0 \le x - \frac{\pi}{3} \le 2\pi$$
$$+\frac{\pi}{3} \quad\quad +\frac{\pi}{3} \quad\quad +\frac{\pi}{3}$$
$$\frac{\pi}{3} \le x \le 2\pi + \frac{\pi}{3}$$

phase shift $= \frac{\pi}{3}$ right

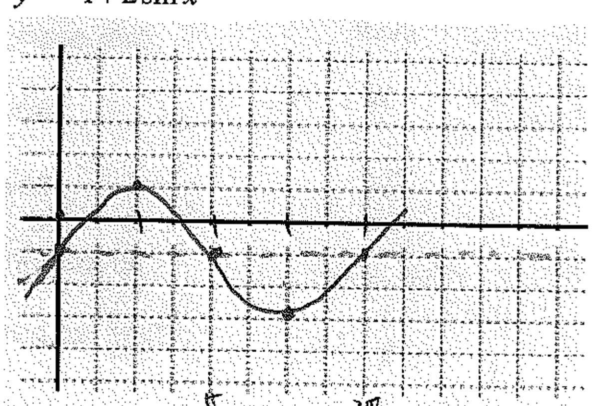
start

period $= 2\pi$

end at $2\pi + \frac{\pi}{3}$

Vertical Shift: d just shifts graph up or down. When graphing, use d to find a new 'baseline'...

$y=-1+2\sin x$



Examples:

#1. Find the period and amplitude of $y=-2\cos 3x$

amplitude $= |a| = |-2| = \boxed{2}$

$\frac{0}{3} \le \frac{3x}{3} \le \frac{2\pi}{3}$

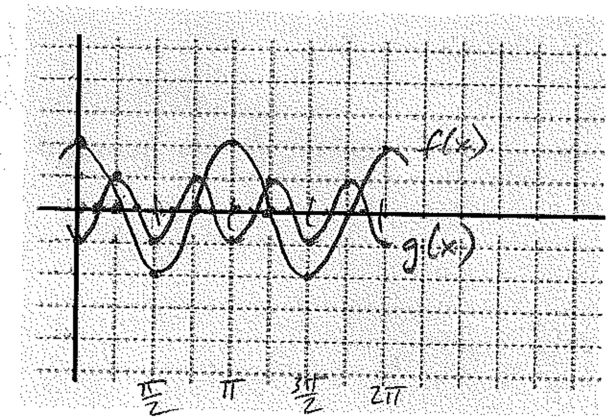
$0 \le x \le \frac{2\pi}{3}$ period $= \boxed{\frac{2\pi}{3}}$

#2. Sketch f(x) and g(x) on the same plane (by hand)

$f(x)=2\cos 2x$

$g(x)=-\cos 4x$

	f(x)	g(x)
$\lvert a \rvert$/reflected?	2/no	1/yes
period:	π	$\frac{\pi}{2}$
phase shift:	none	none
start:	0	0
end:	π	$\frac{\pi}{2}$
intercepts:	~~$\frac{PD}{4}, \frac{7PD}{4}$~~ $\frac{\pi}{4}, \frac{3\pi}{4}$	$\frac{\pi}{8}, \frac{3\pi}{8}$
vertical shift:	none	none

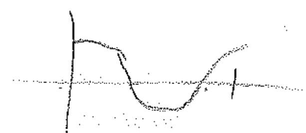


$0 \le 2x \le 2\pi$

$0 \le x \le \pi$

$0 \le 4x \le 2\pi$

$0 \le x \le \frac{\pi}{2}$

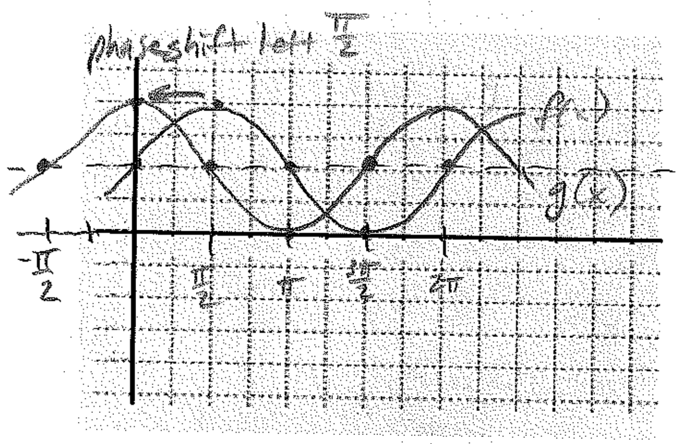


#3. Sketch f(x) and g(x) on the same plane (by hand)

$f(x)=2+2\sin x$

$g(x)=2+2\sin\left(x+\frac{\pi}{2}\right)$

	f(x)	g(x)
$\lvert a \rvert$/reflected?	2/no	2/no
period:	2π	2π
phase shift:	none	$-\frac{\pi}{2}$
start:	0	$-\frac{\pi}{2}$
end:	2π	$2\pi-\frac{\pi}{2}$
intercepts:	ends, middle	ends, middle
vertical shift:	2 up	2 up

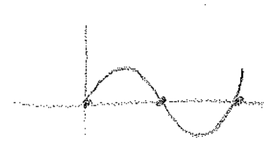


$0 \le x \le 2\pi$

$0 \le x+\frac{\pi}{2} \le 2\pi$

$-\frac{\pi}{2} \quad -\frac{\pi}{2} \quad -\frac{\pi}{2}$

$-\frac{\pi}{2} \le x \le 2\pi - \frac{\pi}{2}$



HAlg3-4, 4.5 day 2 Notes – Graphs of Sine and Cosine functions

Sketch by hand:

$y = 2 + 2\sin\left(x + \frac{\pi}{2}\right)$

$|a|$ /reflected? 2, no

period: 2π

phase shift: $-\frac{\pi}{2}$

start: $-\frac{\pi}{2}$

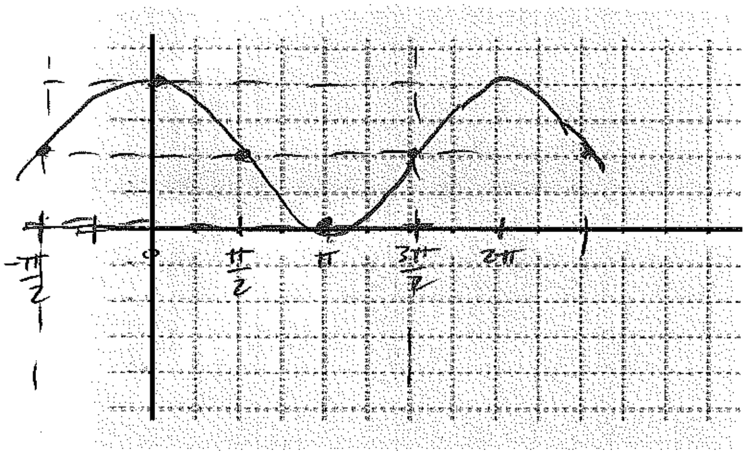
end: $\frac{3\pi}{2}$

intercepts: $-\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}$

vertical shift: 2

$0 \le x + \frac{\pi}{2} \le 2\pi$

$-\frac{\pi}{2} \le x \le 2\pi - \frac{\pi}{2}$



Sketch by hand:

$y = 2\sin \pi x + 1$

$|a|$ /reflected? 2, no

period: 2

phase shift: 0

start: 0

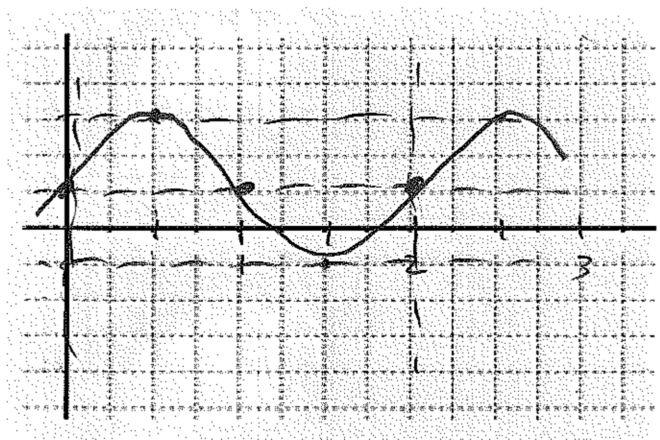
end: 2

intercepts: 0, 2, 1

vertical shift: 1

$0 \le \pi x \le 2\pi$

$0 \le x \le 2$



Sketch by hand:

$y = 3\cos\frac{x}{2}$

$|a|$ /reflected? 3, no

period: 4π

$0 \le \frac{x}{2} \le 2\pi$

$0 \le x \le 4\pi$

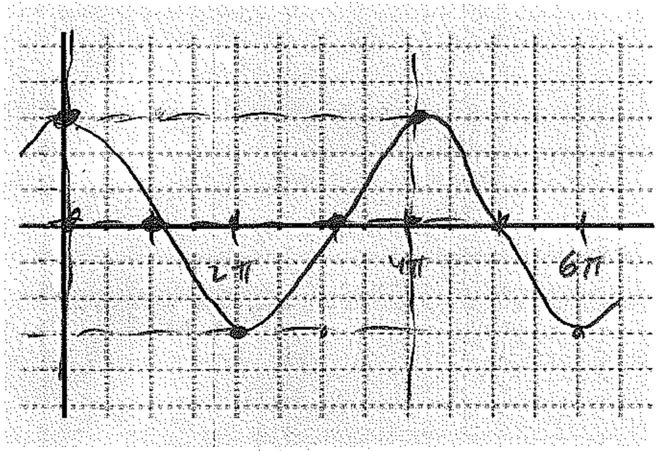
phase shift: 0

start: 0

end: 4π

intercepts: π, 3π

vertical shift: 0



Sketch by hand:

$y = \frac{1}{2} - \frac{3}{2}\cos 2\left(x - \frac{\pi}{4}\right)$

$|a|$ /reflected? $\frac{3}{2}$, yes

period: π

phase shift: $\frac{\pi}{4}$

start: $\frac{\pi}{4}$

end: $\frac{5\pi}{4}$

intercepts: $\frac{\pi}{2}$, π

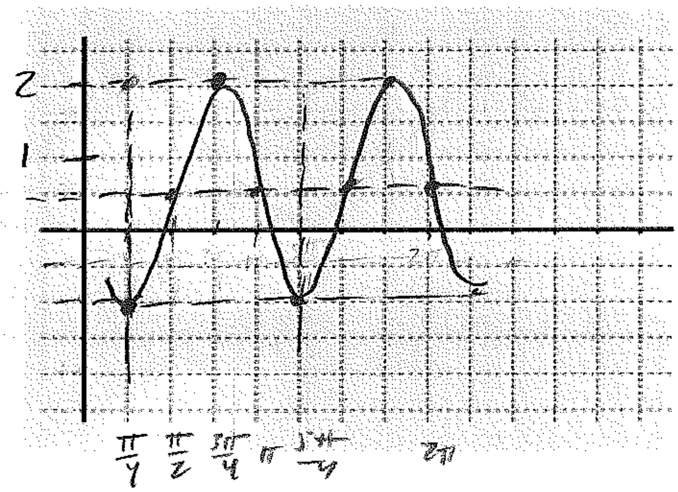
vertical shift: $\frac{1}{2}$

$0 \le 2(x - \frac{\pi}{4}) \le 2\pi$

$0 \le 2x - \frac{\pi}{2} \le 2\pi$

$\frac{\pi}{2} \le 2x \le 2\pi + \frac{\pi}{2}$

$\frac{\pi}{4} \le x \le \pi + \frac{\pi}{4}$



HAlg3-4, 4.6 Notes – Graphs of Other Trig functions

Graph of tangent function: $y = \tan x$

$\tan 0 = \frac{0}{1} = 0$

$\tan\frac{\pi}{4} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

$\tan\frac{\pi}{2} = \frac{1}{0} = \text{undef}$

$\tan\frac{3\pi}{4} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$

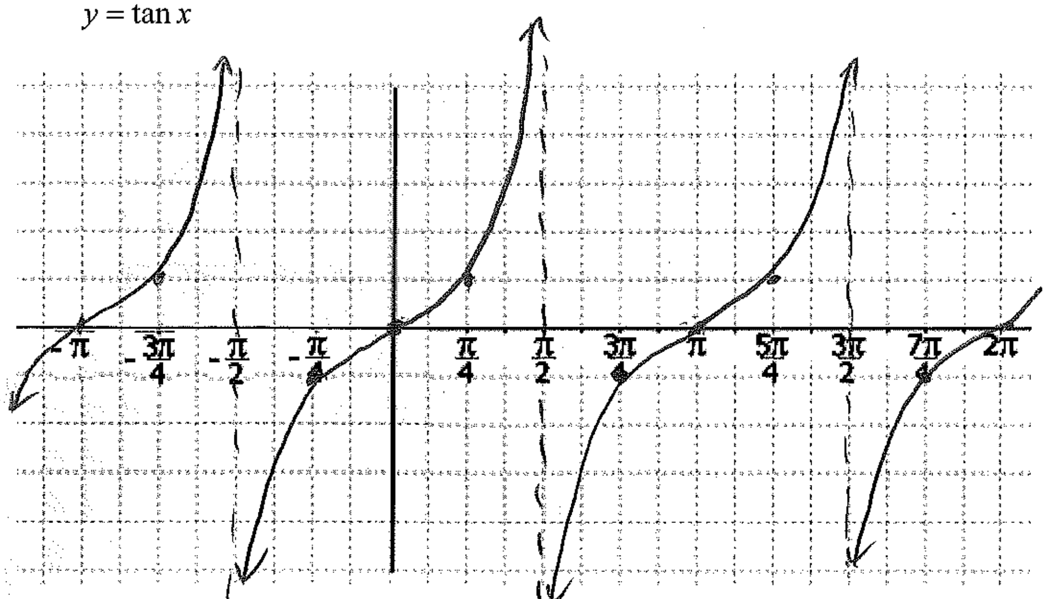
$\tan\pi = \frac{0}{-1} = 0$

$\tan\frac{5\pi}{4} = \frac{-\sqrt{2}/2}{-\sqrt{2}/2} = 1$

$\tan\frac{3\pi}{2} = \frac{-1}{0} = \text{und}$

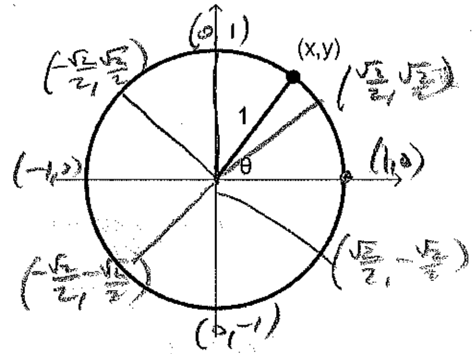
$\tan\frac{7\pi}{4} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

$\tan 2\pi = \frac{0}{1} = 0$



$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{\to 1}{\to 0}$



Things to note:

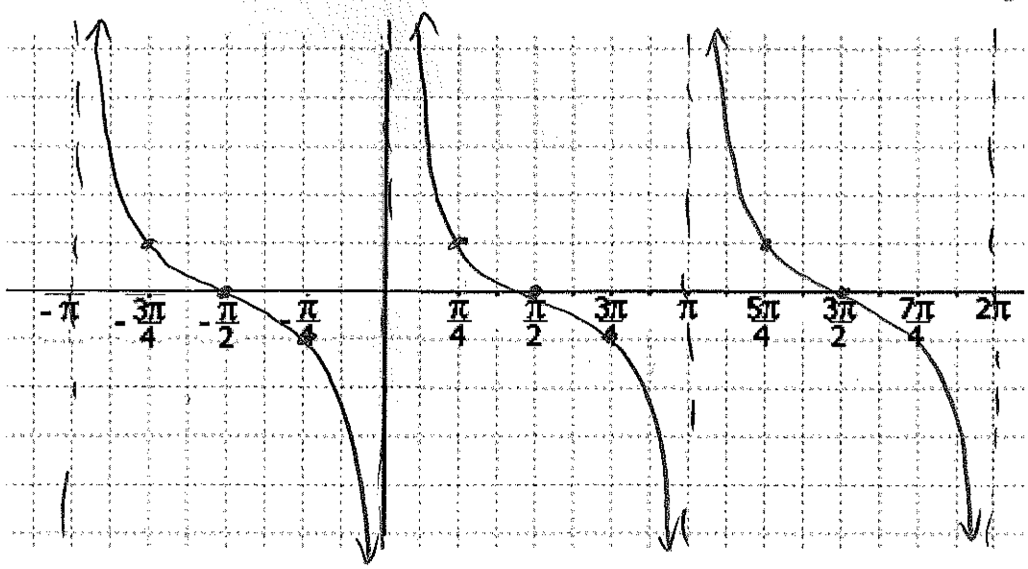
Period: π

Domain / Range: D: $\mathbb{R}$, $x \neq \frac{\pi}{2} \pm n\pi$ R: $(-\infty, \infty)$

Keypoints: 0 @ $n\pi$

± 1 @ $\frac{\pi}{4} \pm n\pi$

Graph of cotangent function: $y = \cot x$ $\cot x = \frac{1}{\tan x}$



Things to note:

Period: π

Domain / Range:
D: $\mathbb{R}$, $x \neq n\pi$
R: $(-\infty, \infty)$

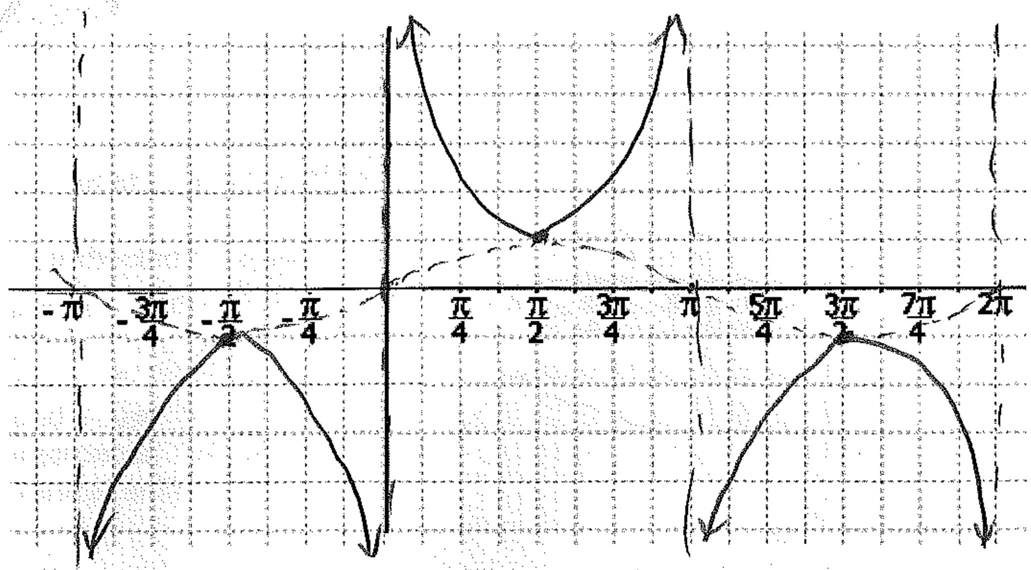
Keypoints:

0 @ $\frac{\pi}{2} \pm n\pi$

± 1 @ $\frac{\pi}{4} \pm n\pi$

graph from tan curve

raph of cosecant function: $y = \csc x$ $\csc x = \frac{1}{\sin x}$



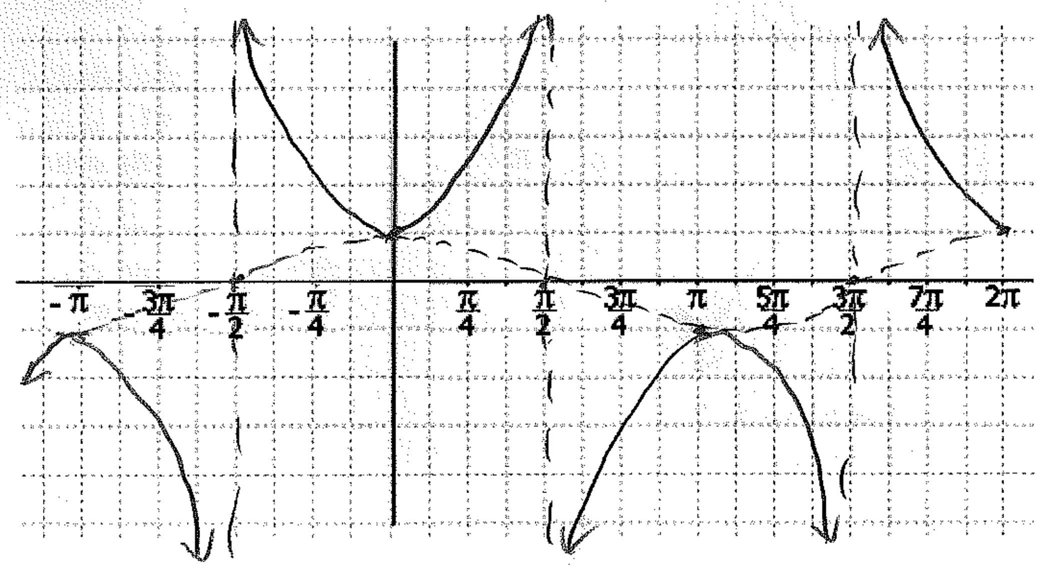
Things to note:

Period: 2π

Domain / Range:
D: $\mathbb{R}$ $x \neq \pm n\pi$
R: $(-\infty,-1] \cup [1,\infty)$

Keypoints:
graph from sin curve.

Graph of secant function: $y = \sec x$ $\sec x = \frac{1}{\cos x}$



Things to note:

Period: 2π

Domain / Range:
D: $\mathbb{R}$, $x \neq \frac{\pi}{2} \pm n\pi$
R: $(-\infty,-1] \cup [1,\infty)$

Keypoints:
graph from cos curve

Examples:

Graph the function:

$y = 2\csc\left(\frac{x}{2}\right)$

Consider: $y = 2\sin\left(\frac{x}{2}\right)$

$|a|$ /reflected? 2/no

period: 4π

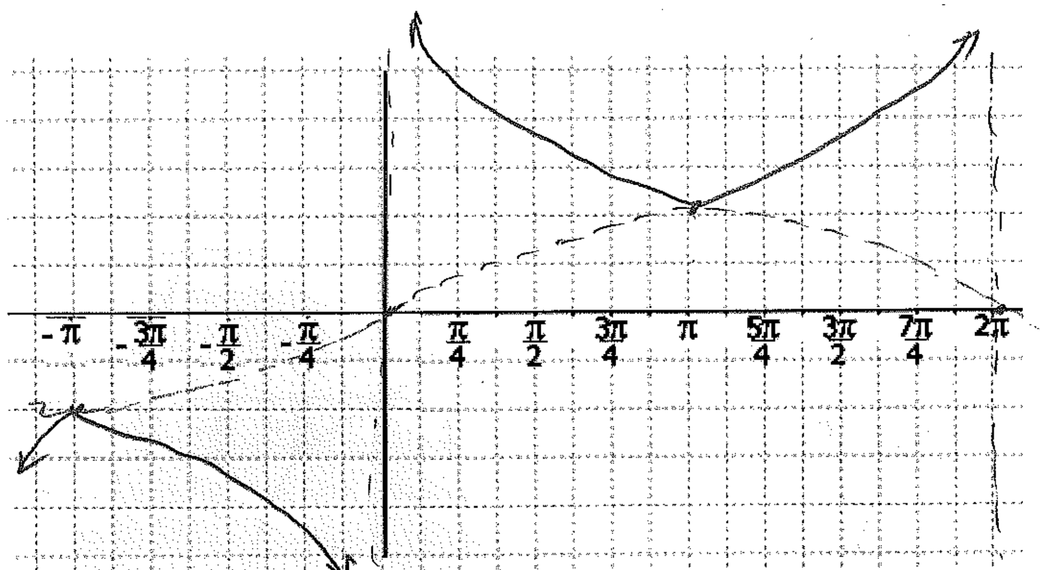
phase shift: none

start: 0

end: 4π

intercepts: start, end, middle

vertical shift: none



$0 \le \frac{x}{2} \le 2\pi$

$0 \le x \le 4\pi$

Graph the function:

$y = \sec(x + \pi)$

Consider: $\cos(x + \pi)$

$|a|$ /reflected? 1/no

period: 2π

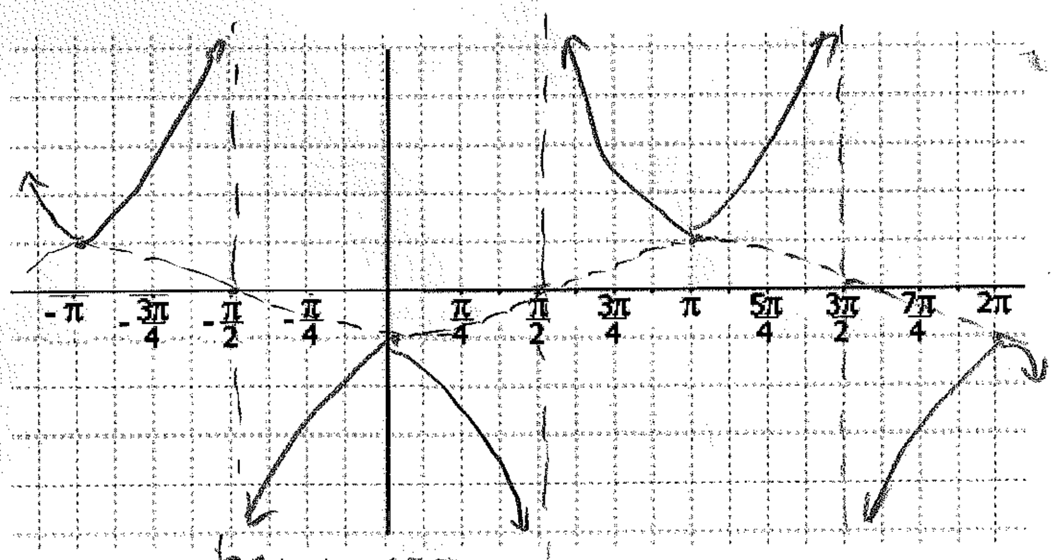
phase shift: $-\pi$

start: $-\pi$

end: $2\pi - \pi$ (π)

intercepts: PD/4, 3PD/4

vertical shift: none



$0 \le x + \pi \le 2\pi$

$-\pi \le x \le 2\pi - \pi$

Graph the function:

$y = \frac{1}{3}\sec\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$

Consider: $\frac{1}{3}\cos\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$

$|a|$ /reflected? $\frac{1}{3}$/no

period: 4

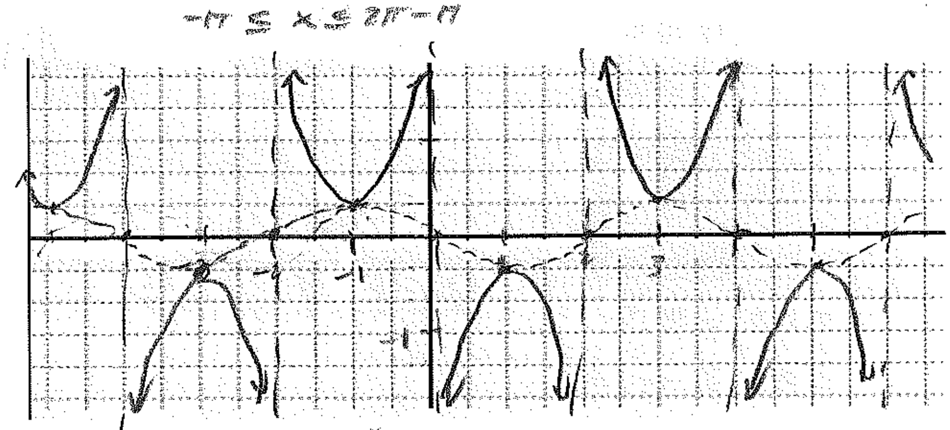
phase shift: -1

start: -1

end: 3

intercepts: PD/4, 3PD/4

vertical shift: none



$0 \le \frac{\pi}{2}x + \frac{\pi}{2} \le 2\pi$

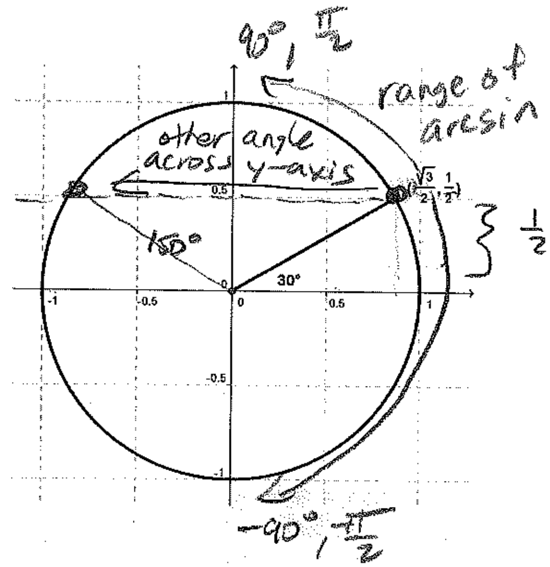
$-\frac{\pi}{2} \le \frac{\pi}{2}x \le 2\pi - \frac{\pi}{2}$

$-1 \le x \le 4 - 1$

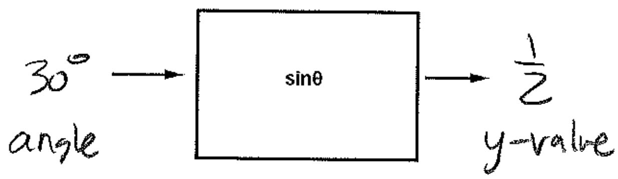
HAlg3-4, 4.7 day 1 Notes – Inverse Trig Functions

In solving right triangle problems, we sometimes needed to find an angle: $\sin\theta = 0.5$ and we used our inverse trig calculator functions:

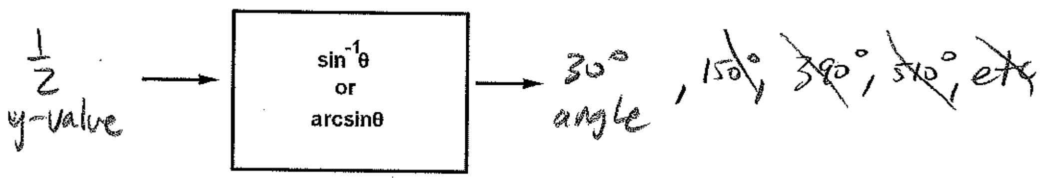
Let's look at this from our definition of the sine function with the unit circle:



The sine function takes an angle ($30°$) as input and returns as output the number $\frac{1}{2}$ which is the y-coordinate on the unit circle. We could think of the sine function like a 'machine':



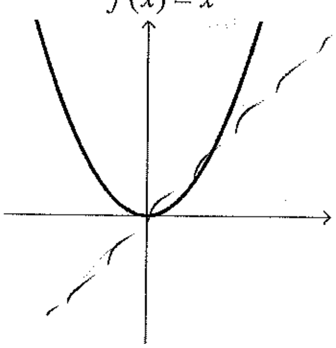
What would an inverse sine function do? The reverse:



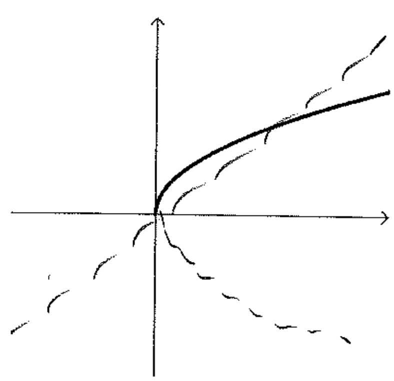
The input would be the 'height' (y-coordinate) and the inverse function would return the angle. But there are multiple angles that have this y-value (sine value). Which one do we use?

