HAlg3-4, 9.1 Notes – Sequences and Series

Sequence – a list of numbers in a specific order.

$$\overset{1^{st}\text{ term}}{1},\ \overset{2^{nd}\text{ term}}{3},\ \overset{3^{rd}\text{ term}}{5},\ 7,\ 9,\ 11$$

$$a_1, a_2, a_3, a_4, a_5, a_6$$

Sequences are like functions where 'n' is the input and the term is the output:

$a_1 = 1$

$a_2 = 3$

$a_3 = 5$

Infinite sequence: infinite number of terms (goes on forever) $1, 3, 5, 7, 9, \ldots$ ← continues

Finite sequence: finite number of terms (only n terms) $1, 3, 5, 7$ ← only 4 terms

Some sequences have a 'rule' or 'expression' or 'formula' for finding a term given n:

$a_n = 2n - 1$

$a_1 = 2(1) - 1 = 1$

$a_2 = 2(2) - 1 = 3$

$a_3 = 2(3) - 1 = 5$

$1, 3, 5, \ldots$

$a_n = \dfrac{(-1)^n}{2n-1}$

$a_1 = \dfrac{(-1)^1}{2(1)-1} = \dfrac{-1}{2-1} = -1$

$a_2 = \dfrac{(-1)^2}{2(2)-1} = \dfrac{1}{4-1} = \dfrac{1}{3}$

$a_3 = \dfrac{(-1)^3}{2(3)-1} = \dfrac{-1}{6-1} = \dfrac{-1}{5}$

$-1, \frac{1}{3}, \frac{-1}{5}, \ldots$

Some sequences have a rule for finding a term from previous terms (instead of from n)
These are called recursive sequences:

Example: The Fibonacci sequence... 1, 1, 2, 3, 5, 8, ...

What is the rule? each term is sum of 2 previous terms. $a_k = a_{k-1} + a_{k-2}$ (for $k \geq 2$)

Factorials For positive integer n,

$$n! = \overset{n\cdot(n-1)\cdot\ldots\cdot 3\cdot 2\cdot 1}{\cancel{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot\ldots\cdot(n-1)\cdot n}}$$

special case: $0! = 1$

Examples: $4! = 4\cdot 3\cdot 2\cdot 1 = 24$

$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 720$

Simplifying factorials: $\dfrac{8!}{2!\cdot 6!} = \dfrac{8\cdot 7\cdot\cancel{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{2\cdot 1\cdot\cancel{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}} = \dfrac{\overset{4}{\cancel{8}}\cdot 7}{\cancel{2}\cdot 1} = 7\cdot 4 = 28$

$\dfrac{(n+1)!}{n!} = \dfrac{(n+1)\cdot\cancel{n(n-1)\cdots 2\cdot 1}}{1\cdot\cancel{n(n-1)\cdots 2\cdot 1}} = n+1$

On calculator: to find 6! 6, MATH, right arrow to PRB menu, down arrow to !, enter twice

Finding terms of a sequence

Given a formula for nth term – just plug in n:

Example: Write the first 5 terms if $a_n = 5n - 2$

$a_1 = 5(1) - 2 = 3$
$a_2 = 5(2) - 2 = 8$
$a_3 = 5(3) - 2 = 13$
$a_4 = 5(4) - 2 = 18$
$a_5 = 5(5) - 2 = 23$

$3, 8, 13, 18, 23, \ldots$

Given a rule for recursive sequence, write starting term(s), use rule to find next terms: $a_1 = 5$

Example: Write the first 5 terms of recursive sequence: $a_1 = 5, \quad a_{k+1} = 3(a_k + 2)$

$a_2 = 3(5+2) = 21$
$a_3 = 3(21+2) = 69$
$a_4 = 3(69+2) = 213$
$a_5 = 3(213+2) = 645$

$5, 21, 69, 213, 645, \ldots$

Finding a formula, given the sequence

Sometimes easy to see...can help writing a line of 'n' above matching terms:

Examples: Write an expression for the most apparent nth term of the sequence:

