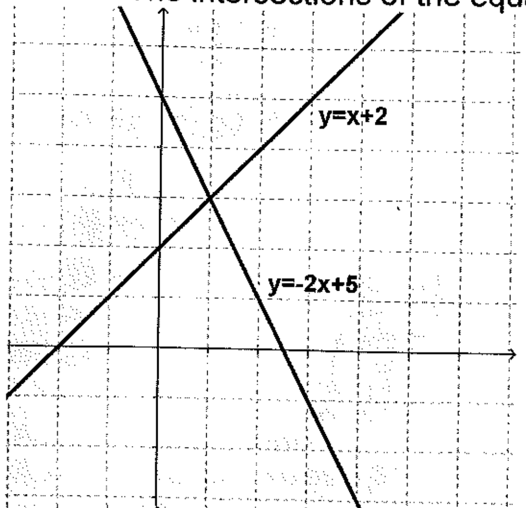
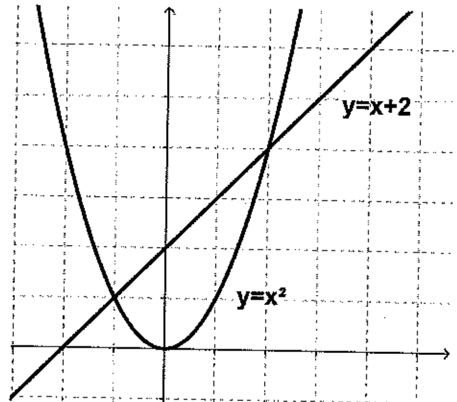
HAlg3-4, 7.1 day 1 Notes – Solving Systems of Equations

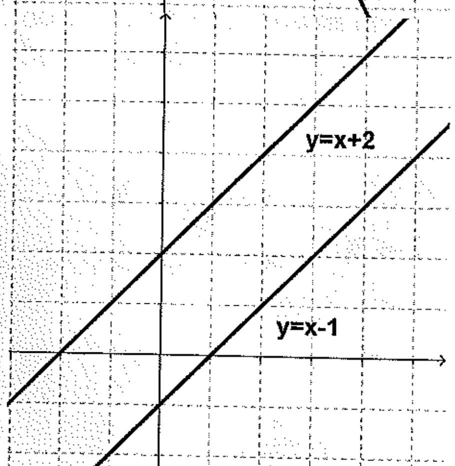
System of Equations – 2 or more equations that describe a system.

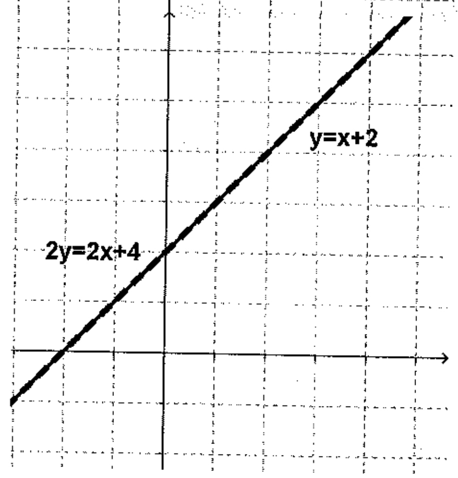
What is a solution of a system of equations?

- The ordered pairs which satisfy all equations in the system.
- The intersections of the equation curves.









Solving Systems of Equations – Graphically

1) Enter each equation in calculator (Y1=)
2) Find intersections using Calc – Intersection

Example: Solve system graphically: $\begin{cases} y = x^2 \\ y = x+2 \end{cases}$

Solving Systems of Equations – Algebraically using method of substitution

1) Solve one of the equations for one variable in terms of the other.
2) Substitute the expression found in step 1 into the other equation to obtain an equation in one variable.
3) Solve the equation from step 2.
4) Back-substitute the solution into the expression obtained in step 1.

Examples: Solve the systems by substitution

$$\begin{cases} 30x - 40y - 33 = 0 \\ 10x + 20y - 21 = 0 \end{cases}$$

$40y = 30x - 33$

$y = \frac{30x}{40} - \frac{33}{40}$

$10x + 20\left(\frac{30x}{40} - \frac{33}{40}\right) - 21 = 0$

$10x + 15x - \frac{33}{2} - 21 = 0$

$25x - \frac{33}{2} - 21 = 0$

$50x - 33 - 42 = 0$

$50x - 75 = 0$

$50x = 75$

$x = \frac{75}{50} = \frac{3}{2}$

$y = \frac{30}{40}\cdot\frac{3}{2} - \frac{33}{40}$

$= \frac{90}{80} - \frac{66}{80} = \frac{24}{80} = \frac{3}{10}$

$\boxed{\left(\frac{3}{2}, \frac{3}{10}\right)}$

$$\begin{cases} x^2 + y^2 = 5 \\ x + y = 1 \end{cases}$$

$y = 1 - x$

$x^2 + (1-x)^2 = 5$

$x^2 + 1 - 2x + x^2 = 5$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

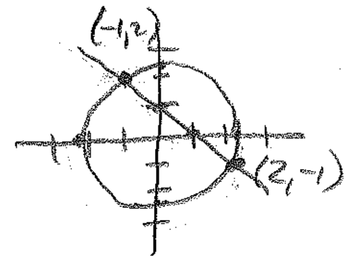
$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$y = 1 - 2$ $\quad y = 1 + 1$

