HAlg3-4, 8.1 Notes – Matrices and Systems of Equations

A matrix – is a rectangular arrangement of real numbers:

$$\begin{array}{r c}
\text{row1} & \\
\text{row2} & \begin{pmatrix} 2 & -3 & 4.5 \\ \frac{3}{2} & 8.3 & -15 \\ 0 & 2 & 0 \end{pmatrix} \\
\text{row3} & \\
 & \text{col1} \quad \text{col2} \quad \text{col3}
\end{array}
\qquad
\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Individual elements denoted by subscripts: $a_{row,column}$

Dimension or Order of a matrix: #rows x #columns

$$\begin{pmatrix} 2 & -3 & 4.5 \\ \frac{3}{2} & 8.3 & -15 \\ 0 & 2 & 0 \end{pmatrix} \quad \begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix} \quad \begin{bmatrix} 2 & -5 & 3 \\ 0 & 1 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 0 & 8 \\ 9 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & -4 & 6 \end{bmatrix} \quad \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix}$$

3x3   2x2   2x3   4x2   1x3   3x1

Representing systems of equations with a matrix:

$$\begin{cases} x+3y \qquad = 9 \\ \quad -y+4z = -2 \\ x \qquad -5z = 0 \end{cases} \longrightarrow \begin{array}{c} x \quad\; y \quad\; z \quad \text{constant} \\ \left[\begin{array}{ccc:c} 1 & 3 & 0 & 9 \\ 0 & -1 & 4 & -2 \\ 1 & 0 & -5 & 0 \end{array}\right] \end{array}$$

augmented matrix

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & 4 \\ 1 & 0 & -5 \end{bmatrix} \qquad \begin{bmatrix} 9 \\ -2 \\ 0 \end{bmatrix}$$

coefficient matrix   constant matrix

Solving a system of equations using matrices (Gaussian Elimination, back-substitution):

$$\begin{cases} x-2y+3z=9 \\ -x+3y \qquad =-4 \\ 2x-5y+5z=17 \end{cases} \qquad \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

Same elementary row operations from last chapter

$$R_1+R_2 \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

$$-2R_1+R_3 \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{array}\right]$$

$$R_2+R_3 \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

$$\tfrac{1}{2}R_1+R_3 \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\boxed{\text{row-echelon form (REF)}}$

- leading 1 top left
- zeros below
- 1 diagonal

then, back substitute

$z=2$

$y+3(2)=5 \quad y=-1$

$x-2(-1)+3(2)=9$

$x+2+6=9$

$x=1 \qquad (1,-1,2)$

Solving a system of equations using matrices (Gauss-Jordan Elimination):

$$\begin{cases} x-2y+3z=9 \\ -x+3y \quad =-4 \\ 2x-5y+5z=17 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & & -4 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

(example above to row-echelon form)

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Keep going until only 1's on diagonal, zero's elsewhere on left

$$-2R_2+R_1 \left[\begin{array}{ccc|c} 1 & 0 & 9 & 19 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$-9R_3+R_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$-3R_3+R_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} x=1 \\ y=-1 \\ z=2 \end{array}$$

$(1,-1,2)$

reduced row echelon form

RREF

(1's diagonal, 0's everywhere else)

Examples/Practice: Solve system using Gaussian or Gauss-Jordan elimination

$$\begin{cases} 2x+6y=14 \\ 2x+3y=2 \end{cases}$$

$$\frac{1}{2} \left[\begin{array}{cc|c} 2 & 6 & 14 \\ 2 & 3 & 2 \end{array}\right]$$

$$\tfrac{1}{2}R_1 \left[\begin{array}{cc|c} 1 & 3 & 7 \\ 2 & 3 & 2 \end{array}\right]$$

$$-2R_1+R_2 \left[\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & -3 & -12 \end{array}\right]$$

$$-\tfrac{1}{3}R_2 \left[\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & 1 & 4 \end{array}\right]$$

$$-3R_2+R_1 \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 4 \end{array}\right]$$

$x=-5$
$y=4$ $\quad \boxed{(-5,4)}$

$$\begin{cases} 2x \quad +3z=3 \\ 4x-3y+7z=5 \\ 8x-9y+15z=9 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 4 & -3 & 7 & 5 \\ 8 & -9 & 15 & 9 \end{array}\right]$$

$$-2R_1+R_2 \rightarrow \left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 0 & -3 & 1 & -1 \\ 8 & -9 & 15 & 9 \end{array}\right]$$

$$-4R_1+R_3 \rightarrow \left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 0 & -3 & 1 & -1 \\ 0 & -9 & 3 & -3 \end{array}\right]$$

$$\begin{array}{r} \\ -R_2 \rightarrow \\ -\tfrac{1}{3}R_3 \rightarrow \end{array} \left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 0 & 3 & -1 & 1 \\ 0 & 3 & -1 & 1 \end{array}\right]$$

$$-R_2+R_3 \rightarrow \left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

consistent

write equations:

$2x+3z=3$

$3y-z=1$

make $z=a$ (real #)

$3y-a=1$

$y=\frac{a+1}{3}$

$2x+3(a)=3$

$2x=3-3a$

$x=\frac{3-3a}{2}$

$$\boxed{\left(\frac{3-3a}{2}, \frac{a+1}{3}, a\right)}$$

$$\begin{cases} 2x - y + 3z = 24 \\ 2y - z = 14 \\ 7x - 5y = 6 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 24 \\ 0 & 2 & -1 & 14 \\ 7 & -5 & 0 & 6 \end{array}\right]$$

$$\begin{array}{r} 7R_1 \to \\ \\ -2R_3 \to \end{array}\left[\begin{array}{ccc|c} 14 & -7 & 21 & 168 \\ 0 & 2 & -1 & 14 \\ -14 & 10 & 0 & -12 \end{array}\right]$$

$$\begin{array}{r} \\ \\ R_1+R_3 \to \end{array}\left[\begin{array}{ccc|c} 14 & -7 & 21 & 168 \\ 0 & 2 & -1 & 14 \\ 0 & 3 & 21 & 156 \end{array}\right]$$

$$\begin{array}{r} \\ \\ \frac{1}{3}R_3 \to \end{array}\left[\begin{array}{ccc|c} 14 & -7 & 21 & 168 \\ 0 & 2 & -1 & 14 \\ 0 & 1 & 7 & 52 \end{array}\right]$$

$$\begin{array}{r} \\ R_3 \\ R_2 \end{array}\left[\begin{array}{ccc|c} 14 & -7 & 21 & 168 \\ 0 & 1 & 7 & 52 \\ 0 & 2 & -1 & 14 \end{array}\right]$$

$$\begin{array}{r} \\ \\ -2R_2+R_3 \to \end{array}\left[\begin{array}{ccc|c} 14 & -7 & 21 & 168 \\ 0 & 1 & 7 & 52 \\ 0 & 0 & -15 & -90 \end{array}\right]$$

$-15z = -90$

$z = 6$

$y + 7(6) = 52$

$y = 10$

$14x - 7(10) + 21(6) = 168$

$x = 8$

$\boxed{(8, 10, 6)}$

faster techniques

swap columns to get a leading 1 in upper left

$$\begin{array}{c} \begin{array}{ccc} y & x & z \end{array} \\ \left[\begin{array}{ccc|c} 2 & -1 & 3 & 24 \\ 0 & 2 & -1 & 14 \\ 7 & -5 & 0 & 6 \end{array}\right] \end{array}$$

$$\left[\begin{array}{ccc|c} -1 & 2 & 3 & 24 \\ 2 & 0 & -1 & 14 \\ -5 & 7 & 0 & 6 \end{array}\right]$$

2 operations at once

$$\begin{array}{r} \\ 2R_1+R_2 \to \\ -5R_1+R_3 \to \end{array}\left[\begin{array}{ccc|c} -1 & 2 & 3 & 24 \\ 0 & 4 & 5 & 62 \\ 0 & -3 & -15 & -114 \end{array}\right]$$

$$\begin{array}{r} \\ 3R_2 \to \\ 4R_3 \to \end{array}\left[\begin{array}{ccc|c} -1 & 2 & 3 & 24 \\ 0 & 12 & 15 & 186 \\ 0 & -12 & -60 & -456 \end{array}\right]$$

$$\begin{array}{r} \\ \\ R_2+R_3 \to \end{array}\left[\begin{array}{ccc|c} -1 & 2 & 3 & 24 \\ 0 & 12 & 15 & 186 \\ 0 & 0 & -45 & -270 \end{array}\right]$$

$-45z = -270$

$z = 6$

$12x + 15(6) = 186$

$x = 8$

$-y + 2(8) + 3(6) = 24$

$y = 10$

$\boxed{(8, 10, 6)}$

HAlg3-4, 8.2 day 1 Notes – Operations with matrices

Matrix notation: A matrix is represented by a capital letter: $A=\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$

Equality of matrices: Two matrices are equal if:
- they have the same order (dimensions)
- all corresponding entries are equal.

Homework example: Given the following matrices are equal, find x, y, and z.

$$\begin{bmatrix} x+4 & 8 & -3 \\ 1 & 22 & 2y \\ 7 & -2 & z+2 \end{bmatrix} = \begin{bmatrix} 2x+9 & 8 & -3 \\ 1 & 22 & -2 \\ 7 & -2 & 11 \end{bmatrix}$$

$x+4 = 2x+9$
$-5 = x$

$2y = -2$
$y = -1$

$z+2 = 11$
$z = 9$

Matrix addition, subtraction: *to add or subtract, order of matrices must be the same*

Add (or subtract) corresponding entries:

$$\begin{bmatrix} 2 & -3 & 1 \\ 0 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 & -7 \\ 2 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -1 & -6 \\ 2 & -1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 6 & -11 \\ 2 & -1 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$$ not possible (undefined)