$n: \; 1 \quad 2 \quad 3 \quad 4 \quad 5$

$\text{term}: \; 1, \; \frac{1}{4}, \; \frac{1}{9}, \; \frac{1}{16}, \; \frac{1}{25}, \; \ldots \qquad a_n = \frac{1}{n^2}$

n^2

$1 \quad 2 \quad 3 \quad 4$

$\frac{1}{2}, \; \frac{1}{6}, \; \frac{1}{24}, \; \frac{1}{120}, \; \ldots \qquad a_n = \frac{1}{(n+1)!}$

Sometimes difficult to see a pattern, so we look for matches in a table of patterns:

$n = 1, \; 2, \; 3, \; 4, \; 5, \; 6, \ldots$

$n^2 = 1, \; 4, \; 9, \; 16, \; 25, \; 36, \ldots$

$(n+1)^2 = 4, \; 9, \; 16, \; 25, \; 36, \; 49, \ldots$

$(n-1)^2 = 0, \; 1, \; 4, \; 9, \; 16, \; 25, \ldots$

$n^3 = 1, \; 8, \; 27, \; 64, \; 125, \; 216, \ldots$

$(n+1)^3 = 8, \; 27, \; 64, \; 125, \; 216, \; 343, \ldots$

$2n = 2, \; 4, \; 6, \; 8, \; 10, \; 12, \ldots$

$2^n = 2, \; 4, \; 8, \; 16, \; 32, \; 64, \ldots$

$2^{n-1} = 1, \; 2, \; 4, \; 8, \; 16, \; 32, \ldots$

$2^{n+1} = 4, \; 8, \; 16, \; 32, \; 64, \; 128, \ldots$

$3n = 3, \; 6, \; 9, \; 12, \; 15, \; 18, \ldots$

$3^n = 3, \; 9, \; 27, \; 81, \; 243, \; 729, \ldots$

$3^{n-1} = 1, \; 3, \; 9, \; 27, \; 81, \; 243, \ldots$

$3^{n+1} = 9, \; 27, \; 81, \; 243, \; 729, \; 2187, \ldots$

$n! = 1, \; 2, \; 6, \; 24, \; 120, \; 720, \ldots$

$(n-1)! = 1, \; 1, \; 2, \; 6, \; 24, \; 120, \ldots$

$(n+1)! = 2, \; 6, \; 24, \; 120, \; 720, \; 5040, \ldots$

More examples:

$1+\frac{1}{3}, \; 1+\frac{7}{9}, \; 1+\frac{25}{27}, \; 1+\frac{79}{81}, \; 1+\frac{241}{243}, \; \ldots$

$a_n = 1 + \frac{3^n - 2}{3^n}$

$n^2 \quad 1 \quad 4 \quad 9 \quad 16$

$n \quad 1 \quad 2 \quad 3 \quad 4$

$0, \; 3, \; 8, \; 15, \; \ldots$

$a_n = n^2 - 1$

Series = the sum of the terms in a sequence

Sequence: 1, 3, 5, 7

Series: 1 + 3 + 5 + 7 = 16

Summation (Sigma) Notation

finite series $a_1 + a_2 + a_3 + a_4 + ... + a_i + ... + a_n = \sum_{i=1}^{n} a_i$

i = 'index of summation'

n = 'upper limit of summation'

infinite series $a_1 + a_2 + a_3 + a_4 + ... + a_i + ... = \sum_{i=1}^{\infty} a_i$

1 = 'lower limit of sumamtion'

(doesn't have to be 1 sometimes 0 or other numbers)

Properties:

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n}(a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

Examples: Find $\sum_{i=1}^{5} 4i-3 = 4(1)-3 + 4(2)-3 + 4(3)-3 + 4(4)-3 + 4(5)-3$

$= 1 + 5 + 9 + 13 + 17$

$= 45$

$n = 1 \quad 2 \quad 3 \quad 4$

Use Sigma notation to write the sum: $\cancel{1} - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} .... - \frac{1}{128} = \boxed{\sum_{i=1}^{8} \frac{(-1)^{i+1}}{2^{i-1}}}$

Find rule for a_i first

$a_i = \frac{(-1)^{i+1}}{2^{i-1}}$ ← makes sign alternate

2^{n-1} pattern on bottom

how many terms?

