HAlg3-4, 5.1 day 1 Notes – Using Fundamental Identities

***Start Memorizing These...*

Reciprocal identities:

$\sin x = \dfrac{1}{\csc x}$ $\cos x = \dfrac{1}{\sec x}$ $\tan x = \dfrac{1}{\cot x}$

$\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient identities:

$\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean identities:

$\sin^2 x + \cos^2 x = 1$ $1 + \tan^2 x = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

$\sin^2 x = 1 - \cos^2 x$ $\sec^2 x - 1 = \tan^2 x$ $\csc^2 x - 1 = \cot^2 x$

$\cos^2 x = 1 - \sin^2 x$ $\sec^2 x - \tan^2 x = 1$ $\csc^2 x - \cot^2 x = 1$

Cofunction identities:

$\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$ $\cos\left(\dfrac{\pi}{2} - x\right) = \sin x$

$\tan\left(\dfrac{\pi}{2} - x\right) = \cot x$ $\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$

$\sec\left(\dfrac{\pi}{2} - x\right) = \csc x$ $\csc\left(\dfrac{\pi}{2} - x\right) = \sec x$

Even/Odd identities:

$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$ $\tan(-x) = -\tan x$

$\csc(-x) = -\csc x$ $\sec(-x) = \sec x$ $\cot(-x) = -\cot x$

We can use identities to simplify a trigonometric expression or to verify a more complex identity

General procedure: start with more complicated expression and make it simpler, or to verify, turn more complicated expression into simpler expression.

Example: Simplify $\sin x \cos^2 x - \sin x$ (given)

$\sin x(\cos^2 x - 1)$ (factor $\sin x$)

$-\sin x(1 - \cos^2 x)$ (factor -1)

$-\sin x(\sin^2 x)$ ($1-\cos^2 x = \sin^2 x$)

$\boxed{-\sin^3 x}$ (multiply)

notation

$\sin x^2 = \sin(x^2)$

$\sin^2 x = (\sin x)^2$

Strategies for steps:

1) Factor out trig functions as if they were variables

Ex: Simplify $\sin x\cos^2 x-\sin x$ (given)

$\sin x(\cos^2 x-1)$ (factor)

solved below

2) When there is a squared term, think 'Pythagorean identity':

Ex: Factor the expression: $\sin x\cos^2 x-\sin x$ (given)

$\sin x(\cos^2 x-1)$ (factor $\sin x$)

$-\sin x(1-\cos^2 x)$ (factor -1)

$-\sin x(\sin^2 x)$ ($\sin^2 x+\cos^2 x=1$) or (Pythagorean identity)

$\boxed{-\sin^3 x}$ (multiply)

3) Convert everything to sin or cos

Ex: Simplify $\cot x\sin x$ (given)

$\frac{\cos x}{\sin x}\sin x$ (quotient identity)

$\boxed{\cos x}$ (cancel $\sin x$)

4) If you have fractions, combine with common denominator

Ex: Verify $\frac{\sin x}{1+\cos x}+\frac{\cos x}{\sin x}=\csc x$ (given)

$\frac{\sin^2 x}{(1+\cos x)\sin x}+\frac{\cos x(1+\cos x)}{(1+\cos x)\sin x}=\csc x$ (common denominator)

$\frac{\sin^2 x}{(1+\cos x)\sin x}+\frac{\cos x+\cos^2 x}{(1+\cos x)\sin x}=\csc x$ (multiply)

$\frac{\sin^2 x+\cos x+\cos^2 x}{(1+\cos x)\sin x}=\csc x$ (combine fractions)

$\frac{1+\cos x}{(1+\cos x)\sin x}=\csc x$ ($\sin^2 x+\cos^2 x=1$)

$\frac{1}{\sin x}=\csc x$ (cancel $1+\cos x$)

$\csc x=\csc x$ ✓ (reciprocal identity)

5) If there is a binomial in the denominator, multiply by the 'conjugate' if it creates a Pythagorean identity:

Ex: Rewrite $\dfrac{1}{1-\sin x}$ so it is not in fractional form (given)

$\dfrac{1}{1-\sin x}\cdot\dfrac{1+\sin x}{1+\sin x}$ (multiply by conjugate)

$\dfrac{1+\sin x}{1-\sin^2 x}$ (multiply)

$\dfrac{1+\sin x}{\cos^2 x}$ ($\sin^2 x+\cos^2 x=1$)

$\dfrac{1}{\cos^2 x}+\dfrac{\sin x}{\cos^2 x}$ (separate fractions)

$\dfrac{1}{\cos^2 x}+\dfrac{\sin x}{\cos x}\cdot\dfrac{1}{\cos x}$ (factor)

$\boxed{\sec^2 x+\tan x\sec x}$ (reciprocal, quotient identities)

6) Look for factoring patterns treating trig functions like variables:

Ex: Factor the expression: $\sec^2 x-1$ (given)

$a^2-b^2=(a+b)(a-b)$

$\boxed{(\sec x+1)(\sec x-1)}$ (factor a^2-b^2)