$y = -1$ $\quad y = 2$

$(2, -1)$ $\quad (-1, 2)$



Application examples:

A small business invests \$10,000 in equipment to produce a product. Each unit of the product costs \$0.65 to produce and is sold for \$1.20. How many items must be sold before the business breaks even?

$C = 0.65x + 10000$ ← cost function

$R = 1.2x$ ← revenue (sales) function

breakeven when $R = C$

$1.2x = 0.65x + 10000$

$0.55x = 10000$

$x = 18181.8$

$\boxed{18182 \text{ units}}$

A small business has an initial investment of \$5000. The unit cost of the product is \$21.60, and the selling price is \$34.10.

(a) Write the cost and revenue functions for x units of product.

(b) Find the break-even point algebraically.

a) $C = 21.60x + 5000$

$R = 34.10x$

b) $R = C$

$34.10x = 21.60x + 5000$

$12.5x = 5000$

$x = \boxed{400 \text{ units}}$

Choice of 2 jobs: You are offered two different jobs selling college textbooks.
- One company offers an annual salary of $25,000 plus a year-end bonus of 1% of your total sales.
- The other company offers an annual salary of $20,000 plus a year-end bonus of 2% of your total sales.

Determine the annual sales that would make the second offer better.

Earnings $E_1 = 25000 + .01S$

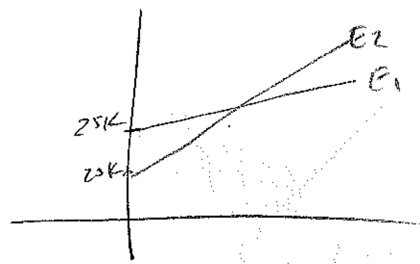
$E_2 = 20000 + .02S$

$E_2 = E_1$

$20000 + .02S = 25000 + .01S$

$.01S = 5000$

$S = \boxed{\$500{,}000}$



Geometry: Find the dimensions of the rectangle meeting the following conditions:
- The perimeter is 42 inches.
- The width is three-fourths the length.

L, W variables

$P = 2L + 2W$

$2L + 2W = 42$

$W = \frac{3}{4}L$

$2L + 2\left(\frac{3}{4}L\right) = 42$

$2L + \frac{3}{2}L = 42$

$\frac{4}{2}L + \frac{3}{2}L = 42$

$\frac{2}{7} \cdot \frac{7}{2}L = 42 \cdot \frac{2}{7}$

$L = \frac{84}{7} = 12, \quad W = \frac{3}{4}L = 9$

$\boxed{9 \times 12}$

HAlg3-4, 7.1 / 7.2 Notes – Solving Systems of Equations

Graphical interpretation of 2-variable systems

Number of Solutions	Graphical meaning	
Exactly one solution	Two lines intersect at one point	'consistent' (at least one solution)
Infinitely many solutions	Two lines are identical (coinciding)	
No solution	Two lines are parallel	'inconsistent' (no solution)

Solving System of Equations – Algebraically using Method of Elimination

1) Obtain coefficients for x (or y) that differ only in sign by multiplying all terms of one or both equations by suitably chosen constants.
2) Add the equations to eliminate one variable. Solve the resulting equation.
3) Back-substitute the value obtained into either original equation to solve for the other variable.
4) optional: Check your solution in both of the original equations.

Examples: Solve each system using elimination

$$\begin{cases} 2x-3y=-15 \\ 5x+2y=10 \end{cases}$$

$2(2x-3y=-15)$
$3(5x+2y=10)$

$4x-6y=-30$
$15x+6y=30$
$19x=0$
$x=0$
$2(0)-3y=-15$
$y=\frac{-15}{-3}=5$
$\boxed{(0,5)}$

$$\begin{cases} 3x+4y=11 \\ x+2y=5 \end{cases}$$

$-3(x+2y=5)$

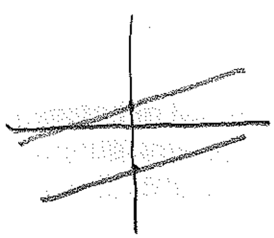
$3x+4y=11$
$-3x-6y=-15$
$-2y=-4$
$y=2$

$x+2(2)=5$
$x+4=5$
$x=1$
$\boxed{(1,2)}$

$$\begin{cases} x-2y=3 \\ -2x+4y=1 \end{cases}$$

$2(x-2y=3)$

$2x-4y=6$
$-2x+4y=1$
$0=7$
contradiction
$\boxed{\text{No Solutions case}}$
'inconsistent'



$x-2y=3$:
$-2y=-x+3$
$y=\frac{1}{2}x-\frac{3}{2}$
$-2x+4y=1$:
$4y=2x+1$
$y=\frac{1}{2}x+\frac{1}{4}$

$$\begin{cases} 2x-y=1 \\ 4x-2y=2 \end{cases}$$

$2(2x-y=1)$

$4x-2y=2$
$4x-2y=2$
$0=0$
Same line
$\boxed{\text{infinitely many solutions}}$

More application examples:

A man in a boat can row 8 miles downstream in one hour. He can row 6 miles upstream in three hours. How fast can the man row in still water and what is the rate of the current?