$2^{i-1} = 128 = 2^7$ $\quad i-1=7$

$i=8$

A deposit of \$100 is made each month in an account that earns 12% interest compounded monthly. The balance in the account after n months is:

$A_n = 100(101)\left[(1.01)^n - 1\right], \quad n = 1,2,3...$

(a) compute the first six terms of this sequence.

(b) Find the balance in this account after 5 years by computing the 60th term of the sequence.

(a) $A_1 = 100(101)[(1.01)^1 - 1] = 101,00$

$A_2 = 100(101)[(1.01)^2 - 1] = 203,01$

$A_3 = 100(101)[(1.01)^3 - 1] = 306,04$

$A_4 = 100(101)[(1.01)^4 - 1] = 410,10$

$A_5 = 100(101)[(1.01)^5 - 1] = 515,20$

(b) 5 years $\frac{12 \text{ mo.}}{\text{yr}}$ = 60 month.

$A_{60} = 100(101)[(1.01)^{60} - 1] = 8248.64$

HAlg3-4, 9.2 Notes – Arithmetic Sequences and Partial Sums

Consider this sequence:

1st term → a_1

$$1, \underset{+3}{\curvearrowright} 4, \underset{+3}{\curvearrowright} 7, \underset{+3}{\curvearrowright} 10, \underset{+3}{\curvearrowright} 13, \underset{+3}{\curvearrowright} 16, \ldots$$

↑ d = common difference

This is an **arithmetic sequence**. A sequence is arithmetic if the differences between consecutive terms is a constant, which is called the **common difference**.

Examples: Determine if the sequences are arithmetic and find the common differences.

#1. -12, -8, -4, 0, 4,....

+4 +4 +4 4

$d=4$

#2. 9, 6, 3, 0, -3, ...

−3 −3 −3 −3

$d=-3$

#3. Find the first 5 terms and determine if the sequence is arithmetic: $a_n = (2^n)n$

$a_1 = (2^1)1 = 2$ ↓ +6

$a_2 = (2^2)2 = 8$ ↓ +16

$a_3 = (2^3)3 = 24$

$a_4 = (2^4)4 = 64$

$a_5 = (2^5)5 = 160$

<u>not</u> arithmetic

Formulas for nth term of arithmetic sequences: Three formulas...

1) $a_n = a_1 + (n-1)d$

↖ $n-1$ so $a_n = a_1$ when $n=1$

2) $a_n = dn + c, \quad where\ c = a_1 - d$ c is the 'zeroth' term (textbook's formula)

$a_n = a_1 + (n-1)d$

$= a_1 + dn - d$

$= dn + (a_1 - d)$ ← 'zeroth' term

3) $a_{n+1} = a_n + d$ (recursive formula)

Example: Find a formula for the nth term of the arithmetic sequence whose common difference is 3 and whose first term is 2.

$d = 3$

$a_1 = 2$

$a_n = a_1 + (n-1)d$

$\boxed{a_n = 2 + (n-1)3}$

or

$a_n = 2 + 3n - 3$

$\boxed{a_n = -1 + 3n}$

Example: The 4th term of an arithmetic sequence is 20, and the 13th term is 65. Find a formula for the nth term.

$a_4\ a_5\ a_6\ a_7\ a_8\ a_9\ a_{10}\ a_{11}\ a_{12}\ a_{13}$

$d\ d\ d\ d\ d\ d\ d\ d\ d$

$a_{13} = 65$

$- a_4 = 20$

45 difference $= (13-4)d = 9d$

$9d = 45$

$d = 5$

$a_n = a_1 + (n-1)d$

$20 = a_1 + (4-1)5$

$\cancel{20} = a_1 + \cancel{20} - 5 \quad a_1 = 5$

$\boxed{a_n = 5 + (n-1)5}$

or

$a_n = 5 + 5n - 5$

$\boxed{a_n = 5n}$

Example: Find the 7th term of the arithmetic sequence whose first two terms are 2 and 9.