Ex: Factor the expression: $4\tan^2\theta+\tan\theta-3$ (given)

mult add

-12 | 1

$(4)(-3)$ | $4-3$

$\dfrac{(4x+4)}{4}\dfrac{(4x-3)}{1}$

$(x+1)(4x-3)$

$4x^2+x-3$ (substitute $x=\tan\theta$)

$(4x-3)(x+1)$ (factor)

$\boxed{(4\tan\theta-3)(\tan\theta+1)}$ (substitute)

7) Don't forget the 'simple' identities:

Ex: Simplify $\dfrac{\sin\left(\dfrac{\pi}{2}-x\right)}{\cos\left(\dfrac{\pi}{2}-x\right)}$ (given)

$\dfrac{\cos x}{\sin x}$ (cofunction identities)

$\cot x$ (quotient identity)

If expressions can't be verified analytically (as above) two expressions can be shown to be equal by calculator. Enter each expression as an equation (for Y1 and Y2) and show by graph or table that they are equivalent. (See example 3 in the textbook) ractice: Simplify...

#1. $(1-\sin^2 x)\sec x$ (given)

$\cos^2 x \sec x$ (pyth. id.)

$\cos^2 x \frac{1}{\cos x}$ (recip. id.)

$\frac{\cos^2 x}{\cos x}$ (combine fractions)

$\boxed{\cos x}$ (divide)

#2. $\cot\theta\sec\theta$ (given)

$\frac{\cos\theta}{\sin\theta}\frac{1}{\cos\theta}$ (recip., quot. ident.)

$\frac{1}{\sin\theta}$ (cancel)

$\boxed{\csc\theta}$ (recip. id.)

#3. $\dfrac{\cos^2\left(\frac{\pi}{2}-x\right)}{\cos x}$ (given)

$\frac{\sin^2 x}{\cos x}$ (cofunction id.)

$\sin x \frac{\sin x}{\cos x}$ (separate)

$\boxed{\sin x \tan x}$ (quotient id.)

Verify...

#4. $\cos x \sec x - \cos^2 x = \sin^2 x$ (given)

$\cos x \frac{1}{\cos x} - \cos^2 x = \sin^2 x$ (reciprocal id.)

$1 - \cos^2 x = \sin^2 x$ (cancel)

$\sin^2 x = \sin^2 x$ ✓ (pythagorean id.)

#5. $\dfrac{\sec^2\theta - \tan^2\theta + \tan\theta}{\sec\theta} = \cos\theta + \sin\theta$ (given)

$\frac{1 + \tan\theta}{\sec\theta} = \cos\theta + \sin\theta$ ($1+\tan^2 x = \sec^2 x$)

$\frac{1}{\sec\theta} + \frac{\tan\theta}{\sec\theta} = \cos\theta + \sin\theta$ (separate fractions)

$\cos\theta + \cos\theta \frac{\sin\theta}{\cos\theta} = \cos\theta + \sin\theta$ (recip., quotient ids)

$\cos\theta + \sin\theta = \cos\theta + \sin\theta$ (cancel)

HAlg3-4, 5.1 day 2 Notes – Using Fundamental Identities – more examples

Simpler problems using just basic identities (1-13 in hw):

Ex: Use the given values to evaluate the remaining trig functions

$\csc\theta = \frac{5}{3}, \quad \tan\theta = \frac{3}{4}$

$\frac{1}{\sin\theta} = \frac{5}{3}$

$\sin\theta = \frac{3}{5}$

$\frac{\sin\theta}{\cos\theta} = \frac{3}{4}$

$\frac{\left(\frac{3}{5}\right)}{\cos\theta} = \frac{3}{4}$

$3\cos\theta = 4\left(\frac{3}{5}\right)$

$\frac{3\cos\theta}{3} = \frac{4 \cdot \frac{3}{5}}{3}$

$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$ $\qquad \csc\theta = \frac{5}{3}$

$\cos\theta = \frac{4}{5}$ $\qquad \sec\theta = \frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\qquad \cot\theta = \frac{4}{3}$

Perform an operation, then simplify: Ex: $\frac{1}{\sec x+1} - \frac{1}{\sec x-1}$ (given)

$\frac{\sec x-1}{(\sec x+1)(\sec x-1)} - \frac{\sec x+1}{(\sec x+1)(\sec x-1)}$ (common denominator)

$\frac{\sec x-1}{\sec^2 x-1} - \frac{\sec x+1}{\sec^2 x-1}$ (multiply)

$\frac{\sec x-1-\sec x-1}{\sec^2 x-1}$ (combine fractions)

$\frac{-2}{\sec^2 x-1}$ (combine terms)

$\boxed{\frac{-2}{\tan^2 x}}$ ($1+\tan^2 x = \sec^2 x$)

Use trig substitution to write an algebraic expression:

Ex: $\sqrt{16-4x^2}$, $x = 2\sin\theta$ (given)

$\sqrt{16-4(2\sin\theta)^2}$ (substitute)

$\sqrt{16-16\sin^2\theta}$ (multiply)

$\sqrt{16(1-\sin^2\theta)}$ (factor)

$\sqrt{16\cos^2\theta}$ ($\sin^2 x+\cos^2 x=1$)

$\sqrt{16}\sqrt{\cos^2\theta}$ (separate)

$4\cos\theta$ square root

(technically: $4|\cos\theta|$)

HAlg3-4, ~~5.1 day 2~~ 5.2 Notes – Verifying Trig Identities

What is an identity?