Now we'll more formally define the inverse trig functions for sin, cos and tan. Inverse functions in general are symmetric over the line y=x:

$f(x) = x^2$

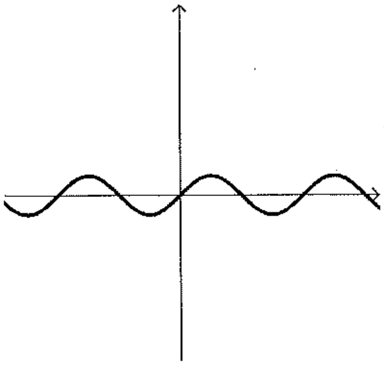


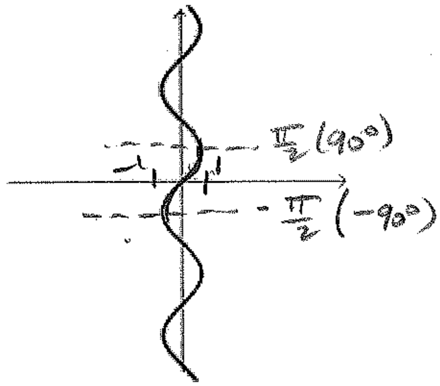
$f^{-1}(x) = \sqrt{x}$



By reflecting the function f(x)=sinx over the line y=x, we get the inverse sine curve:

$f(x)=\sin(x)$ $\qquad\qquad\qquad$ $f^{-1}(x)=\sin^{-1}(x)$



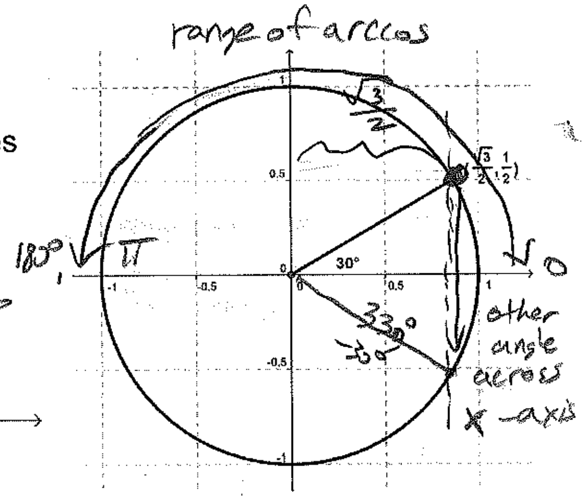


Inverse Sine Function:

$y=\sin^{-1}(x)$ or $y=\arcsin(x)$ 'arc sine of x' = 'the arc (angle) whose sine is x'

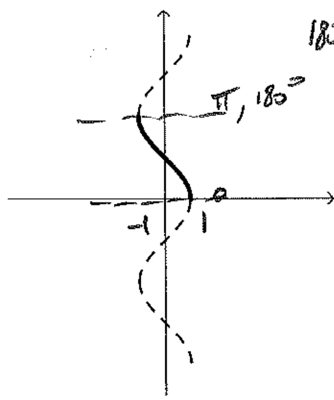
−90° 90°

$D:[-1,1]$ $R:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$



The same thing happens with cosine…there are multiple angles that share the same cosine value:

Inverse Cosine Function:



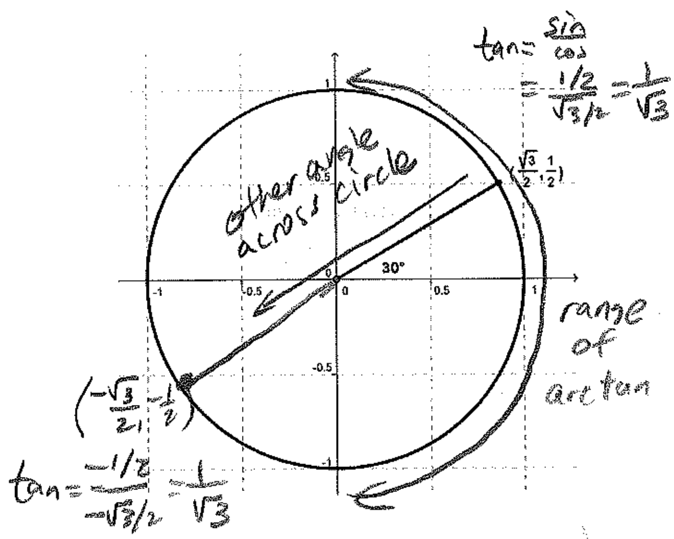
$y=\cos^{-1}(x)$ or $y=\arccos(x)$

0 180°

$D:[-1,1]$ $R:[0,\pi]$

Inverse Tangent Function:

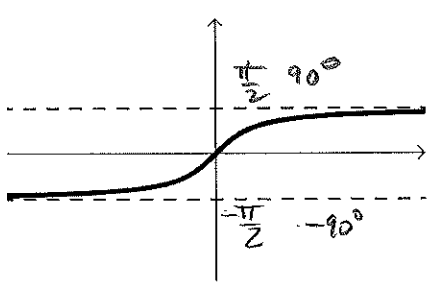
Tangent is defined by sine and cosine



$y=\tan^{-1}(x)$ or $y=\arctan(x)$

−90° 90°

$D:[-\infty,\infty]$ $R:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$



Using inverse trig functions:

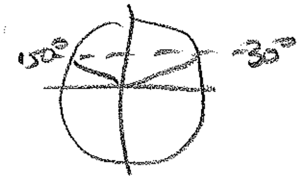
We can use inverse trig function to solve equations for an angle, but we have to be aware that using a calculator will only provide one value – the one in the range of the inverse sine function:

$\sin\theta = 0.5$ $\quad (y = \frac{1}{2})$

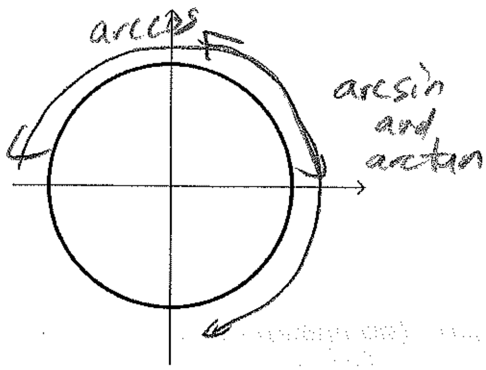
$\arcsin(\sin\theta) = \arcsin(0.5)$

$\theta = 30^\circ$ or 150°

$\frac{\pi}{6}$ or $\frac{5\pi}{6}$



Ranges of the inverse trig functions



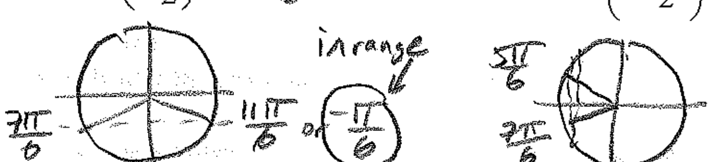
Real-world problems: may need other angle
Evaluate problems: give only the angle in the range.