$d = 9 - 2 = 7$

$a_1 = 2$

$a_n = a_1 + (n-1)d$

$a_n = 2 + (n-1)7$

$a_7 = 2 + (7-1)7$

$a_7 = 2 + 6 \cdot 7$

$a_7 = 2 + 42 = \boxed{44}$

Example: The first two terms are given, find the missing term. $a_1 = 3, \quad a_2 = 9, \quad a_9 = ?$

$d = 9 - 3 = 6$

$a_n = 3 + (n-1)6$

$a_9 = 3 + (9-1)6$

$a_9 = 3 + 8 \cdot 6 = \boxed{51}$

Practice: Find formulas for the arithmetic sequences:

#1. $a_1 = 15, \quad d = 4$

$a_n = a_1 + (n-1)d$

$\boxed{a_n = 15 + (n-1)4}$

or

$a_n = 15 + 4n - 4$

$\boxed{a_n = 11 + 4n}$

#2. $-6, -2, 2, 6$

a_1

$+4\ +4\ +4 = d$

$\boxed{a_n = -6 + (n-1)4}$

or

$a_n = -6 + 4n - 4$

$\boxed{a_n = -10 + 4n}$

#3. $a_{\cancel{4}} = \cancel{20}, \quad a_{\cancel{13}} = \cancel{65}$

$3 \quad 94 \quad 6 \quad 85$

-9 for $6-3$ terms

$-9 = 3d \quad d = -3$

$a_n = a_1 + (n-1)(-3)$

$94 = a_1 + (3-1)(-3)$

$94 = a_1 - 6$

$a_1 = 100$

$\boxed{a_n = 100 - (n-1)3}$

or

$a_n = 100 - 3n + 3 = \boxed{103 - 3n}$

Sum of a finite arithmetic series (partial sum of an infinite arithmetic series)

$S = 1 + 3 + 5 + 7 + 9 + 11 \quad = 36$

$S = 11 + 9 + 7 + 5 + 3 + 1$

$2S = 12 + 12 + 12 + 12 + 12 + 12$

$2S = 6(12)$

$S = \frac{1}{2}6(12) = \frac{1}{2}n(a_1 + a_n)$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$a_1 = -6$

Example: Find the partial sum: -6, -2, 2, 6, ... n=50

$\underset{+4}{\rightarrow}\ \underset{+4}{\rightarrow}\ \underset{+4}{\rightarrow}$ $d=4$

$a_n = a_1 + (n-1)d$

$a_n = -6 + (n-1)4$

$a_{50} = -6 + (50-1)4 = 190$

$S_{50} = \frac{n}{2}(a_1 + a_n)$

$S_{50} = \frac{50}{2}(-6 + 190) = \boxed{4600}$

Try these:
#4. What is the sum of integers from 1 to 100.

$a_1 = 1$

$a_{100} = 100$

$S_{100} = \frac{100}{2}(1 + 100) = \boxed{5050}$

$S_n = \frac{n}{2}(a_1 + a_n)$

#5. Find the sum: $\sum_{n=1}^{100} \frac{8-3n}{16}$

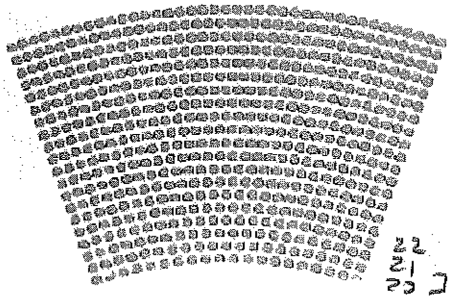
$a_1 = \frac{8-3}{16} = \frac{5}{16}$

$a_{100} = \frac{8-300}{16} = \frac{-292}{16}$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{100} = \frac{100}{2}\left(\frac{5}{16} - \frac{292}{16}\right) = \boxed{\frac{100}{2} \cdot \frac{-287}{16}}$?

Example: An auditorium has 40 rows of seats. There are 20 seats in the 1st row, 21 in the 2nd, 22 in the 3rd, etc. How many total seats are there?