$\sin x = 0$ is a conditional equation

and is true only for some values of x $(x = n\pi)$

Finding values of x where equation is true is called solving the equation.

$\sin^2 x = 1 - \cos^2 x$ is an identity

and is true for all values of x.

More general tips for verifying identities:

1) Work with one side of the equation at a time. Usually best to try to turn more complicated side into less complicated side.

2) Look for opportunities to factor, add fractions.

3) Look for opportunities to use the fundamental identities.

4) Use simple side as the 'goal' to help guide what to do next. Example: if goal has secants, try converting what you have to secants, if goal has two terms and you are starting with one, look for ways to split fractions, etc.

5) Try converting everything to sin or cos and see if anything cancels or combines.

6) Try something! The path to a dead end still reveals insights.

More strategies...

Work with each side separately: Ex: Verify $\dfrac{\cot^2 x}{1+\csc x} = \dfrac{1-\sin x}{\sin x}$ (given)

(pyth. id.) $\dfrac{\csc^2 x - 1}{1+\csc x} = \dfrac{1}{\sin x} - \dfrac{\sin x}{\sin x}$ (separate fractions)

(a^2-b^2 factoring) $\dfrac{(\csc x + 1)(\csc x - 1)}{1+\csc x} = \csc x - 1$ (reciprocal identi)

(cancel) $\csc x - 1 = \text{csx} - 1$ ✓

Powers greater than 2, separate factors: Ex: Verify $\tan^4 x = \tan^2 x \sec^2 x - \tan^2 x$

(separate factors) $\tan^2 x \tan^2 x = \tan^2 x \sec^2 x - \tan^2 x$

(pyth. identi) $\tan^2 x(\sec^2 x - 1) = \tan^2 x \sec^2 x - \tan^2 x$

(distribute) $\tan^2 x \sec^2 x - \tan^2 x = \tan^2 x \sec^2 x - \tan^2 x$ ✓

Square roots – multiply by conjugates to get something squared underneath:

Ex: $\sqrt{\dfrac{1-\cos x}{1+\cos x}} = \dfrac{1-\cos x}{|\sin x|}$ (given)

$\sqrt{\dfrac{(1-\cos x)(1-\cos x)}{(1+\cos x)(1-\cos x)}} = \dfrac{1-\cos x}{|\sin x|}$ (multiply conjugate)

$\sqrt{\dfrac{(1-\cos x)^2}{1-\cos^2 x}} = \dfrac{1-\cos x}{|\sin x|}$ (multiply)

$\sqrt{\dfrac{(1-\cos x)^2}{\sin^2 x}} = \dfrac{1-\cos x}{|\sin x|}$ (pyth. id.)

$\dfrac{\sqrt{(1-\cos x)^2}}{\sqrt{\sin^2 x}} = \dfrac{1-\cos x}{|\sin x|}$ (split square root)

$\dfrac{1-\cos x}{|\sin x|} = \dfrac{1-\cos x}{|\sin x|}$ ✓ (square root of square)

A complex example: Verify $\dfrac{\tan\theta}{1+\sec\theta} + \dfrac{1+\sec\theta}{\tan\theta} = 2\csc\theta$ (given)

$\dfrac{\tan^2\theta}{\tan\theta(1+\sec\theta)} + \dfrac{(1+\sec\theta)^2}{\tan\theta(1+\sec\theta)} = 2\csc\theta$ (common denom.)

$\dfrac{\tan^2\theta}{\tan\theta(1+\sec\theta)} + \dfrac{1+2\sec\theta+\sec^2\theta}{\tan\theta(1+\sec\theta)} = 2\csc\theta$ (multiply)

$\dfrac{\tan^2\theta+1+2\sec\theta+\sec^2\theta}{\tan\theta(1+\sec\theta)} = 2\csc\theta$ (combine fractions)

$\dfrac{\sec^2\theta+2\sec\theta+\sec^2\theta}{\tan\theta(1+\sec\theta)} = 2\csc\theta$ ($\tan^2\theta+1=\sec^2\theta$)

$\dfrac{2\sec^2\theta+2\sec\theta}{\tan\theta(1+\sec\theta)} = 2\csc\theta$ (combine terms)

$\dfrac{2\sec\theta(\sec\theta+1)}{\tan\theta(1+\sec\theta)} = 2\csc\theta$ (factor $2\sec\theta$)

(cancel)

$\dfrac{2\sec\theta}{\tan\theta} = 2\csc\theta$

(reciprocal identi)

$2\dfrac{1}{\cos\theta}\dfrac{\cos\theta}{\sin\theta} = 2\csc\theta$

(cancel)

$2\dfrac{1}{\sin\theta} = 2\csc\theta$

(reciprocal ident)

$2\csc\theta = 2\csc\theta$ ✓

Identities Misconceptions (day of 5/1/5/12 worksheets)