$d=rt$

r = rowing speed still H_2O
w = water speed (current)

$d=rt$

$8=(r+w)1$ $\quad 3(8=r+w)$ $\rightarrow 3$

$6=(r-w)3$ $\quad 6=3r-3w$ $\rightarrow 2$

$24=3r+3w$

$30=6r$

$5=r$

$8=5+w$

$w=3$

$\boxed{\text{row still } H_2O = 5 \text{ mph}}$ $\quad \boxed{\text{Current} = 3\text{ mph}}$

An airplane flying into a headwind travels the 1800-mile flying distance between two cities in 3 hours and 36 minutes. On the return flight, the distance is traveled in 3 hours. Find the airspeed of the plane and the speed of the wind, assuming both remain constant.

$d=rt$

a = airspeed
w = wind speed

$1800=(a-w)3.6$ $\quad (1800=3.6a-3.6w)3$

$1800=(a+w)3$ $\quad (1800=3a+3w)3.6$

$5400=10.8a-10.8w$

$6480=10.8a+10.8w$

$11880=21.6a$

$\boxed{550 = a \text{ mph}}$

$1800=3(550)+3w$

$\boxed{w=50 \text{ mph}}$

Five hundred gallons of 89 octane gasoline is obtained by mixing 87 octane gas with 92 octane gas. (a) Write equations for total amount of fuel, and octane of fuel mix. (b) How much of each type of gasoline is required to obtain the 500 gallons of 89 octane gas?

x = amt of 87 octane
y = amt of 92 octane

amt of fuel: $x+y=500$

octane: $87x+92y=89(500)$

$-87x-87y=-43500$

$87x+92y=44500$

$5y=1000$

$\boxed{y=200 \text{ gallons of 92 octane}}$

$x+200=500$

$\boxed{x=300 \text{ gallons of 87 octane}}$

A total of $32,000 in invested in two municipal bonds that pay 5.75% and 6.25% simple interest. The investor wants an annual interest income of $1900 from the investments. What is the most that can be invested in the 5.75% bond?

x = amt in 5.75% bond
y = amt in 6.25% bond

total invested: $(x+y=32000)\,{-575}$

interest income: $(.0575x+.0625y=1900)\,10000$

$-575x-575y=-18400000$

$575x+625y=19000000$

$50y=600000$

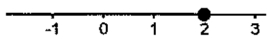
$y=\$12000$

$\boxed{x=\$20000}$

HAlg3-4, 7.3 day 1 Notes – Multivariable Linear Systems, Gaussian Elimination

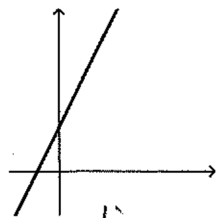
Multivariable Linear Systems:

1-dimension
x=2



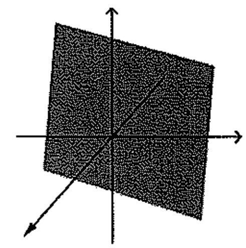
point

2-dimensions
4x – 2y = -2



line

3-dimensions
4x – 2y + 3z = 5

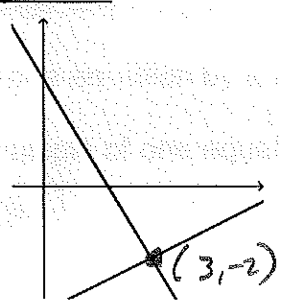


plane

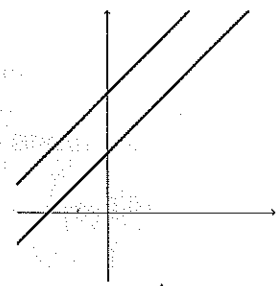
2-D systems of equations:

$$\begin{cases} 5x+3y=9 \\ 2x-4y=14 \end{cases}$$

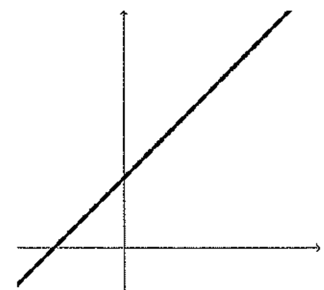
solution: (3, -2)



point



no solution

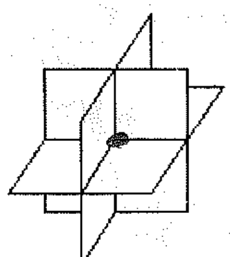


infinite solutions
(same line)