Examples: If possible, find the exact value:

$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ $\quad y = -\frac{1}{2}$

$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ $\quad x = -\frac{\sqrt{3}}{2}$

$\arcsin(2)$ $\quad y = 2$



not possible, 2 is not in domain of arcsin [−1, 1]

Write each trigonometric expression in inverse function form, or vice versa:

$\sin(45^\circ) = \frac{\sqrt{2}}{2} \leftrightarrow$ $\underline{\arcsin\left(\frac{\sqrt{2}}{2}\right) = 45^\circ}$ $\qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \leftrightarrow$ $\underline{\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}}$

$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} \leftrightarrow$ $\underline{\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}}$ $\qquad \tan\left(\frac{\pi}{4}\right) = 1 \leftrightarrow$ $\underline{\arctan(1) = \frac{\pi}{4}}$

Evaluate the expression using a calculator:

$\arcsin\frac{1}{2} = 0.5236$

$\arctan\frac{\sqrt{3}}{3} = 0.5236$

Use a calculator to approximate the value of the expression:

$\arcsin(-0.125) = -0.1253$

$\arctan(2.8) = 1.2278$

HAlg3-4, 4.7 day 2 Notes – Inverse Trig Functions

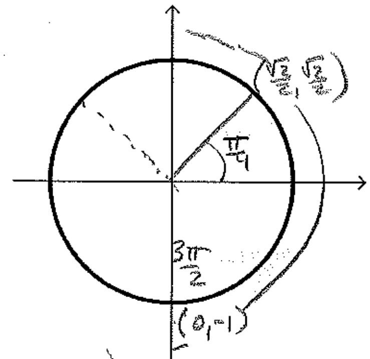
Properties of Inverse Trig Functions:

arcsin and sin are inverses, so, in general, they 'cancel each other out':

$$\sin\left(\arcsin\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}$$

also...

$$\arcsin\left(\sin\frac{\pi}{4}\right)=\frac{\pi}{4}$$



..but only if the argument is in the range of arcsine.

$$\arcsin\left(\sin\frac{3\pi}{2}\right)=-\frac{\pi}{2}$$ ($\frac{3\pi}{2}$ is not in range of arcsine) R: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\arcsin(-1)=-\frac{\pi}{2}$

Same is true for arccos and arctan...they 'cancel cos and tan' but be careful that inverse functions can only give values in their ranges.

Examples:

always ok

$\tan(\arctan 25)=25$

always ok

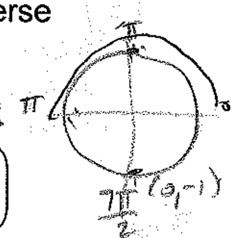
$\sin(\arcsin(-0.2))=-0.2$

inverse on outside! ! caution!

$\arccos\left(\cos\left(\frac{7\pi}{2}\right)\right)$

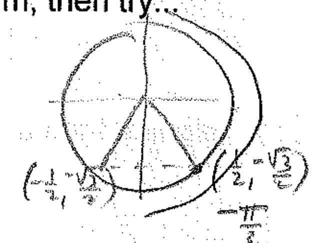
$\arccos(0)$ R: $[0,\pi]$

$=\boxed{\frac{\pi}{2}}$



Evaluate the expression without using a calculator: rewrite in non-inverse form, then try...

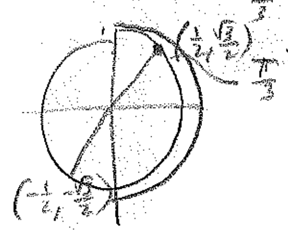
...unit circle look-up: $\arcsin\left(-\frac{\sqrt{3}}{2}\right) \overset{=\theta}{\Longleftrightarrow} \sin\theta=\frac{-\sqrt{3}}{2}$ $\theta=\boxed{\frac{-\pi}{3}}$



...divide top & bottom by 2 'trick': $\arctan\left(\sqrt{3}\right) \overset{=\theta}{\Longleftrightarrow} \tan\theta=\sqrt{3}=\boxed{\frac{\pi}{3}}$

$=\frac{\sqrt{3}/2}{1/2}$ ← sin ← cos

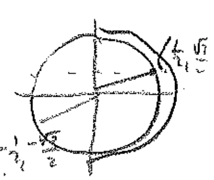
or $\frac{-\sqrt{3}/2}{-1/2}$



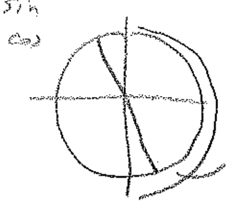
...moving radical to other side 'trick': $\arctan\left(\frac{\sqrt{3}}{3}\right) \overset{=\theta}{\Longleftrightarrow} \tan\theta=\frac{\sqrt{3}}{3}$

$=\boxed{\frac{\pi}{6}}$

$=\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{3}{3\sqrt{3}}=\frac{1/2}{\sqrt{3}/2}$ ← sin ← cos



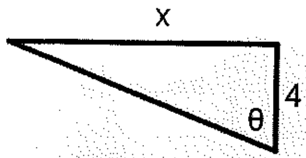
Practice: $\arctan\left(-\sqrt{3}\right) \overset{=\theta}{\Longleftrightarrow} \tan\theta=\frac{-\sqrt{3}/2}{1/2}$ sin cos



$\boxed{-\frac{\pi}{3}}$

Evaluating inverse functions using triangle sketches:

Example: Use an inverse trigonometric function to write θ as a function of x: on board:



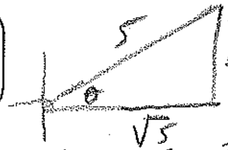
$\tan\theta = \frac{x}{4}$

$\theta = \arctan\left(\frac{x}{4}\right)$

- set 'inside' $= \theta$ and write in other form
- make a sketch to match (make sure angle is in range of inverse trig function)
- find missing side
- evaluate outside trig function

Find the exact value of the expression (hint: make a sketch of a right triangle):

$\sin\left(\underbrace{\arccos\left(\frac{\sqrt{5}}{5}\right)}_{\theta}\right)$ adj, hyp



$b = 2\sqrt{5}$

$\cos\theta = \frac{\sqrt{5}}{5} \quad \frac{x}{r}$

$(\sqrt{5})^2 + b^2 = 5^2$

$5 + b^2 = 25$

$b^2 = 20$

$b = \sqrt{20}$

$b = 2\sqrt{5}$

$\sin\theta = \frac{\text{off}}{\text{hyp}} = \boxed{\frac{2\sqrt{5}}{5}}$

$\csc\left(\underbrace{\arctan\left(-\frac{5}{12}\right)}_{\theta}\right)$

$\tan\theta = \frac{-5}{12} \quad \frac{\sin}{\cos}$

$(-5)^2 + 12^2 = r^2$

$25 + 144 = r^2$

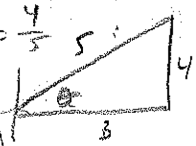
$169 = r^2$

$13 = r$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{-5}{13}\right)} = \boxed{\frac{-13}{5}}$

$\sec\left(\underbrace{\arcsin\left(\frac{4}{5}\right)}_{\theta}\right)$

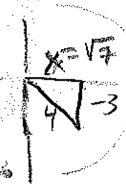
$\sin\theta = \frac{4}{5}$



$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{3}{5}\right)} = \boxed{\frac{5}{3}}$

$\tan\left(\underbrace{\arcsin\left(-\frac{3}{4}\right)}_{\theta}\right)$

$\sin\theta = \frac{-3}{4}$



arcsin

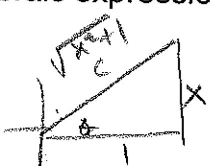
$x^2 + 9 = 16$

$x^2 = 7$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-3/4}{\sqrt{7}/4} = \frac{-3}{\sqrt{7}} = \boxed{\frac{-3\sqrt{7}}{7}}$

Write an algebraic expression that is equivalent to the expression (hint: sketch a right triangle):

$\cot(\underbrace{\arctan x}_{\theta})$



$\tan\theta = \frac{x}{1} \quad \frac{y}{x}$

$c^2 = 1^2 + x^2$

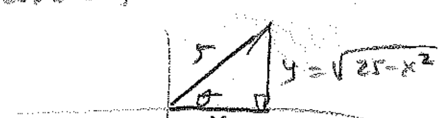
$c = \sqrt{x^2+1}$

$\cot\theta = \frac{\text{adj}}{\text{off}} = \boxed{\frac{1}{x}}$

$\tan\left(\arccos\left(\frac{x}{5}\right)\right)$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \boxed{\frac{\sqrt{25-x^2}}{x}}$