$a_{40} = 59$ seats

22
21
20 $\}$ $d=1$

$a_1 = 20$

$a_n = a_1 + (n-1)d$

$a_n = 20 + (n-1)1$

$a_n = 20 + n - 1$

$a_n = 19 + n$

$a_{40} = 19 + 40 = 59$

$S = \frac{40}{2}(20 + 59) = \boxed{1580 \text{ seats}}$

Example: Consider a job offer with a starting salary of $36,800 and an annual raise of $1,750.
a) Determine the salary during the 6th year of employment.
b) Determine the total compensation from the company through 6 full years of employment.

$a_1 = 36800$

$d = 1750$

a) $a_n = a_1 + (n-1)d$

$a_n = 36800 + (n-1)1750$

$a_6 = 36800 + (6-1)1750 = \boxed{\$45,550}$

b) $S_6 = \frac{6}{2}(36800 + 45550) = \boxed{\$247,050}$

HAlg3-4, 9.3 Notes – Geometric Sequences and Series

Consider this sequence:

2, 4, 8, 16, 32, ...

$\times 2\ \times 2\ \times 2\ \times 2 \leftarrow r = 2$ common ratio

$a_1,\quad a_1 r,\quad a_1 r^2,\quad a_1 r^3,\quad a_1 r^4, \ldots$

This is a **geometric sequence**. A sequence is geometric if the ratios of consecutive terms is a constant, which is called the **common ratio**. (r)

Examples: Determine if the sequences are geometric and find the common ratio.

#1. 12, 36, 108, 324, ...

$\frac{36}{12} = 3\quad \times 3\quad \times 3\quad \times 3$

$\frac{108}{36} =$ yes, $r = 3$

#2. 1, 4, 9, 16, ...

$\times 4\quad \times \frac{9}{4}\quad \times \frac{16}{9}$

no

Formula for nth term of geometric sequences: $\boxed{a_n = a_1 r^{n-1}}$

Example: Write the 1st 5 terms of the geometric sequence whose 1st term is 3 with common ratio of 2.

$a_1 = 3$

$r = 2$

$a_n = 3(2)^{n-1}$

$a_1 = 3(2)^{1-1} = 3$

$a_2 = 3(2)^{2-1} = 6$

$a_3 = 3(2)^{3-1} = 12$

$a_4 = 3(2)^{4-1} = 24$

$a_5 = 3(2)^{5-1} = 48$

$\boxed{3, 6, 12, 24, 48, \ldots}$

Example: Find the 15th term of the geometric sequence whose 1st term is 20 and whose common ratio is 1.05.

$a_n = 20(1.05)^{n-1}$

$a_{15} = 20(1.05)^{15-1} = 39.5986\ldots$

Try it

~~Example~~: Find a formula~~s~~ for the nth term of the geometric sequence: 5, 15, 45,...
What is the 9th term of the sequence?

$\times 3\quad \times 3\quad r = 3$

$a_1 = 5$

$a_n = a_1 r^{n-1}$

$a_n = 5(3)^{n-1}$

$a_9 = 5(3)^{9-1} = 5 \cdot 3^8 = 32805$

Example: $a_4 = 125, \quad a_{10} = \frac{125}{64}$ Find the 14th term (assume terms of sequence are positive)

$\frac{a_{10}}{a_4} = \frac{a_1 r^{10-1}}{a_1 r^{4-1}} = \frac{a_1 r^9}{a_1 r^3} = r^6$