1) numerator split: ok: $\frac{2+3}{6} = \frac{2}{6}+\frac{3}{6}$

not ok: $\frac{2}{3+6} = \frac{2}{3}+\frac{2}{6}$

2) squared → fourth: $1+\cot^2 x = \csc^2 x$, $1+\cot^4 x = \csc^4 x$ not true

instead use patterns or split a^2-b^2 or maybe $1+\cot^2 x\cot^2 x$

3) big jumps: $\frac{\tan x(\csc x-\cot x)}{\csc^2 x-\cot^2 x}$ or $\cos^2\beta+\cos^2\beta\tan^2\beta$

$\downarrow$ $\leftarrow \cos^2\beta(1+\tan^2\beta)$

$\frac{\sec x-1}{1}$ $\cos^2\beta\sec^2\beta$

4) cancel part of fractions: $\frac{1+\cos^2 x\sin^2 x}{\cos^2 x}$

5) denom extended: $\cos x+\sin x\tan x = \cos x+\sin x\frac{\sin x}{\cos x}$

not $\frac{\cos x+\sin x\sin x}{\cos x}$

6) replacement misplaced: $\frac{\tan x}{\csc x+\cot x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\sin x}+\frac{\cos x}{\sin x}}$

not $\frac{1}{\csc x}\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}$

HAlg3-4, 5.3 Notes – Solving Trig Equations

Solving Trig Equations: Primary goal is to get a single trig function on one side of the equation so you can find x.

Example...solve: $2\sin x - 1 = 0$

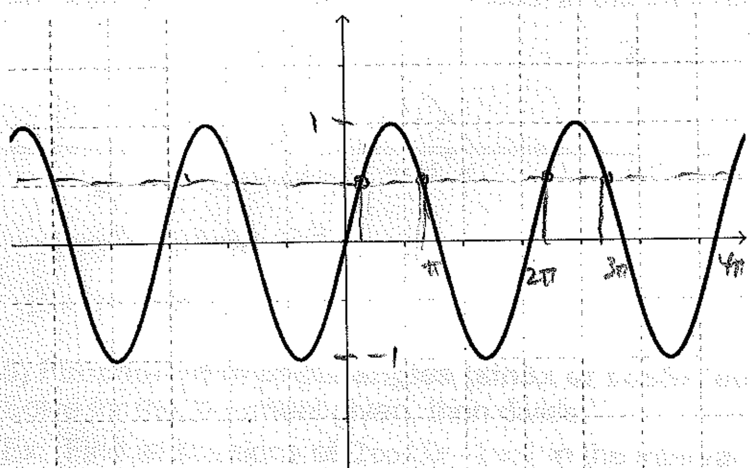
$2\sin x = 1$

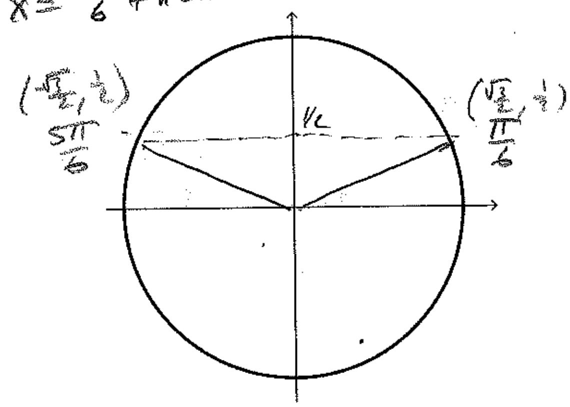
$\sin x = \frac{1}{2}$

Once you've isolated a single trig function, you can think of solving in a couple of ways:

$x = \frac{\pi}{6} + n2\pi$

$x = \frac{5\pi}{6} + n2\pi$





Strategies...

1) Collect like terms

Ex: Find all solutions of $\sin x + \sqrt{2} = -\sin x$ in the interval $[0, 2\pi)$

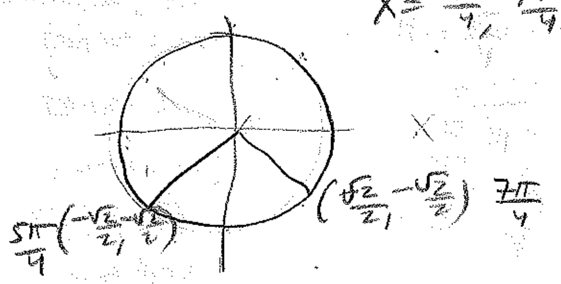
$\sin x + \sqrt{2} = -\sin x$

$2\sin x + \sqrt{2} = 0$

$2\sin x = -\sqrt{2}$

$\sin x = -\frac{\sqrt{2}}{2}$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}$



2) Extract square roots

Ex: Find all solutions of $3\tan^2 x - 1 = 0$ in the interval $[0, 2\pi)$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