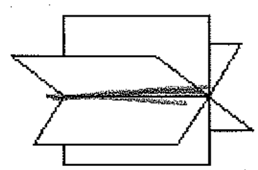
3-D systems of equations:

$$\begin{cases} x-2y+3z=9 \\ -x+3y \qquad =-4 \\ 2x-5y+5z=17 \end{cases}$$

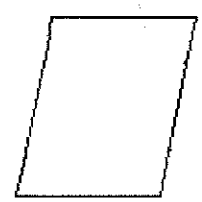
solution: (1, -1, 2)



point

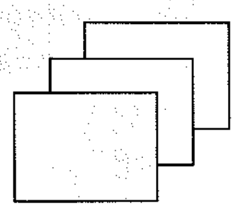


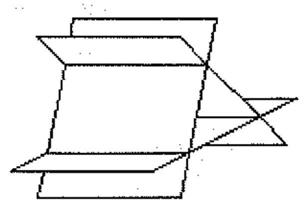
line

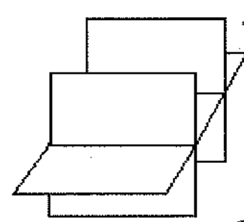


plane
(coinciding planes)

'consistent'







'inconsistent'
no solutions

Row-Echelon Form / Back-Substitution

Systems of equations are <u>equivalent</u> if they have the same solution set.

$$\begin{cases} x-2y+3z=9 \\ -x+3y \quad =-4 \\ 2x-5y+5z=17 \end{cases} \xrightarrow[\text{equivalent}]{} \begin{cases} x-2y+3z=9 \\ y+3z=\cancel{4}\,5 \\ z=2 \end{cases}$$

back substitue

$x-2(-1)+3(2)=9 \quad x+2+6=9 \quad x=1$

$y+3(2)=5 \quad y+6=5 \quad y=-1$

$\boxed{(1,-1,2)}$

Gaussian Elimination – a way to get equivalent systems that are easier to solve

<u>Elementary Row Operations</u> – each of these produces an equivalent system

1) Interchange any 2 equations.

2) Multiply one equation by a nonzero constant.

3) Add a multiple of one equation to another equation.

Example: (teacher)

$$\begin{cases} x-2y+3z=9 \\ -x+3y \quad =-4 \\ 2x-5y+5z=17 \end{cases}$$

$$R_1+R_2 \begin{cases} x-2y+3z=9 \\ y+3z=5 \\ 2x-5y+5z=17 \end{cases}$$

$$-2R_1+R_3 \begin{cases} x-2y+3z=9 \\ y+3z=5 \\ -y-z=-1 \end{cases}$$

$$-R_2+R_3 \begin{cases} x-2y+3z=9 \\ y+3z=5 \\ 2z=4 \end{cases}$$

$2z=4 \quad z=2$

$y+3(2)=5 \quad y+6=5 \quad y=-1$

$x-2(-1)+3(2)=9 \quad x+2+6=9 \quad x=1$

$\boxed{(1,-1,2)}$

Try these: (together)

$$\begin{cases} 4x+\ y-3z=11 \\ 2x-3y+2z=9 \\ x+\ y+\ z=-3 \end{cases}$$

$$R_3 \leftrightarrow R_1 \begin{cases} x+y+z=-3 \\ 2x-3y+2z=9 \\ 4x+y-3z=11 \end{cases}$$

$$-2R_1+R_2 \begin{cases} x+y+z=-3 \\ -5y=15 \\ 4x+y-3z=11 \end{cases}$$

$$-4R_1+R_3 \begin{cases} x+y+z=-3 \\ -5y=15 \\ -3y-7z=23 \end{cases}$$

$$R_3 \leftrightarrow R_2 \begin{cases} x+y+z=-3 \\ -3y-7z=23 \\ -5y=15 \end{cases}$$

$-5y=15 \quad y=-3$

$-3(-3)-7z=23 \quad 9-7z=23 \quad -7z=14 \quad z=-2$

$-x-3-2=-3 \quad x=2$

$\boxed{(2,-3,-2)}$

(students)

$$\begin{cases} 2x+\quad 2z=6 \\ 5x+3y \quad =11 \\ 3y-4z=1 \end{cases}$$

$$-R_1+R_2 \begin{cases} 2x+2z=6 \\ 5x+3y=11 \\ -5x-4z=-10 \end{cases}$$

$$2R_1+R_3 \begin{cases} 2x+2z=6 \\ 5x+3y=11 \\ -x=2 \end{cases}$$

$-x=2 \quad x=-2$

$5(-2)+3y=11 \quad -10+3y=11 \quad 3y=21 \quad y=7$

$2(-2)+2z=6 \quad -4+2z=6 \quad 2z=10 \quad z=5$

$\boxed{(-2,7,5)}$

Hints:

- Try to eliminate x's in all but the first row.
- Try to use the 2nd row to eliminate y's (or sometimes eliminating z's is easier).
- Make 2nd and 3rd rows have the same 2 terms.
- Good to have 1st row start with x (not 2x, -4x, etc)

Example: an inconsistent system

$$\begin{cases} x-3y\ +z=1 \\ 2x\ -y-2z=2 \\ x+2y-3z=-1 \end{cases}$$

$$\begin{matrix} 1 \\ -2R_1+R_2 \\ 1 \end{matrix}\begin{cases} x-3y+z=1 \\ 5y-4z=0 \\ x+2y-3z=-1 \end{cases}$$

$$\begin{matrix} \\ 1 \\ -R_1+R_3 \end{matrix}\begin{cases} x-3y+z=1 \\ 5y-4z=0 \\ 5y-4z=-2 \end{cases}$$

$$\begin{cases} x-3y+z=1 \\ 5y-4z=0 \\ 5y-4z=-2 \end{cases}$$

$$\begin{matrix} \\ \\ -R_2+R_3 \end{matrix}\begin{cases} x-3y+z=1 \\ 5y-4z=0 \\ \boxed{0=-2} \end{cases}$$

inconsistent

Application examples (setup only):

Find a quadratic equation, $y=ax^2+bx+c$ whose graph passes through the points (-1,3), (1,1), and (2,6).

x y

$3=a(-1)^2+b(-1)+c$

$3=a-b+c$

$$\begin{cases} a-b+c=3 \\ a+b+c=1 \\ 4a+2b+c=6 \end{cases}$$

Find the position equation: $s=\frac{1}{2}at^2+v_0t+s_0$

for an object at the given heights moving vertically at the specified times.

At t = 1 second, s = 128 feet
At t = 2 seconds, s = 80 feet
At t = 3 seconds, s = 0 feet

$128=\frac{1}{2}a(1)^2+v_0(1)+s_0$

$$\begin{cases} \frac{1}{2}a+v_0+s_0=128 \\ 2a+2v_0+s_0=80 \\ \frac{9}{2}a+3v_0+s_0=0 \end{cases}$$

HAlg3-4, 7.3 day 2 Notes – Multivariable Linear Systems, Gaussian Elimination

Example: A system with infinitely many solutions

$$\begin{cases} x + y - 3z = -1 \\ y - z = 0 \\ -x + 2y = 1 \end{cases}$$

$$\begin{cases} x + y - 3z = -1 \\ y - z = 0 \\ 3y - 3z = 0 \end{cases} \quad R_1 + R_3$$

$$\begin{cases} x + y - 3z = -1 \\ y - z = 0 \\ 0 = 0 \end{cases} \quad -3R_2 + R_3$$

not inconsistent, but no new information.

Really only 2 equations in this system.

$$\begin{cases} x + y - 3z = -1 \\ y - z = 0 \end{cases}$$

solve for everything in terms of last variable, z;

$y - z = 0$

$y = z$

$x + y - 3z = -1$

$x + (z) - 3z = -1$

$x - 2z = -1$

$x = 2z - 1$

if $z = a$ (any real number)

Solutions would be

$\boxed{(2a-1, a, a)}$

where a is any real number (infinitely many points)

$x(a) = 2a - 1$

$y(a) = a$

$z(a) = a$

Nonsquare System = a system in which # of equations is different than # of variables

Example: nonsquare system

$$\begin{cases} x - 2y + z = 2 \\ 2x - y - z = 1 \end{cases}$$

$$\begin{cases} x - 2y + z = 2 \\ 3y - 3z = -3 \end{cases} \quad -2R_1 + R_2$$

$$\begin{cases} x - 2y + z = 2 \\ y - z = -1 \end{cases} \quad \tfrac{1}{3}R_2$$

infinitely many solutions

solve in terms of z;

$y = z - 1$

$x - 2(z - 1) + z = 2$

$x - 2z + 2 + z = 2$

$x - z = 0$

$x = z$

let $z = a$

$\boxed{(a, a-1, a)}$

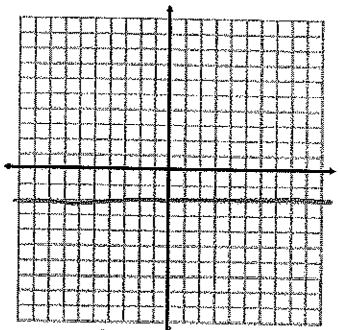
If you have too many equations; check answer in all

(#93)

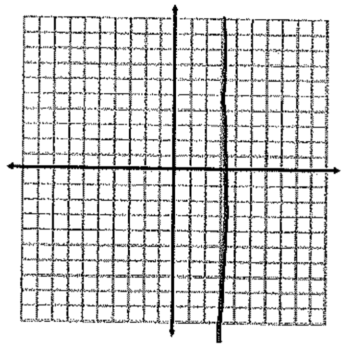
HAlg3-4, 7.4 day 1 Notes – Systems of Inequalities

Quick review of graphing: Graph each equation.