$r^6 = \frac{125/64}{125} = \frac{125}{64} \cdot \frac{1}{125} = \frac{1}{64}$

$r = \left(\frac{1}{64}\right)^{1/6} = \frac{1}{64^{1/6}} = \frac{1}{2}$

$a_n = a_1 r^{n-1}$

$a_4 = a_1 r^{4-1}$

$125 = a_1 \left(\frac{1}{2}\right)^3 = a_1 \left(\frac{1}{8}\right)$

$\therefore a_1 = 1000$

$a_n = 1000\left(\frac{1}{2}\right)^{n-1}$

$a_{14} = 1000\left(\frac{1}{2}\right)^{14-1}$

$= 1000\left(\frac{1}{2^{13}}\right)$

$= \frac{1000}{8192} = \boxed{\frac{125}{1024}}$

Sum of a finite geometric sequence

$$S_n = a_1 + a_1 r + a_1 r^2 + a_1 r^3 ..... + a_1 r^{n-1}$$

$$-rS_n = \quad a_1 r + a_1 r^2 + a_1 r^3 ..... + a_1 r^{n-1} + a_1 r^n$$

$$S_n - rS_n = a_1 - a_1 r^n$$

$$S_n(1-r) = a_1(1-r^n)$$

$$S_n = a_1 \frac{(1-r^n)}{(1-r)}$$

$$\hookrightarrow \boxed{S_n = a_1\left(\frac{1-r^n}{1-r}\right)}$$

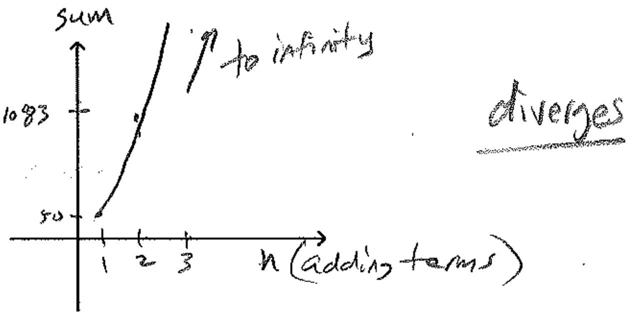
Example: Find the sum: $\sum_{n=1}^{12} 4(0.3)^n = a_1\left(\frac{1-r^n}{1-r}\right) = 1{,}2\left(\frac{1-0{,}3^{12}}{1-0{,}3}\right) \approx 1{,}714...$

$a_1 = 4(0{,}3)^1 = 1{,}2$, $r = 0{,}3$, $n = 12$

Sum of an infinite geometric sequence

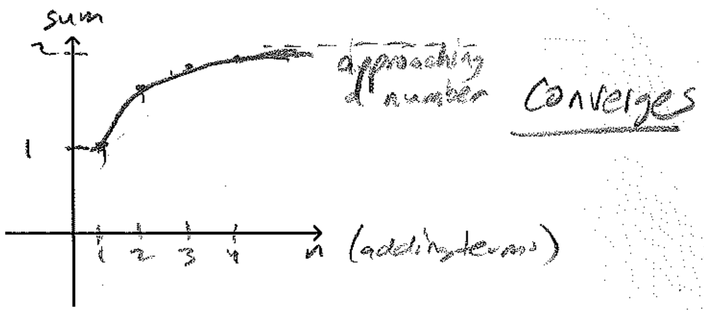
$$\sum_{n=1}^{\infty} 50(1.4)^{n-1} = 50 + 1033 + 29881 + ...$$

$|r| \geq 1$



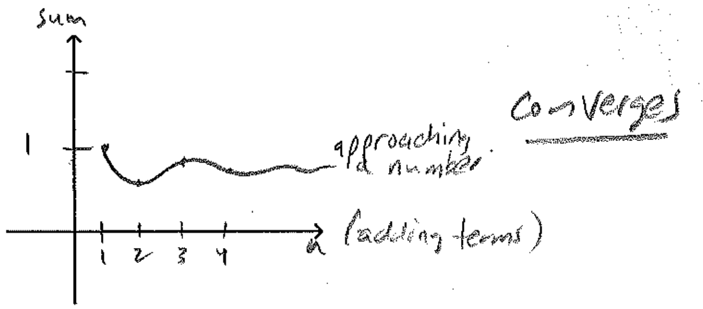
$$\sum_{n=1}^{\infty} 1\left(\frac{1}{2}\right)^{n-1} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$$

$|r| < 1$



$$\sum_{n=1}^{\infty} 1\left(-\frac{1}{2}\right)^{n-1} = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...$$