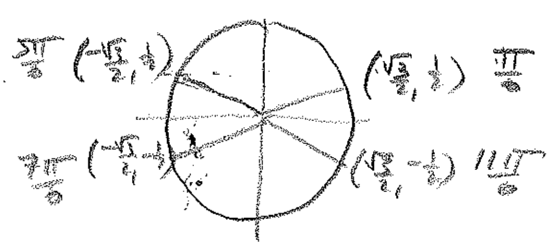
$3\tan^2 x - 1 = 0$

$\tan x = \pm\frac{1/2}{\sqrt{3}/2}$ ← sin (y) ← cos (x)

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm\frac{1}{\sqrt{3}}$



Factoring (simple factoring, patterns, quadratic factoring)

Ex: Find all solutions of $\cot x \cos^2 x = 2\cot x$ in the interval $[0, 2\pi)$

$\cot x \cos^2 x = 2\cot x$

$\cot x \cos^2 x - 2\cot x = 0$

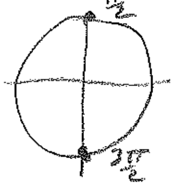
$\cot x(\cos^2 x - 2) = 0$

$\cot x = 0$ $\quad$ $\cos^2 x - 2 = 0$

$\cot x = 0$

$\frac{\cos x}{\sin x} = 0$ when $\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$



$\cos^2 x - 2 = 0$

$\cos^2 x = 2$

$\cos x = \pm\sqrt{2} > 1$

no sol'n

4) Square both sides to get a quadratic to factor

Ex: Find all solutions of $\cos x + 1 = \sin x$ in the interval $[0, 2\pi)$

$(\cos x + 1)^2 = (\sin x)^2$

$\cos^2 x + 2\cos x + 1 = \sin^2 x$

$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$

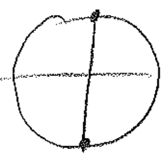
$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$2\cos x = 0$

$\cos x = 0$

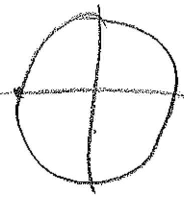
$x = \frac{\pi}{2}, \frac{3\pi}{2}$



$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$



5) Function of multiple angles (sin3x or cos5x, etc.)

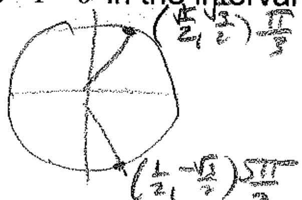
Solve for the argument given, then divide:

Ex: Find all solutions of $2\cos 3x - 1 = 0$ in the interval $[0, 2\pi)$

$2\cos 3x = 1$

$\cos 3x = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$



$\theta = \frac{\pi}{3}$

$3x = \frac{\pi}{3}$

$x = \frac{\pi}{9}$

$\theta = \frac{5\pi}{3}$

$3x = \frac{5\pi}{3}$

$x = \frac{5\pi}{9}$

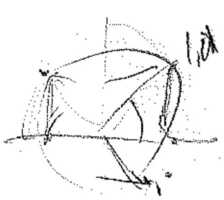
6) Using inverse functions (use calculator if not a unit circle value)

Ex: Find all solutions of $\sec^2 x - 2\tan x = 4$ in the interval $[0, 2\pi)$

$1 + \tan^2 x - 2\tan x - 4 = 0$

$\tan^2 x - 2\tan x - 3 = 0$

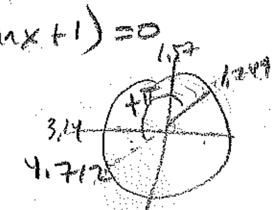
$(\tan x - 3)(\tan x + 1) = 0$



$\tan x - 3 = 0$

$\tan x = 3$

$x = \arctan 3$



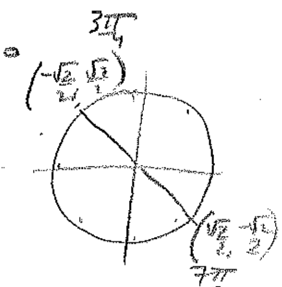
$\boxed{x = 1.249 \text{ and } 4.7124}$

$\tan x + 1 = 0$

$\tan x = -1$

$\frac{\sin x}{\cos x} = -1$

$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}}$

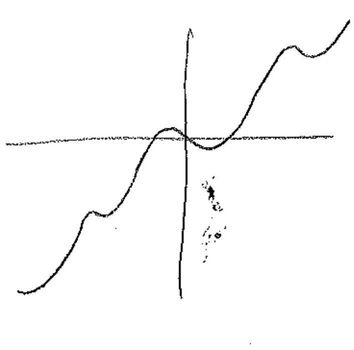


7) Move everything to one side and use graphing calculator to find zeros

Some problems have no reasonable algebraic solution.