Multiplying a matrix by a scalar number: Multiply each element in the matrix by the number

$$3\begin{bmatrix} 2 & -3 & 1 \\ 0 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -9 & 3 \\ 0 & 3 & 12 \end{bmatrix}$$

Properties of Matrix Addition and Scalar Multiplication:

For matrices $A, B,$ and C, and scalars c and d:

1. $A+B=B+A$ *(commutative property of addition)*
2. $A+(B+C)=(A+B)+C$ *(associative property of addition)*
3. $(cd)A=c(dA)$ *(associative property of scalar multiplication)*
4. $1A=A$ *(scalar identity)*
5. $c(A+B)=cA+cB$ *(distributive property)*
6. $(c+d)A=cA+dA$ *(distributive property)*

<u>Summary:</u> *For simple operations, matrices behave the same as numbers.*
Order of operations is the same.

Example: For the following matrices, find 3A – B:

$$A=\begin{bmatrix}2 & 0 & 4\\ 3 & -2 & 5\\ -1 & 0 & 2\end{bmatrix} \qquad B=\begin{bmatrix}1 & 2 & 7\\ -1 & 3 & 2\\ -3 & 2 & 1\end{bmatrix}$$

$$3\begin{bmatrix}2 & 0 & 4\\ 3 & -2 & 5\\ -1 & 0 & 2\end{bmatrix}-\begin{bmatrix}1 & 2 & 7\\ -1 & 3 & 2\\ -3 & 2 & 1\end{bmatrix}$$ multiplication before subtraction

$$\begin{bmatrix}6 & 0 & 12\\ 9 & -6 & 15\\ -3 & 0 & 6\end{bmatrix}-\begin{bmatrix}1 & 2 & 7\\ -1 & 3 & 2\\ -3 & 2 & 1\end{bmatrix}=\begin{bmatrix}5 & -2 & 5\\ 10 & -9 & 13\\ 0 & -2 & 5\end{bmatrix}$$

Solving matrix equations:

1. Solve the equation using capital letter symbols for matrices.
2. Replace the symbols with matrices and find the final answer matrix.

Solve the matrix equation: $A-2X=B$ for $A=\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}$ $B=\begin{bmatrix}-3 & -6\\ 5 & 1\end{bmatrix}$

$$A-2X=B$$
$$-A \qquad -A$$
$$\frac{-2X}{-2}=\frac{B-A}{-2}$$

$$X=-\frac{1}{2}(B-A)$$

$$X=-\frac{1}{2}\left(\begin{bmatrix}-3 & -6\\ 5 & 1\end{bmatrix}-\begin{bmatrix}2 & -1\\ 4 & 3\end{bmatrix}\right)$$ parentheses 1st

$$=-\frac{1}{2}\left(\begin{bmatrix}-5 & -5\\ 1 & -2\end{bmatrix}\right)$$ scalar multiply

$$=\begin{bmatrix}\frac{5}{2} & \frac{5}{2}\\ -\frac{1}{2} & 1\end{bmatrix}$$

HAlg3-4, 8.2 day 2 Notes – Operations with matrices

Matrix multiplication (multiplying a matrix by another matrix):

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \bullet \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Easier to see with an example:

$$\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix} \bullet \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & \\ & \end{bmatrix} \qquad (1)(-1) + (2)(3) = -1 + 6 = 5$$

$$\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix} \bullet \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ & \end{bmatrix} \qquad (1)(5) + (2)(2) = 5 + 4 = 9$$

$$\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix} \bullet \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 7 & \end{bmatrix} \qquad (5)(-1) + (4)(3) = -5 + 12 = 7$$

$$\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix} \bullet \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 7 & 33 \end{bmatrix} \qquad (5)(5) + (4)(2) = 25 + 8 = 33$$

$\leftarrow$ answer

To compute an element:
1. Take the matching <u>row from the left matrix</u> and <u>column from the right matrix</u>,
2. Moving across row from left to right, and down column from top to bottom, multiply each pair of numbers.
3. Add the results...the sum is what you put in the corresponding space in the answer matrix.

Same procedure, regardless of the size of the matrices. But, for it to work:
<u>the number of columns of the left matrix</u> must match <u>the number of rows of the right matrix</u>.

Examples – multiply if possible: $\underbrace{2 \times 3 \cdot 3 \times 3}_{}$ (ok) — dim of answer

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \bullet \begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -5 & & \\ & & \end{bmatrix} \qquad (1)(-2) + (0)(1) + (3)(-1) = -2 + 0 - 3$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \bullet \begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 7 & \\ & & \end{bmatrix} \qquad (1)(4) + (0)(0) + (3)(1) = 4 + 0 + 3 = 7$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \bullet \begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 7 & -1 \\ & & \end{bmatrix} \qquad (1)(2) + (0)(0) + (3)(-1) = 2 + 0 - 3 = -1$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \bullet \begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -5 & 7 & -1 \\ -3 & & \end{bmatrix} \qquad (2)(-2) + (-1)(1) + (-2)(-1) = -4 - 1 + 2 = -3$$

$$\begin{bmatrix}1 & 0 & 3\\ 2 & -1 & -2\end{bmatrix}\cdot\begin{bmatrix}-2 & 4 & 2\\ 1 & 0 & 0\\ -1 & 1 & -1\end{bmatrix}=\begin{bmatrix}-5 & 7 & -1\\ -3 & 6 & \end{bmatrix}$$