$y=-2$

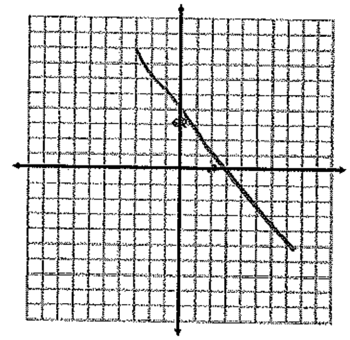


$x=3$



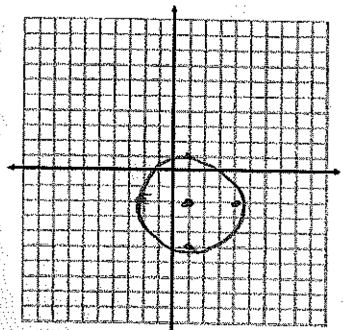
$3x+\cancel{2y}=6$

$3x+2y=6$

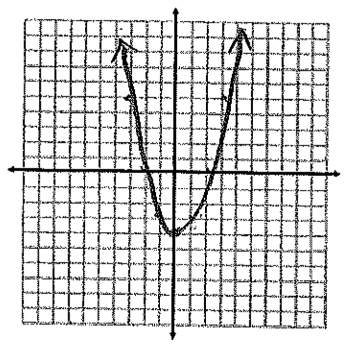


$(x-h)^2+(y-k)^2=r^2$

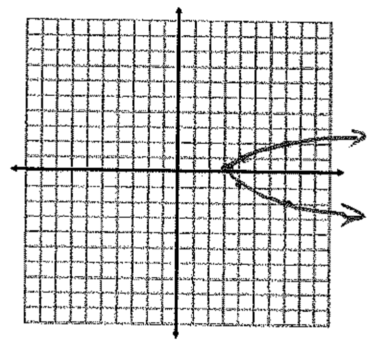
$(x-1)^2+(y+2)^2=9$



$x^2=y+4$ $y=x^2-4$



$y^2=x-3$ $x=y^2+3$



Graphing inequalities – Turn into an equation, and graph, then shade appropriate side.

$\geq or \leq$ solid line

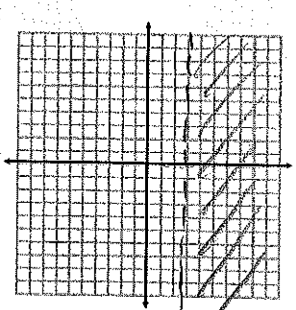
$> or <$ dotted line

Shading: $\geq or >$ above or to the right -or- use a test point

$\leq or <$ below or to the left

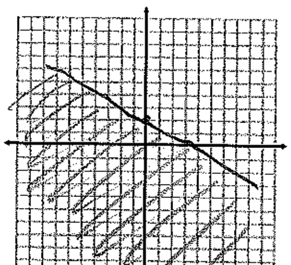
Examples/practice:

$x>3$



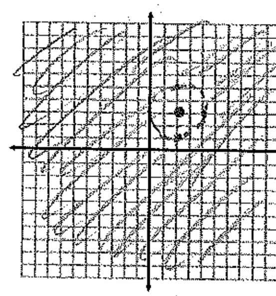
test(0,0)

$2x+3y\leq 6$

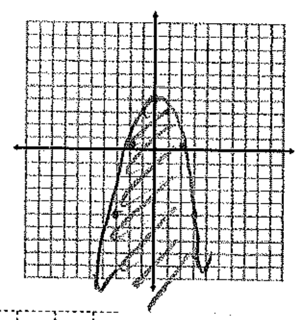


test(0,0)

$(x-2)^2+(y-3)^2>4$



$y\leq 4-x^2$ $-x^2+4$



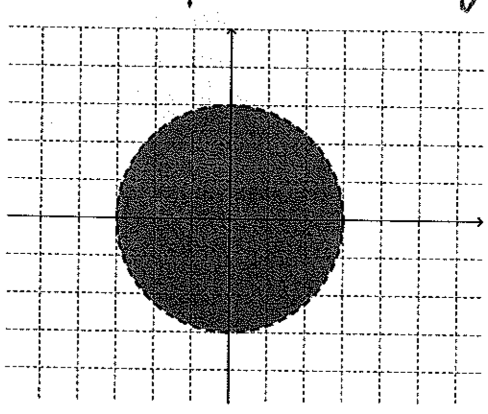
Write an inequality for the shaded region:

test(0,0) $x^2+y^2=9$

$0^2+0^2 \,?\, 9$

$0 \,?\, 9$

$\boxed{x^2+y^2<9}$



HAlg3-4, 7.4 day ~~1~~ 2 Notes – Systems of Inequalities

Graphing systems of inequalities:

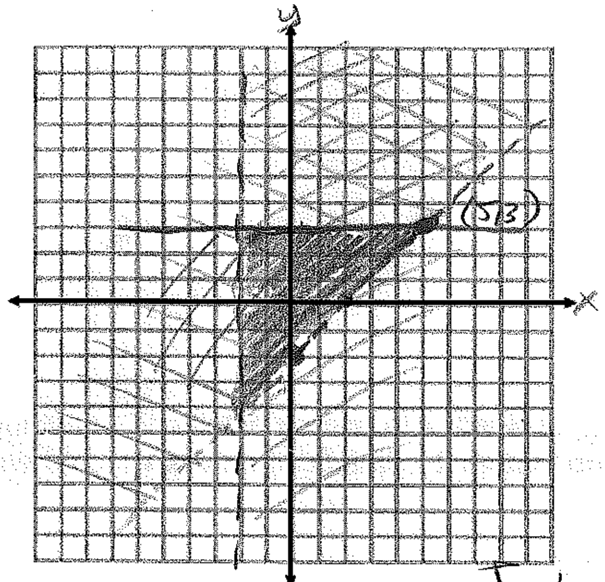
- Graph each inequality in the system, including shading (use different colors, or different cross-hatching marks.)
- The solution is a region – the area that 'overlaps' (is shaded for all inequalities in the system.)
- The solution may be: bounded or unbounded, or there may be no solution (no overlap.)

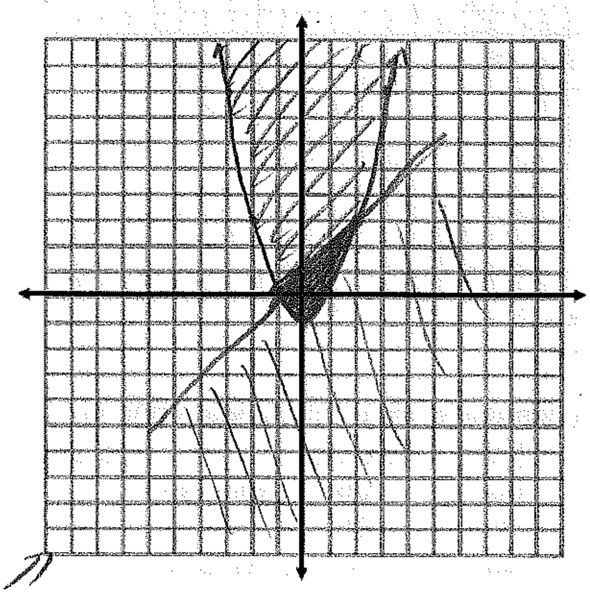
$$\begin{cases} x - y < 2 \\ x > -2 \\ y \le 3 \end{cases}$$