Ex: Find all solutions of $x = 2\sin x$

$Y1 = X - 2\sin X = 0$



trace; $x = -1.895...$

or $\quad x = 0$

zero feature $\quad x = 1.895...$

HAlg3-4, 5.4 Notes – Sum and Difference Formulas

Sum and Difference Formulas (do not need to memorize)

$$\sin(u+v)=\sin u\cos v+\cos u\sin v$$

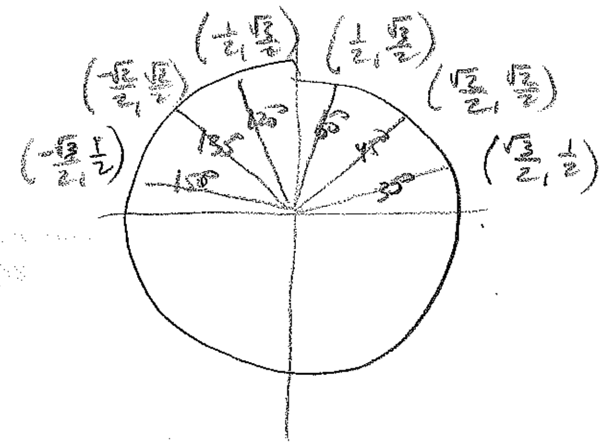
$$\sin(u-v)=\sin u\cos v-\cos u\sin v$$

$$\cos(u+v)=\cos u\cos v-\sin u\sin v$$

$$\cos(u-v)=\cos u\cos v+\sin u\sin v$$

$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v}$$

$$\tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$$



Examples:

Find $\sin 165^\circ = \sin(135^\circ+30^\circ)$ (hint: 165°=135°+30°)

$= \sin 135^\circ\cos 30^\circ + \cos 135^\circ \sin 30^\circ$

$= \left(\frac{\sqrt2}{2}\right)\left(\frac{\sqrt3}{2}\right)+\left(\frac{-\sqrt2}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt6}{4}-\frac{\sqrt2}{4} = \frac{\sqrt6-\sqrt2}{4} = \frac{\sqrt2(\sqrt3-1)}{4} = \boxed{\frac{\sqrt2}{4}(\sqrt3-1)}$

$\cos 165^\circ = \cos(135^\circ+30^\circ)$

$= \cos 135^\circ\cos 30^\circ - \sin 135^\circ \sin 30^\circ = \frac{-\sqrt6-\sqrt2}{4} = \frac{-\sqrt2(\sqrt3+1)}{4} = \boxed{-\frac{\sqrt2}{4}(\sqrt3+1)}$

$= \left(-\frac{\sqrt2}{2}\right)\left(\frac{\sqrt3}{2}\right)-\left(\frac{\sqrt2}{2}\right)\left(\frac{1}{2}\right) =$

$\tan 135^\circ = \frac{\sqrt2/2}{-\sqrt2/2} = -1$

$\tan 30^\circ = \frac{1/2}{\sqrt3/2} = \frac{1}{\sqrt3} = \frac{\sqrt3}{3}$

$\tan 165^\circ = \tan(135^\circ+30^\circ) = \frac{\tan 135^\circ + \tan 30^\circ}{1-\tan 135^\circ \tan 30^\circ} = \frac{-1+\frac{\sqrt3}{3}}{1-(-1)\left(\frac{\sqrt3}{3}\right)}$

$= \frac{-1+\frac{\sqrt3}{3}}{1+\frac{\sqrt3}{3}} = \frac{(-3+\sqrt3)(3-\sqrt3)}{(3+\sqrt3)(3-\sqrt3)} = \frac{-9+6\sqrt3-3}{9-3} = \frac{-12+6\sqrt3}{6} = \boxed{-2+\sqrt3}$

Find $\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{6}-\frac{\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{\pi}{4} - \cos\frac{\pi}{6}\sin\frac{\pi}{4}$ (hint: $-\frac{\pi}{12}=\frac{\pi}{6}-\frac{\pi}{4}$)

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt2}{2}\right)-\left(\frac{\sqrt3}{2}\right)\left(\frac{\sqrt2}{2}\right)$

$= \frac{\sqrt2}{4}-\frac{\sqrt6}{4} = \boxed{\frac{\sqrt2}{4}(1-\sqrt3)}$

$\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6}-\frac{\pi}{4}\right) = \cos\frac{\pi}{6}\cos\frac{\pi}{4}+\sin\frac{\pi}{6}\sin\frac{\pi}{4} = \left(\frac{\sqrt3}{2}\right)\left(\frac{\sqrt2}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{\sqrt2}{2}\right) = \frac{\sqrt6}{4}+\frac{\sqrt2}{4} = \boxed{\frac{\sqrt2}{4}(\sqrt3+1)}$

$\tan\left(-\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{6}-\frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{6}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{6}\tan\frac{\pi}{4}} = \frac{\frac{\sqrt3}{3}-1}{1+\left(\frac{\sqrt3}{3}\right)(1)} = \frac{\frac{\sqrt3}{3}-1}{1+\frac{\sqrt3}{3}} = \frac{(\sqrt3-3)(\sqrt3-1)}{(\sqrt3+1)(\sqrt3-1)} = \frac{3-4\sqrt3+3}{3-1}$

$= \frac{6-4\sqrt3}{2} = \boxed{3-2\sqrt3}$

$\tan\frac{\pi}{6} = \frac{1/2}{\sqrt3/2} = \frac{\sqrt3}{3}$

$\tan\frac{\pi}{4} = 1$

Use sum or difference formulas to write the expression as sin, cos or tan or an angle:

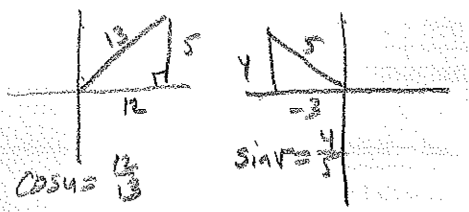
$\sin 140° \cos 50° + \cos 140° \sin 50°$ $= \sin(140° + 50°) = \boxed{\sin 190°}$

$\cos 3x \cos 2y + \sin 3x \sin 2y$ $= \boxed{\cos(3x - 2y)}$

Find the exact value of $\cos(v-u)$ given that:

$\sin u = \frac{5}{13}, \left(\frac{y}{r}\right)$ $where\ 0 < u < \frac{\pi}{2}$ and $\cos v = \frac{-3}{5}, \frac{x}{r}$ $where\ \frac{\pi}{2} < v < \pi$

- Make 2 sketches (one for u, one for v)
- Use sketches to find the 'other' sine and cosine (sinv, cosu this problem)
- Use sum/difference formula and plug in values.



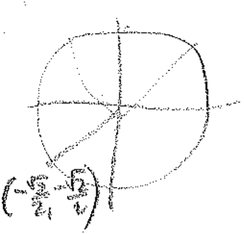
$\cos(v-u) = \cos u \cos v + \sin u \sin v$
$= \left(\frac{12}{13}\right)\left(\frac{-3}{5}\right) + \left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$
$= \frac{-36}{65} + \frac{20}{65} = \boxed{\frac{-16}{65}}$

Practice: Find the exact value of $\sin(u+v)$ given that:

$\sin u = \frac{7}{25},\ where\ \frac{\pi}{2} < u < \pi$ and $\cos v = \frac{4}{5},\ where\ \frac{3\pi}{2} < v < 2\pi$

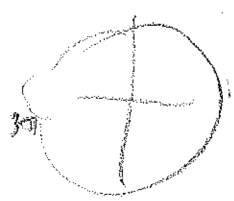
$\sin(u+v) = \sin u \cos v + \cos u \sin v$
$= \left(\frac{7}{25}\right)\left(\frac{4}{5}\right) + \left(\frac{-24}{25}\right)\left(\frac{-3}{5}\right)$ $= \frac{28 + 72}{125} = \frac{100}{125} = \boxed{\frac{4}{5}}$

Example: Verify the identity: $\cos\left(\frac{5\pi}{4} - x\right) = -\frac{\sqrt{2}}{2}(\cos x + \sin x)$



$\cos \frac{5\pi}{4} \cos x + \sin \frac{5\pi}{4} \sin x = -\frac{\sqrt{2}}{2}(\cos x + \sin x)$
$-\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x = -\frac{\sqrt{2}}{2}(\cos x + \sin x)$
$-\frac{\sqrt{2}}{2}(\cos x - \sin x) = -\frac{\sqrt{2}}{2}(\cos x - \sin x)$ ✓

Practice: Verify the identity: $\sin(3\pi - x) = \sin x$



$\sin 3\pi \cos x - \cos 3\pi \sin x = \sin x$
$(0)\cos x - (-1)\sin x = \sin x$
$\sin x = \sin x$ ✓

HAlg3-4, 5.5 Notes – Double and Half Angle Formulas & power-reducing formulas

Double Angle Formulas (do not need to memorize)

$$\sin 2u = 2\sin u\cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\cos 2u = 2\cos^2 u - 1$$

$$\cos 2u = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

Examples:

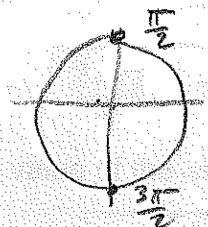
Solve: $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\quad$ $2\sin x - 1 = 0$

$\cos x = 0$



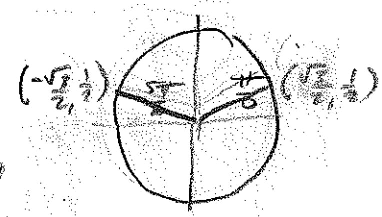
$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

$2\sin x - 1 = 0$

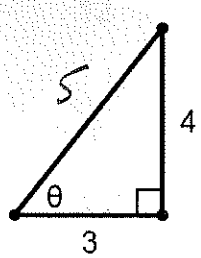
$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\boxed{\frac{\pi}{6}, \frac{5\pi}{6}}$



Use the figure to find the exact value of the $\cot 2\theta$



$\cot 2\theta = \frac{1}{\tan 2\theta}$ $\qquad$ $\tan\theta = \frac{4}{3}$

$\cot 2\theta = \frac{1}{\tan 2\theta} = \boxed{\frac{-7}{24}}$

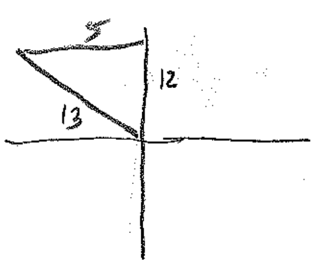
$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2} = \frac{2\left(\frac{4}{3}\right)}{1} \div \left(1-\left(\frac{4}{3}\right)^2\right)$