$(2)(4)+(-1)(0)+(-2)(1)=8+0-2=6$

$$\begin{bmatrix}1 & 0 & 3\\ 2 & -1 & -2\end{bmatrix}\cdot\begin{bmatrix}-2 & 4 & 2\\ 1 & 0 & 0\\ -1 & 1 & -1\end{bmatrix}=\begin{bmatrix}-5 & 7 & -1\\ -3 & 6 & 6\end{bmatrix}$$

$(2)(2)+(-1)(0)+(-2)(-1)=4+0+2=6$

Example: $\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}\cdot\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\underbrace{2\times\underbrace{2 \cdot 2}_{\text{OK}}\times 2}_{\text{ans}}$

Example: $\begin{bmatrix}1 & -2 & -3\end{bmatrix}\cdot\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}=\begin{bmatrix}1\end{bmatrix}$ $\quad 2+2-3$

$1\times\underbrace{3 \cdot 3}_{\text{OK}}\times 1$

Example: $\begin{bmatrix}1 & -2 & -3\end{bmatrix}\cdot\begin{bmatrix}5 & -4 & 1\end{bmatrix}=$

$1\times 3 \cdot 1\times 3$ ← doesn't match, not possible

Identity Matrix: An identity matrix has 1's along the main diagonal, and 0's elsewhere:

main diagonal $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $\qquad$ $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ main diagonal

Called the identity matrix because if you multiply a matrix by an identity matrix, you get the original matrix:

$$\begin{bmatrix}3 & 4 & -2\\ 1 & -6 & 2\\ 7 & 0 & 5\end{bmatrix}\cdot\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}3 & 4 & -2\\ 1 & -6 & 2\\ 7 & 0 & 5\end{bmatrix}$$

Properties of Matrix Multiplication:

For matrices $A, B,$ and C, and scalar c:

1. $A(BC)=(AB)C$ (*associative property of matrix multiplication*)
2. $A(B+C)=AB+AC$ (*distributive property*)
3. $(A+B)C=AC+AB$ (*distributive property*)
4. $c(AB)=(cA)B=A(cB)$ (*associative property of scalar multiplication*)

Note: no commutative property. In general, $AB \neq BA$

---- Read textbook example 10, p. 576, to see how matrix multiplication might be used ---

Matrices on the graphing calculator:

You can enter matrices into your calculator, then have the calculator perform operations, including matrix multiplication (and other more complex things we'll cover later.)

Examples to show capabilities and procedure:

Find 3A-B for $A = \begin{bmatrix} 2 & 0 & 4 \\ 3 & -2 & 5 \\ -1 & 0 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 2 & 7 \\ -1 & 3 & 2 \\ -3 & 2 & 1 \end{bmatrix}$

Enter matrix A:
1. 2nd x^{-1} (MATRIX)
2. -> -> EDIT, with cursor on 1: [A] line, press 'enter'
3. '3', enter, '3', enter (specifying a 3x3 matrix)
4. type in each element and enter it by pressing the enter key, the cursor automatically jumps to the next element: e.g., '2', enter, '0', enter, '4', enter, '3', enter, etc. until all elements are entered. (use (-) key, not subtract, to enter negative numbers).
5. 2nd MODE (QUIT) to exit entry mode.

Enter matrix B:
6. 2nd x^{-1} (MATRIX)
7. -> -> EDIT, with cursor on 2: [B] line, press 'enter'
8. '3', enter, '3', enter (specifying a 3x3 matrix)
9. type in each element and enter it by pressing the enter key, the cursor automatically jumps to the next element: e.g., '1', enter, '2', enter, '7', enter, '-1', enter, etc. until all elements are entered. (use (-) key, not subtract, to enter negative numbers).
10. 2nd MODE (QUIT) to exit entry mode.

Compute expression 3A-B:
11. press '3'
12. 2nd x^{-1} (MATRIX)
13. with cursor on 1:[A], press 'enter'
14. press the subtraction key '-'
15. 2nd x^{-1} (MATRIX)
16. with cursor on 2:[B], press 'enter'
17. press 'enter' again to compute. Should display the answer matrix: $\begin{bmatrix} 5 & -2 & 5 \\ 10 & -9 & 13 \\ 0 & -2 & 5 \end{bmatrix}$

Using the same matrices, A and B, try to compute AB and BA. Answers should be:

$$AB = \begin{bmatrix} -10 & 12 & 18 \\ -10 & 10 & 22 \\ -7 & 2 & -5 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & -4 & 28 \\ 5 & -6 & 15 \\ -1 & -4 & 0 \end{bmatrix}$$

You can also explore 2nd x^{-1} (MATRIX), -> MATH menu. There are lots of matrix related functions here. Scroll down to rowSwap, *row+...these are elementary row operations. Later, we'll be doing determinants...that is also found here (det()).