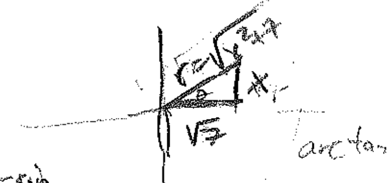


$x^2 + y^2 = 5^2$

$y^2 = 25 - x^2$

$y = \sqrt{25-x^2}$

$\csc\left(\underbrace{\arctan\frac{x}{\sqrt{7}}}_{\theta}\right)$



arctan

$\tan\theta = \frac{x}{\sqrt{7}} \quad \begin{matrix}\leftarrow \sin \\ \leftarrow \cos\end{matrix}$

$r^2 = x^2 + 7$

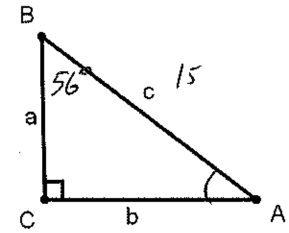
$r = \sqrt{x^2+7}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{x}{\sqrt{x^2+7}}} = \boxed{\frac{\sqrt{x^2+7}}{x}}$

HAlg3-4, 4.8 day 1 Notes – Trig applications

Many real-world problems can be modeled using right triangles, and missing information can be found by 'solving the triangle'.

Sometimes, the problem just gives you a generic triangle to 'solve'.
Convention for side, angle naming:



Example: B=56°, c=15, solve the triangle.

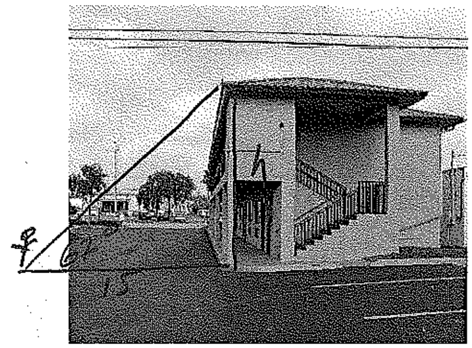
$\cos 56° = \frac{a}{15}$ $\quad a = 15\cos 56° = \boxed{8.4}$

$\sin 56° = \frac{b}{15}$ $\quad b = 15\sin 56° = \boxed{12.4}$

$A = 90 - 56 = \boxed{34°}$

Example:
A man at ground level measures the angle of elevation to the top of a building to be 67°. If, at this point, he is 15 feet from the building, what is the height of the building?

$\tan 67° = \frac{h}{15}$ $\quad h = 15\tan 67° = \boxed{35.3\ ft}$



Things to remember:

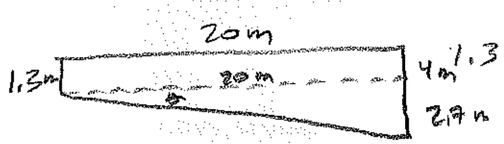
angle of elevation is above a horizontal

angle of depression is below a horizontal

$\sin\theta = \frac{opp}{hyp}$ $\quad \cos\theta = \frac{adj}{hyp}$ $\quad \tan\theta = \frac{opp}{adj}$

Sometimes, you want to find the angle, instead of a side length. Use inverse trig functions:

Example: A swimming pool is 20 meters long and 12 meters wide. The bottom of the pool has a constant slant so that the water depth is 1.3 meters at the shallow end and 4 meters at the deep end. Find the angle of depression of the bottom of the pool.



$\tan\theta = \frac{2.7}{20}$

$\theta = \arctan\frac{2.7}{20}$

$\theta = \boxed{7.7°}$

Practice problems:

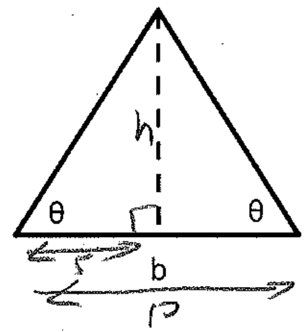
#1. A safety regulation states that the maximum angle of elevation for a rescue ladder is 72°. If a fire department's longest ladder is 110 feet, what is the maximum safe rescue height?

$\sin 72° = \frac{h}{110}$

$h = 110\sin 72°$

$\boxed{h = 104.6\,ft}$

#2. Find the altitude of the isosceles triangle if b=10 meters and $\theta = 18°$



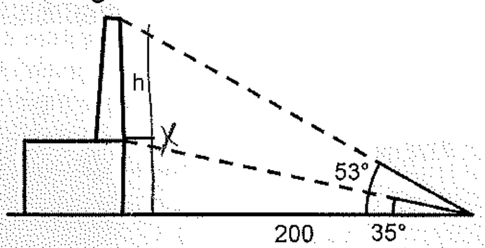
$\tan\theta = \frac{h}{5}$

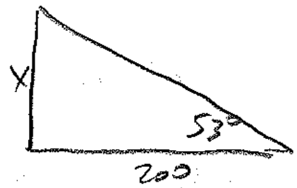
$\tan 18° = \frac{h}{5}$

$h = 5\tan 18°$

$\boxed{h = 1.6\text{ m}}$

#3. At a point 200 feet from the base of a building, the angle of elevation to the bottom of a smokestack is 35°, and the angle of elevation to the top is 53°, as shown. Find the height of the smokestack.





$\tan 53° = \frac{x}{200}$

$x = 200\tan 53°$

$x = 265.4$

$\tan 35° = \frac{y}{200}$

$y = 200 \cdot \tan 35°$

$y = 140.0$

$h = 265.4 - 140 = \boxed{125.4\text{ ft}}$

#4. A shadow of length L is created by an 850-foot building when the sun is $\theta°$ above the horizon.

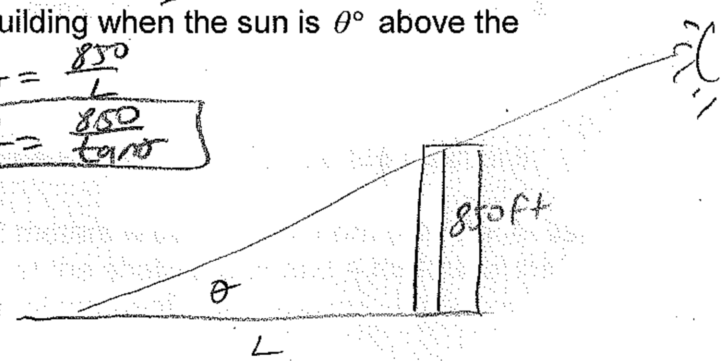
(a) Write L as a function of θ

$\tan\theta = \frac{850}{L}$

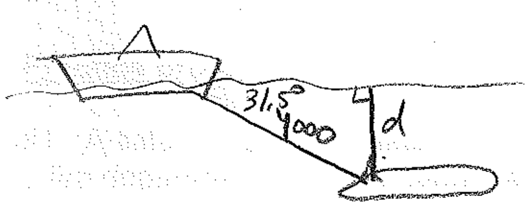
$\boxed{L = \frac{850}{\tan\theta}}$

(b) Use a calculator to complete the table:

θ	10°	20°	30°	40°	50°
L	4820.6	2335.4	1472.2	1013	713.2



#5. The sonar of a navy cruiser detects a nuclear submarine that is 4000 feet from the cruiser. The angle betwen the water level and the submarine is 31.5°. How deep is the submarine?