$y = 3$
$x - (3) = 2$
$x = 5$
$(5, 3)$

$x^2 - y = 1$
$y = x^2 - 1$

$$\begin{cases} x^2 - y \le 1 \\ -x + y \le 1 \end{cases}$$

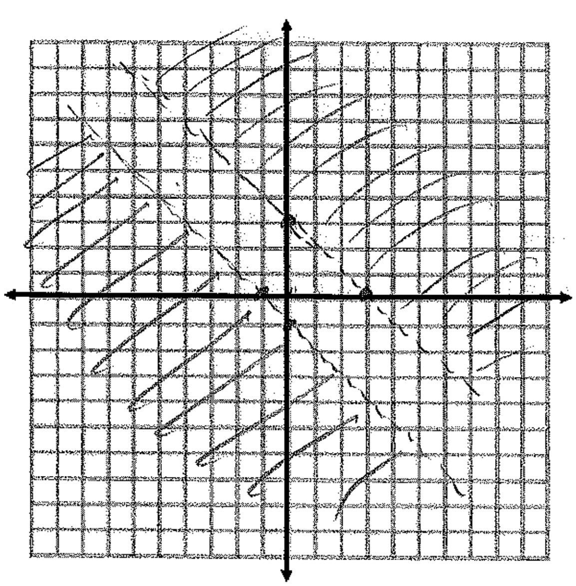




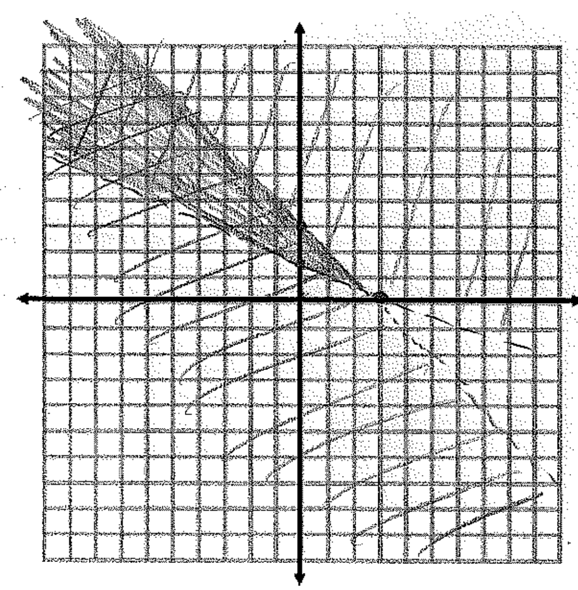
bounded solutions

$$\begin{cases} x + y > 3 \\ x + y < -1 \end{cases}$$

$$\begin{cases} x + y < 3 \\ x + 2y > 3 \end{cases}$$



No solution



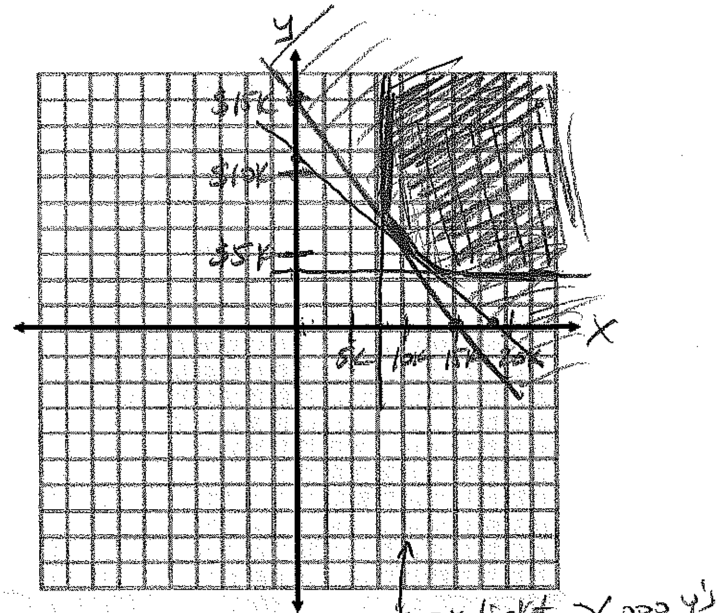
unbounded solution

Applications – Systems of Inequalities

Concert Ticket Sales – One type of concert ticket costs \$15 and another costs \$25. The promoter of a concert must sell at least 15,000 tickets, including at least 8,000 of the \$15 tickets and at least 4,000 of the \$25 tickets, and the gross receipts must total at least \$275,000 in order for the concert to be held.

x – \$15
y – \$25

$$\begin{cases} x + y \geq 15{,}000 \\ x \geq 8{,}000 \\ y \geq 4{,}000 \\ 15x + 25y \geq 275{,}000 \end{cases}$$



10,000 x ticket, >6,000 y's

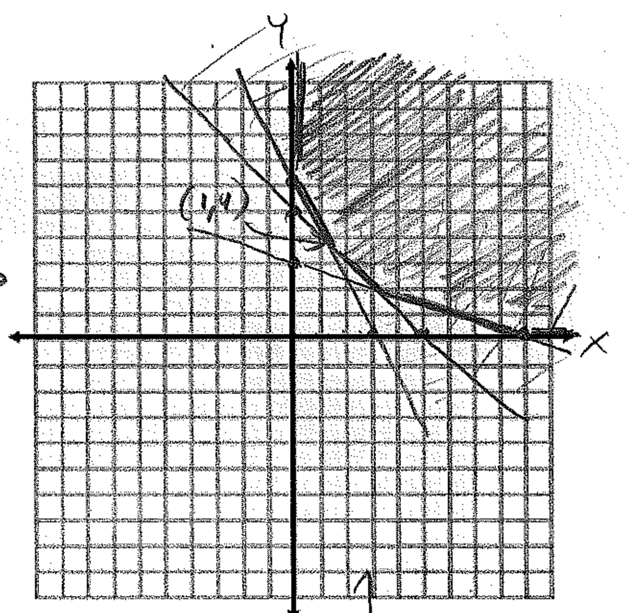
Nutrition - The liquid portion of a diet is to provide at least 300 calories, 36 units of vitamin A, and 90 units of vitamin C daily. 2 dietary drinks are available:

	drink X	drink Y
calories	60	60
vitamin A	12	6
vitamin C	10	30

Set up a system of linear inequalities that must be satisfied in order to meet the minimum daily requirements for calories and vitamins.

$60x + 60y \geq 300$
$12x + 6y \geq 36$
$10x + 30y \geq 90$
$x \geq 0$
$y \geq 0$

$60x + 60y = 300$
$12x + 6y = 36$
$6y = 36 - 12x$
$y = 6 - 2x$
$60x + 60(6 - 2x) = 300$
$60x + 360 - 120x = 300$
$-60x = -60$
$x = 1$
$y = 6 - 2 = 4$
$(1, 4)$



3 x drinks, >2 Y drinks

Economics - supply and demand curves, good example in textbook p. 530