HAlg3-4, 8.3 day 1 Notes – Inverse of a Matrix

What is an inverse?

Solve: $\frac{3x}{3} = \frac{6}{3}$

$x = 2$

really: $\frac{1}{3} 3x = \frac{1}{3} 6$

multiplying 3 by its inverse

$3x = 6$

$\frac{1}{3} 3x = \frac{1}{3} 6$

identity → $1 \cdot x = 2$

$x = 2$

Multiplying a number and its inverse produces identity (1).
Multiplying a matrix and its inverse produced identity (the identity matrix, I)

Example: Show that B is the inverse of A: $A = \begin{bmatrix} -5 & 6 \\ -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -6 \\ 1 & -5 \end{bmatrix}$

$AB = \begin{bmatrix} -5 & 6 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -6 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ ✓

$BA = \begin{bmatrix} 1 & -6 \\ 1 & -5 \end{bmatrix}\begin{bmatrix} -5 & 6 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ ✓

to prove inverse must verify both directions

Note: Inverses are exceptions – inverses do obey the commutative property.

How to find the inverse of a matrix: 3 methods.

(Textbook example 2, p.583, has a general explanation of finding an inverse algebraically from its definition, but we will focus on the more commonly used procedures to find an inverse.)

1) Row-reducing an augmented matrix

1. Write the initial matrix, and augment that matrix with the identity matrix.
2. If possible, row-reduce the augmented matrix to producethe identity matrix on the left side.
3. The right side is the inverse of the initial matrix.

Example: Find the inverse of $A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$

$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ -1 & -3 & 0 & 1 \end{array}\right]$

$R_1 + R_2 \rightarrow \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right]$

$-4R_2 + R_1 \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -3 & -4 \\ 0 & 1 & 1 & 1 \end{array}\right]$

inverse, A^{-1}

$A^{-1} = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$

symbol for inverse of a matrix

check:

$AA^{-1} = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$A^{-1}A = \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

2) Using the determinant

In section 8.4, we'll learn about determinants of matrices. For a 2x2 matrix, the determinant is given by:

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad determinant = \det(A) = |A| = ad - bc$$

Then the inverse of a matrix can be found using the determinant:

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example: Find the inverse of $A=\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix} = \frac{1}{\underbrace{(1)(-3)-(4)(-1)}_{-3\ +4}}\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix} = \frac{1}{1}\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$

3) Using a calculator

1. Enter the initial matrix using the 2nd x^{-1} (MATRIX), EDIT, specifying the size and entering all the matrix entries.
2. 2nd quit, then select the matrix entered by using the 2nd x^{-1} (MATRIX), scrolling with cursor to entered matrix, then hitting enter.
3. Press the x^{-1} key, then hit enter.

Example: Find the inverse of $A=\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$

Do all matrices have inverses?

No inverse if:

- Matrix is not square. ← special 'left inverse' and 'right inverse' matrices but no general inverse
- Matrix has any row that is a multiple of any other row.

because row operations will produce row 0 = 0

HAlg3-4, 8.3 day 2 Notes – Solving Matrix Equations

Writing a system of equations as a matrix equation:

$$\begin{cases} x-2y=1 \\ 2x-3y=-2 \end{cases} \rightarrow \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

coefficient matrix | variable matrix | constants matrix

multiply out left side:

$$\begin{bmatrix} x-2y \\ 2x-3y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x-2y=1$$
$$2x-3y=-2$$

We now know many ways to solve systems of equations

Alg12
- graphing
- substitution
- elimination

Hon Alg34
- Gaussian elimination (equations)
- Gaussian elimination (matrices)
- Gauss-Jordan

New: using matrix inverse

Solving systems of equations using matrices: 2 methods

1) Write system as a matrix equation, solving using matrix inverse

$$\begin{cases} x_1-2x_2=1 \\ 2x_1-3x_2=-2 \end{cases} \qquad \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$A \cdot X = B$$

$$\underbrace{A^{-1} \cdot A}\cdot X = A^{-1} \cdot B$$

$$I \cdot X = A^{-1}B$$

$$X = A^{-1} \cdot B$$

Find A^{-1}:

$$\left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 2 & -3 & 0 & 1 \end{array}\right]$$

$$-2R_1+R_2 \left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$2R_2+R_1 \left[\begin{array}{cc|cc} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} \leftarrow$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \end{bmatrix}$$

$$x_1 = -7$$
$$x_2 = -4$$

2) Write coefficients and answers as augmented matrix, obtain reduced row-echelon form (RREF)

$$\begin{cases} x-2y=1 \\ 2x-3y=-2 \end{cases} \Rightarrow \left[\begin{array}{cc:c} 1 & -2 & 1 \\ 2 & -3 & -2 \end{array}\right]$$

$$-2R_1+R_2 \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & -4 \end{array}\right]$$

$$2R_2+R_1 \left[\begin{array}{cc|c} 1 & 0 & -7 \\ 0 & 1 & -4 \end{array}\right] \leftarrow \text{RREF}$$