$|r| < 1$



For $a_n = a_1 r^{n-1}$, **converges if** $|r| < 1$

diverges if $|r| \geq 1$

If geometric series converges, it converges to:

$$S = \frac{a_1}{1-r}$$

(2nd example above
$r = \frac{1}{2}$; $a_1 = 1$)

$$S = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

check $|r|<1$, converges

Examples: Find the sum: $\sum_{n=1}^{\infty} 40(0.6)^{n-1} = \frac{a_1}{1-r} = \frac{40}{1-0.6} = \boxed{100}$

$a_1 = 40(0.6)^0$

$a_1 = 40$

skip at end if time

$A = P(1+\frac{r}{n})^{nt}$

A deposit of $50 is made on the first day of each month in a savings account that pays 6% compounded monthly. What is the balance of this investment at the end of 2 years? (24 months)

1st deposit: $A_{24} = 50(1+\frac{.06}{12})^{24} = 50(1.005)^{24}$ ← in for 24 months gaining interest

2nd deposit $A_{23} = 50(1+\frac{.06}{24})^{23} = 50(1.005)^{23}$

⋮

last deposit $A_1 = 50(1+\frac{.06}{24})^1 = 50(1.005)^1$

– Finite geometric series (reverse order)

$A_1 + A_2 + A_3 \ldots A_{24}$

– $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ $a_1 = A_1 = 50(1.005)$

$r = 1.005$

$S_{24} = 50(1.005)\left(\frac{1-1.005^{24}}{1-1.005}\right) = \boxed{\$1277.96}$

Find the sum: Finite $\sum_{i=1}^{10} 5\left(-\frac{1}{3}\right)^{i-1} = a_1\left(\frac{1-r^n}{1-r}\right)$

$a_1 = 5$

$r = -\frac{1}{3}$

$n = 10$

$S_{10} = 5\left(\frac{1-(-\frac{1}{3})^{10}}{1-(-\frac{1}{3})}\right) = \boxed{3.75}$

Find the sum: Finite $\sum_{k=0}^{10} 10\left(-\frac{1}{2}\right)^{k} = a_1\left(\frac{1-r^n}{1-r}\right)$

$a_1 = 10$

$r = -\frac{1}{2}$

$n = 11$ * (careful)

$S = 10\left(\frac{1-(-\frac{1}{2})^{11}}{1-(-\frac{1}{2})}\right) = \boxed{6.66992}$

Use summation notation to express the sum: 7+14+~~21~~+28+....+896 $\times 2$ $\times 2$

$r = 2$

$a_1 = 7$

$a_n = a_1 r^{n-1}$

$896 = 7(2)^{n-1}$

$128 = 2^{n-1}$

$\log_2 128 = \log_2 2^{n-1}$

$\log_2 128 = n-1$

$n = 1 + \log_2 128$

$n = 1 + \frac{\ln 128}{\ln 2}$

$n = 8$

$\boxed{\sum_{n=1}^{8} 7(2)^{n-1}}$

or

$128 = 2^{n-1}$

$2^7 = 2^{n-1}$

$7 = n-1$

$n = 8$

Find the sum of the infinite geometric series: $\frac{2}{1} - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \ldots$

$r = \frac{-4/3}{2} = -\frac{4}{3} \div \frac{2}{1} = -\frac{4}{3} \times \frac{1}{2} = \frac{-4}{6} = \frac{-2}{3}$

$|r|<1$ converges, so $S = \frac{a_1}{1-r} = \frac{2}{1-(-\frac{2}{3})} = \frac{2}{1+\frac{2}{3}} = \frac{2}{\frac{3}{3}+\frac{2}{3}} = \frac{2}{\frac{5}{3}} = 2 \times \frac{3}{5} = \boxed{\frac{6}{5}}$

#91 (homework) A company buys a machine for $155,000 and it depreciates at a rate of 30% per year (at the end of each year, the value is 70% of what it was at the start of the year). Find the depreciated value of the machine after 5 full years.

multiplier for change in a year is 0.7 so $r = 0.7$

$a_1, a_2, a_3, a_4, a_5, a_6$ $a_n = a_1 r^{n-1}$

$a_1 = 155{,}000$

after 5 yrs → $a_6 = 155000(.7)^{6-1} = \boxed{\$26{,}050.85}$ $n = 6$ (5 yrs of depreciation)