$\sin 31.5° = \frac{d}{4000}$

$d = 4000\sin 31.5°$

$\boxed{d = 2090\text{ ft}}$

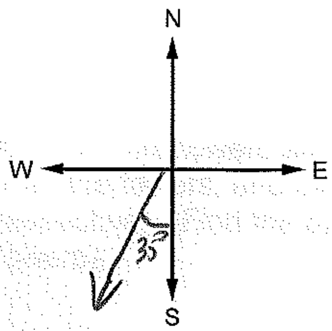
HAlg3-4, 4.8 day 2 Notes – Trig applications

Bearings

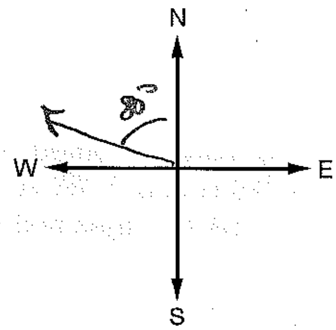
Sometimes, in navigation problems, direction is given as a 'bearing'. Bearing is defined as an acute angle from a North-South reference line, given as 'reference direction' followed by' angle from the reference direction'.

Examples of bearings:

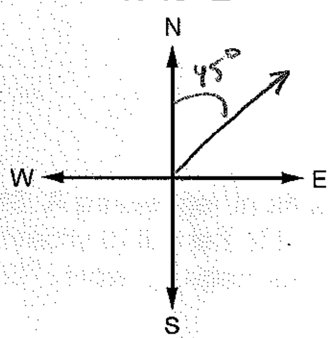
S 35° W



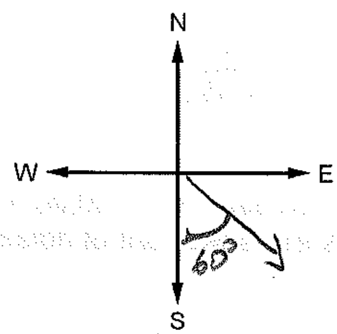
N 80° W



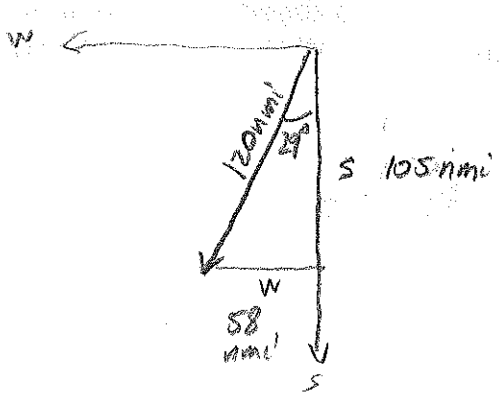
N 45° E



S 60° E



Example:
A ship leaves port at noon and has a bearing of S 29° W. If the ship sails at 20 knots, how many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 p.m?



$\cos 29° = \frac{S}{120}$

$S = 104.95$

$\sin 29° = \frac{W}{120}$

$W = 58$

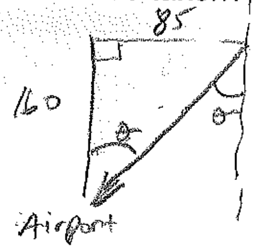
20 knots = $\frac{20 \text{ nautical miles}}{\text{hr}}$

$d = rt$

$d = \left(\frac{20 \text{ nmi}}{\text{hr}}\right)(6 \text{ h}) = 120 \text{ nmi}$

Practice Problems:

#1. A plane is 160 miles north and 85 miles east of an airport. If the pilot wants to fly directly to the airport, what bearing should be taken?

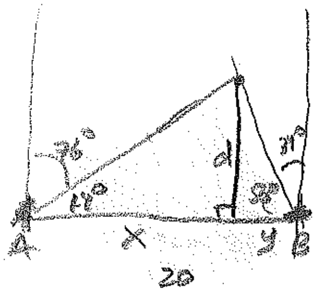


$\tan\theta = \frac{85}{160}$

$\theta = \arctan\frac{85}{160} = 28^\circ$

$\boxed{S\ 28^\circ\ W}$

#2. Two fire towers are 20 kilometers apart, tower A being due west of tower B. A fire is spotted from the towers, and the bearings from A and B are N 76° E and N 34° W, respectively. Find the distance d of the fire from the line segment AB (a line between the 2 towers).



$\tan 14^\circ = \frac{d}{x}$ $\quad \tan 56^\circ = \frac{d}{y}$

$d = x\tan 14^\circ$ $\quad d = y\tan 56^\circ$

$x\tan 14^\circ = y\tan 56^\circ$ $\quad x + y = 20$

$x = y\frac{\tan 56^\circ}{\tan 14^\circ}$ $\quad y\frac{\tan 56^\circ}{\tan 14^\circ} + y = 20$

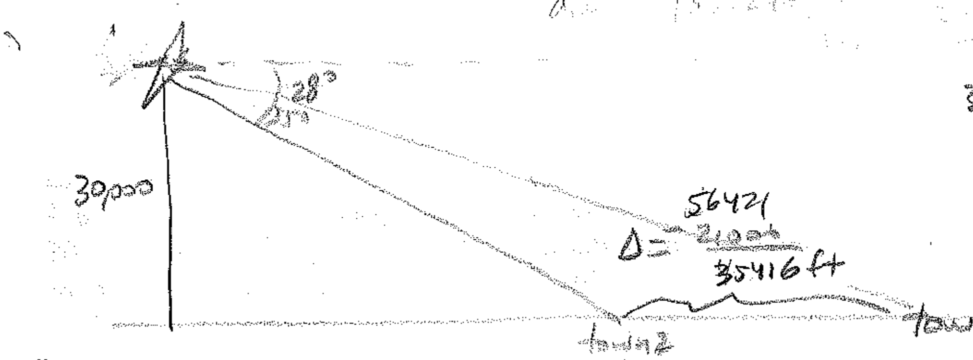
$\boxed{d = 4.3\ \text{Km}}$

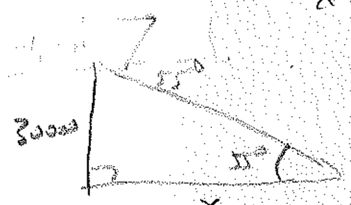
$y\left(1 + \frac{\tan 56^\circ}{\tan 14^\circ}\right) = 20$

$y = 2.88$

$x = 17.12$

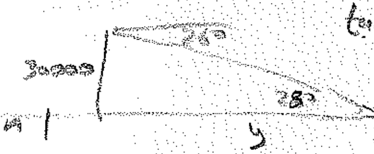
#3. A passenger in an airplane flying at an altitude of 30,000 feet sees two towns directly to the left of the plane. The angles of depression to the towns are 28° and 55°. How far apart are the towns?





$\tan 55^\circ = \frac{30000}{x}$

$x = 21006$



$\tan 28^\circ = \frac{30000}{y}$

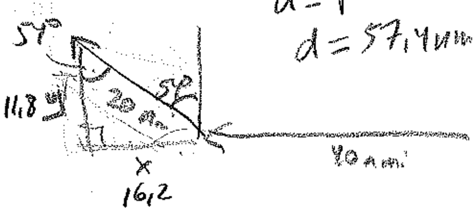
$y = 56421$

#4. A ship leaves port at noon and heads due west at 20 knots (nautical miles per hour). At 2 p.m. the ship changes course to N 54° W. Find the ship's bearing and distance from the port of departure at 3 p.m.

$d = rt$

$d_1 = 20(2) = 40\ \text{nmi}$

$d_2 = 20(1) = 20\ \text{nmi}$



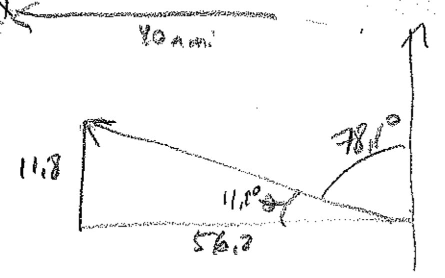
$d = \sqrt{\sim}$

$d = 57.4\ \text{nmi}$

$\boxed{N\ 78.1^\circ\ W}$

$\sin 54^\circ = \frac{x}{20} \Rightarrow x = 16.2$

$\cos 54^\circ = \frac{y}{20} \Rightarrow y = 11.8$



$\tan\theta = \frac{11.8}{56.2}$

$\theta = 11.9^\circ$