$x=-7$
$y=-4$

Examples from homework: Find solution using an inverse matrix. $\begin{cases} x-2y=5 \\ 2x-3y=10 \end{cases}$

$Ax=B$

$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$x=A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad x=5, \; y=0$$

$\boxed{(5,0)}$

$$\left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 2 & -3 & 0 & 1 \end{array}\right]$$

$$-2R_1+R_2 \left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$2R_2+R_1 \left[\begin{array}{cc|cc} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

Use a calculator for large systems: $\begin{cases} 2x+5y \qquad\quad +w=11 \\ x+4y+2z-2w=-7 \\ 2x-2y+5z \quad +w=3 \\ x \qquad\qquad\quad -3w=-1 \end{cases}$

$\boxed{\begin{array}{l} x=6.211 \\ y=-.765 \\ z=-2.671 \\ w=2.404 \end{array}}$

Find solution. $\begin{cases} 5x-3y+2z = 2 \\ 2x+2y-3z = 3 \\ -x+7y-8z = 4 \end{cases}$

$$\begin{bmatrix} 5 & -3 & 2 \\ 2 & 2 & -3 \\ -1 & 7 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

<u>no inverse</u>

try augmented matrix, $(z=a)$

$$\begin{array}{c} R_3 \\ \\ R_1 \end{array}\left[\begin{array}{ccc|c} -1 & 7 & -8 & 4 \\ 2 & 2 & -3 & 3 \\ 5 & -3 & 2 & 2 \end{array}\right]$$

$$\begin{array}{c} \\ 2R_1+R_2 \\ 5R_1+R_3 \end{array}\left[\begin{array}{ccc|c} -1 & 7 & -8 & 4 \\ 0 & 16 & -19 & 11 \\ 0 & 32 & -38 & 22 \end{array}\right]$$

$$\begin{array}{c} \\ \\ -2R_2+R_3 \end{array}\left[\begin{array}{ccc|c} -1 & 7 & -8 & 4 \\ 0 & 16 & -19 & 11 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$16y - 19z = 11$

$y = \dfrac{19z+11}{16}$ $\quad \boxed{z=a}$

$y = \boxed{\dfrac{19a+11}{16}}$

$-x + 7\left(\dfrac{19a+11}{16}\right) - 8(a) = 4$

$-16x + 7(19a+11) - 128a = 64$

$-16x + 133a + 77 - 128a = 64$

$-16x + 5a = -13$

$-16x = -5a - 13$

$\boxed{x = \dfrac{5a+13}{16}}$

$$\boxed{\left(\dfrac{5a+13}{16}, \dfrac{19a+11}{16}, a\right)}$$

HAlg3-4, 8.4 ~~day 1~~ Notes – Determinants

Determinants:

- Only square matrices have determinants.
- Determinant is a scalar number, not a matrix.
- A determinant 'determines' if a matrix has an inverse. If the determinant of a matrix is zero, that matrix has no inverse (is non-invertible).

Finding determinant of a 2x2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \textit{determinant} = \det(A) = |A| = ad - bc$$

Examples/practice – Find the determinant:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 4 & 6 \\ 1 & -3 \end{bmatrix} \qquad \begin{bmatrix} -2 & -3 \\ 4 & x \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$$

$(1)(4)-(2)(3) = \boxed{-2}$ $\qquad (4)(-3)-(6)(1) = \boxed{-18}$ $\qquad \boxed{-2x+12}$ $\qquad 8-8 = \boxed{0}$ (no inverse)

Finding determinants of larger matrices:

cofactors

minors

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

cofactors (signs alternate)

Example: Find determinant of $\begin{bmatrix} 2 & 4 & 6 \\ 0 & 3 & 1 \\ 0 & 0 & -5 \end{bmatrix} = 2\begin{vmatrix} 3 & 1 \\ 0 & -5 \end{vmatrix} - 4\begin{vmatrix} 0 & 1 \\ 0 & -5 \end{vmatrix} + 6\begin{vmatrix} 0 & 3 \\ 0 & 0 \end{vmatrix}$

$= 2(-15-0) - 4(0-0) + 6(0-0)$

$= 2(-15) - 0 + 0$

$= \boxed{-30}$

Can use any row or column.

Cofactors are positive or negative according to this pattern: $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

Example: Find determinant of $\begin{bmatrix} 2 & 4 & 6 \\ 0 & 3 & 1 \\ 0 & 0 & -5 \end{bmatrix} = 2\begin{vmatrix} 3 & 1 \\ 0 & -5 \end{vmatrix} - 0\begin{vmatrix} 4 & 6 \\ 0 & -5 \end{vmatrix} + 0\begin{vmatrix} 4 & 6 \\ 0 & -5 \end{vmatrix}$

$= 2(-15+0) + 0 + 0$

$= \boxed{-30}$

Choose row or column with most zeros...easiest to compute.