$= \frac{8}{3} \div \left(1 - \frac{16}{9}\right)$

$= \frac{8}{3} \div \left(\frac{9}{9} - \frac{16}{9}\right) = \frac{8}{3} \div \left(\frac{-7}{9}\right)$ $\quad$ $\frac{8}{\cancel{3}} \times \left(\frac{\cancel{9}^{3}}{-7}\right) = \boxed{\frac{-24}{7}}$

Given $\sin x = \frac{12}{13}$, $\frac{\pi}{2} < x < \pi$ $\quad$ Find $\sin 2x$



$\sin 2x = 2\sin x\cos x$ $\qquad$ $\cos x = \frac{5}{13}$

$\sin 2x = 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \boxed{\frac{120}{169}}$

Half Angle Formulas (do not need to memorize)

$$\sin\frac{u}{2}=\pm\sqrt{\frac{1-\cos u}{2}}=\pm\sqrt{\frac{1}{2}(1-\cos u)}$$

the sign depends upon the quadrant of u

(graph u, find u/2, determine sign)

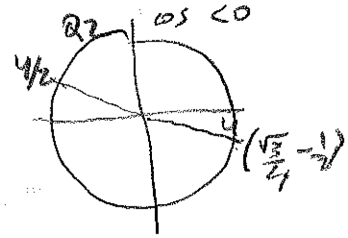
$$\cos\frac{u}{2}=\pm\sqrt{\frac{1+\cos u}{2}}=\pm\sqrt{\frac{1}{2}(1+\cos u)}$$

$$\tan\frac{u}{2}=\frac{1-\cos u}{\sin u}=\frac{\sin u}{1+\cos u}$$

Examples:

Find the exact value of $\cos 165°$

$165=\frac{330}{2}=\frac{\theta}{2}\quad(\theta=330°)$

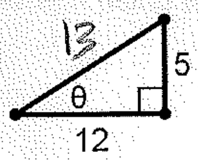


$\cos\frac{u}{2}=\pm\sqrt{\frac{1+\cos u}{2}}$

$\cos 165°=\cos\frac{330°}{2}=\pm\sqrt{\frac{1+\cos 330°}{2}}=\pm\sqrt{\frac{1+(\frac{\sqrt{3}}{2})}{2}}=\pm\sqrt{\frac{2+\sqrt{3}}{4}}$

$=-\sqrt{\frac{2+\sqrt{3}}{4}}=\boxed{-\frac{\sqrt{2+\sqrt{3}}}{2}}$

Use the figure to find the exact value of the $\sin\frac{\theta}{2}$

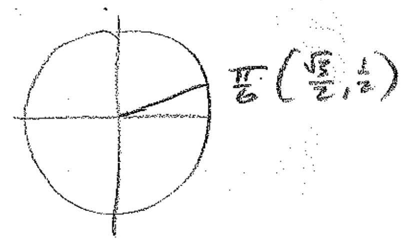


$\sin\frac{\theta}{2}=\pm\sqrt{\frac{1-\cos\theta}{2}}=+\sqrt{\frac{1-(\frac{12}{13})}{2}}=\sqrt{\frac{\frac{13}{13}-\frac{12}{13}}{2}}=\sqrt{\frac{1}{2}(\frac{1}{13})}$

$=\frac{1}{\sqrt{26}}=\boxed{\frac{\sqrt{26}}{26}}$

Find the exact value of $\tan\frac{\pi}{12}$

$\frac{\pi}{12}=\frac{\frac{\pi}{6}}{2}\leftarrow\frac{\theta}{2}$



$\tan\frac{\theta}{2}=\frac{1-\cos\theta}{\sin\theta}$

$\tan\frac{(\frac{\pi}{6})}{2}=\frac{1-\cos(\frac{\pi}{6})}{\sin(\frac{\pi}{6})}\ \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}=2(1-\frac{\sqrt{3}}{2})=2(\frac{2}{2}-\frac{\sqrt{3}}{2})=2(\frac{2-\sqrt{3}}{2})$

$=\boxed{2-\sqrt{3}}$

Power-Reducing Formulas (do not need to memorize)

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Examples:

Rewrite in terms of the first power of the cosine: $\sin^4 x = (\sin^2 x)^2 = \left(\frac{1-\cos 2x}{2}\right)^2$

$$= \frac{(1-\cos 2x)^2}{4} = \frac{1-2\cos 2x+\cos^2 2x}{4}$$

$$= \frac{1}{4}\left[1-2\cos 2x+\left(\frac{1+\cos 4x}{2}\right)\right] = \frac{1}{4}\left[1-2\cos 2x+\frac{1}{2}+\frac{1}{2}\cos 4x\right]$$

$$= \frac{1}{4}\left[\frac{3}{2}-\frac{4}{2}\cos 2x+\frac{1}{2}\cos 4x\right] = \frac{1}{8}\left[3-4\cos 2x+\cos 4x\right]$$

Graph using the power-reducing formulas: $\cos^2 x = \left(\frac{1+\cos 2x}{2}\right)$

$$= \frac{1}{2}+\frac{1}{2}\cos(2x)$$

$0 < 2x < 2\pi$

$0 < x < \pi$ Per