Triangular Matrices and determinants:

Triangular matrices have one or both 'halves' (above or below the main diagonal) zero:

Upper triangular matrix: $\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 3 & 5 & 4 \\ 0 & 0 & 9 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ $\det = 1 \cdot 3 \cdot 9 \cdot 2 = \boxed{54}$

Lower triangular matrix: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 4 & 1 & 9 & 0 \\ 7 & 5 & 2 & 2 \end{bmatrix}$ $\det = \boxed{54}$

Diagonal matrix: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ $\det = \boxed{54}$

For all triangular form matrices, the determinant is the product of the numbers on the main diagonal.

~~HAlg3-4, 8.4 day 2 Notes – Determinants~~ part of 8.5

Alternate (diagonal method) for finding determinants (works with 3x3 only):
1. Rewrite the first two columns at the end of the matrix.
2. Starting from the upper left corner, draw a diagonal to the bottom of the original matrix, and find the product of these 3 numbers.
3. Do the same for the other 2 numbers in the top row (diagonals down and to the right.)
4. Add up these products to a single number.
5. Now starting from the lower left corner, draw a diagonal up and to the right top of the original matrix, and find the product of these 3 numbers.
6. Do the same for the other 2 numbers in the bottom row (diagonals up and to the right.)
7. Add up these products to a single number.
8. The determinant is the number from the first diagonals, minus the number from the second diagonals.

$-112 \quad -30 \quad -54 = -196$

Example: Find the determinant using the diagonal method: $\begin{bmatrix} 5 & 6 & 2 \\ -1 & -8 & 3 \\ 7 & -2 & 9 \end{bmatrix} \begin{matrix} 5 & 6 \\ -1 & -8 \\ 7 & -2 \end{matrix}$

$-360 + 126 + 4 = -230$

$\det = -230 - (-196) = \boxed{-34}$

** Calculator: 2nd x^{-1} (MATRIX), → MATH, det([A])

HAlg3-4, 8.5 Notes – Applications of Matrices and Determinants

Alternate (diagonal method) for finding determinants (works with 3x3 only):

1. Rewrite the first two columns at the end of the matrix.
2. Starting from the upper left corner, draw a diagonal to the bottom of the original matrix, and find the product of these 3 numbers.
3. Do the same for the other 2 numbers in the top row (diagonals down and to the right.)
4. Add up these products to a single number.
5. Now starting from the lower left corner, draw a diagonal up and to the right top of the original matrix, and find the product of these 3 numbers.
6. Do the same for the other 2 numbers in the bottom row (diagonals up and to the right.)
7. Add up these products to a single number.
8. The determinant is the number from the first diagonals, minus the number from the second diagonals.

$-112 \quad -30 \quad -54 = -196$

Example: Find the determinant using the diagonal method: $\begin{bmatrix} 5 & 6 & 2 \\ -1 & -8 & 3 \\ 7 & -2 & 9 \end{bmatrix} \begin{matrix} 5 & 6 \\ -1 & -8 \\ 7 & -2 \end{matrix}$

$-360 \quad 126 \quad 4 = -230$

$$det = -230 - (-196) = \boxed{-34}$$

*CALCULATOR : 2^{nd} x^{-1} (matrix) → MATH → det([A])

Area of a triangle, given coordinates of vertices:

$$Area_{triangle} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Example:

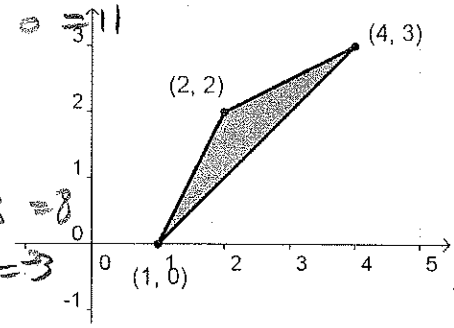
$8 \quad 3 \quad 0 = 11$

$A = \pm\frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & 2 \\ 4 & 3 \end{matrix}$

$2 \quad 0 \quad 6 = 8$

$det = 8 - 11 = -3$

$A = \pm\frac{1}{2}(-3) = \pm\frac{3}{2} = \boxed{\frac{3}{2}}$



Test 3 points for collinearity:

What happens to the area, if the 3 points of a triangle are collinear?

Example: Are the pts (0, 1), (4, 4), and (8,7) collinear?

$$3 \text{ points collinear if}: \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

$\begin{vmatrix} 0 & 1 & 1 \\ 4 & 4 & 1 \\ 8 & 7 & 1 \end{vmatrix} = 0$ yes, collinear

Find equation of a line given 2 points on the line:

Find equation of line passing through (10, 7), (-2, -7)

$$\text{For equation of line, solve}: \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

$-14 - 7x + 10y$

$$\begin{vmatrix} x & y & 1 \\ 10 & 7 & 1 \\ -2 & -7 & 1 \end{vmatrix} \begin{matrix} x & y \\ 10 & 7 \\ -2 & -7 \end{matrix}$$

$7x - 2y - 70$

$7x - 2y - 70 = -14 - 7x + 10y$

$+7x - 10y + 70 \qquad +70 + 7x - 10y$

$14x - 12y = 56$

$\boxed{7x - 6y = 28}$

Cramer's Rule – another way to solve system of linear equations:

$$x_1 = \frac{|A_1|}{|A|}, \quad x_2 = \frac{|A_2|}{|A|}, \quad x_3 = \frac{|A_3|}{|A|}$$

where A_n = matrix with nth column replaced with constants matrix column.

Example: Solve the system using Cramer's Rule: $\begin{cases} 4x - 2y = 10 \\ 3x - 5y = 11 \end{cases}$ $\quad \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & -2 \\ 3 & -5 \end{vmatrix} = -14 \quad |A_1| = \begin{vmatrix} 10 & -2 \\ 11 & -5 \end{vmatrix} = -28 \quad |A_2| = \begin{vmatrix} 4 & 10 \\ 3 & 11 \end{vmatrix} = 14$

$x = \frac{|A_1|}{|A|} = \frac{-28}{-14} = \boxed{2} \qquad y = \frac{|A_2|}{|A|} = \frac{14}{-14} = -1 \qquad \boxed{(2, -1)}$

Example: Solve the system using Cramer's Rule: $\begin{cases} -x + 2y - 3z = 1 \\ 2x \qquad\quad + z = 0 \\ 3x - 4y + 4z = 2 \end{cases}$ $\quad \begin{bmatrix} -1 & 2 & -3 \\ 2 & 0 & 1 \\ 3 & -4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 2 & -3 \\ 2 & 0 & 1 \\ 3 & -4 & 4 \end{vmatrix} = 10$

$|A_1| = \begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & 1 \\ 2 & -4 & 4 \end{vmatrix} = 8$

$|A_2| = \begin{vmatrix} -1 & 1 & -3 \\ 2 & 0 & 1 \\ 3 & 2 & 4 \end{vmatrix} = -15$

$|A_3| = \begin{vmatrix} -1 & 2 & 1 \\ 2 & 0 & 0 \\ 3 & 4 & 2 \end{vmatrix} = -16$

$x = \frac{|A_1|}{|A|} = \frac{8}{10} = \frac{4}{5}$

$y = \frac{|A_2|}{|A|} = \frac{-15}{10} = \frac{-3}{2}$

$z = \frac{|A_3|}{|A|} = \frac{-16}{10} = \frac{-8}{5}$

$\boxed{\left(\frac{4}{5}, \frac{-3}{2}, \frac{-8}{5}\right)}$

Honors Algebra 3-4 Using Matrices to Encode and Decode Messages

Name Key Period ________

Example: Let's encode the message MEET ME MONDAY using the encoding matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}$

1) Break the message into symbol groups each containing 3 letters:

[M E E] [T _ M] [E _ M] [O N D] [A Y _]

2) First, change the message into numbers by replacing each letter with a number as follows: (0 is assigned to a space).

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	15	16	17	18	19	20	21	22	23	24	25	26

[13 5 5] [20 0 13] [5 0 13] [15 14 4] [1 25 0]

3) Multiply each symbol group by the encoding matrix to produce an encoded group: (use C for group) 1x3

[13 -26 21] [33 -53 -12] [18 -23 -42] [5 -20 56] [-24 23 77]

4) Recombine into a single string of numbers. This is the coded message:

13 -26 21 33 -53 -12 18 -23 -42 5 -20 56 -24 23 77

To decode:

1) Find the inverse of the encoding matrix and store it in B to make a decoding matrix.

2) Break the encoded message into symbol groups of 3 numbers:

[13 -26 21] [33 -53 -12] [18 -23 -42] [5 -20 56] [-24 23 77]

3) Multiply each symbol group by the decoding matrix:

[13 5 5] [20 0 13] [5 0 13] [15 14 4] [1 25 0]

4) Use the letter-number map from above to convert the numbers back into letters:

[M E E] [T _ M] [E _ M] [O N D] [A Y _]

5) Recombine the original message.

MEET ME MONDAY

The following message was encoded using the same A matrix. Use the decoding matrix B to decode this message:

5	-5	-33	-1	1	-2	9	-21	24	26	-40	-27
_	M	R	_	F	E	L	L	I	N	G	S
1	-1	-23	22	-43	34	13	-27	12	17	-17	-69
_	S	T	U	D	E	N	T	S	_	A	R
23	-28	-62	8	-13	-3	31	-56	26	-3	-12	84
E	_	R	E	A	D	Y	_	F	O	R	_
11	-30	49	26	-39	-31						
S	U	M	M	E	R						

Try encoding your own message (a minimum of 30 numbers) using matrix [A]. Give the encoded number string along with the translation below: