HAlg3-4, 9.5 Notes – The Binomial Theorem

Binomial = polynomial with two terms, e.g. x+2, 2y-3, x-2y, etc.

Binomial raised to a power: $(x+y)^n$

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2+2xy+y^2$

$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$

$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$

$(x+y)^5 = 1x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+1y^5$

Things to note:

- Exponent of first term (x) starts at n and decreases by 1 each term.
- Exponent of second term (y) starts at 0 and increases by 1 each term.
- Coefficients are symmetrical...called **binomial coefficients.**

The Binomial Theorem:

$$(x+y)^n = {}_nC_0x^n + {}_nC_1x^{n-1}y + {}_nC_2x^{n-2}y^2 + \ldots + {}_nC_rx^{n-r}y^r + \ldots + {}_nC_nx^n$$

← don't forget

where ${}_nC_r = \dfrac{n!}{(n-r)!r!} = \binom{n}{r}$

Example: $(x+y)^4 = {}_4C_0\,x^4 + {}_4C_1\,x^3y + {}_4C_2\,x^2y^2 + {}_4C_3\,xy^3 + {}_4C_4\,y^4$

$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$

$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

${}_4C_0 = \dfrac{4!}{(4-0)!0!} = \dfrac{4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1\cdot1} = \dfrac{1}{1}$

${}_4C_1 = \dfrac{4!}{(4-1)!1!} = \dfrac{4\cdot3\cdot2\cdot1}{3\cdot2\cdot1\cdot1} = 4$

${}_4C_2 = \dfrac{4!}{(4-2)!2!} = \dfrac{4\cdot3\cdot2\cdot1}{2\cdot1\cdot2\cdot1} = \dfrac{12}{2} = 6$

${}_4C_3 = \dfrac{4!}{(4-3)!3!} = \dfrac{4\cdot3\cdot2\cdot1}{1\cdot3\cdot2\cdot1} = 4$

${}_4C_4 = \dfrac{4!}{0!4!} = 1$

* Calculator

Pascal's Triangle – an easier way to compute binomial coefficients

${}_nC_r$ = C (row, column)

${}_6C_4 = \boxed{15}$

0 row → 1 ← starts with a triangle of ones

1 row → 1 1 ← ones on both ends of each row

1 2 1

1 3 3 1 — each number inside is sum of two above it

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 $\boxed{15}$ 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

* start row and column counting with 0

Using the binomial theorem to find expansions

$(x+1)^3 = {}_3C_0\, x^3 + {}_3C_1\, x^2(1) + {}_3C_2\, x(1)^2 + {}_3C_3\, (1)^3$

$1x^3 + 3x^2 + 3x + 1\cdot 1$

$x^3 + 3x^2 + 3x + 1$

$(2x-3)^4 = {}_4C_0\,(2x)^4 + {}_4C_1\,(2x)^3(-3) + {}_4C_2\,(2x)^2(-3)^2 + {}_4C_3\,(2x)(-3)^3 + {}_4C_4\,(-3)^4$

$1(2x)^4 + 4(2x)^3(-3) + 6(2x)^2(-3)^2 + 4(2x)(-3)^3 + 1(-3)^4$

$16x^4 - 96x^3 + 216x^2 - 216x + 81$

Find the 6th term of $(a+2b)^8$

	1	2	3	4	5	6th
a	a^8	7	6	5	4	a^3
2b	$(2b)^0$	1	2	3	4	$(2b)^5$
r	0	1	2	3	4	5

${}_8C_5\; a^3(2b)^5$

$56\, a^3\, 32b^5$

$\boxed{1792a^3b^5}$

$(4-i)^5 = {}_5C_0\,(4)^5 + {}_5C_1\,(4)^4(-i) + {}_5C_2\,(4)^3(-i)^2 + {}_5C_3\,(4)^2(-i)^3 + {}_5C_4\,(4)(-i)^4 + {}_5C_5\,(-i)^5$

$1(1024) + 5\cdot 256\cdot(-i) + 10(64)(i^2) + 10(16)(-i^3) + 5\cdot 4(i^4) + 1(-i^5)$

$i^2 = -1 \qquad -i\,i^2 = +i \qquad i^2 i^2 = -1\cdot -1 = 1 \qquad -i\,i^4 = -i$

$1024 - 1280i - 640 + 160i + 20 - i$

$\boxed{404 - 1121i}$

HAlg3-4, 9.6 Notes – Combinatorics (counting problems)

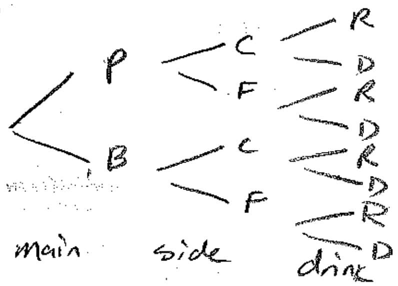
Simple counting problems

List all possibilities

A computer generates integers randomly between 1 and 12. In how many ways can the ~~a~~ number be an even integer?   1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 $\boxed{6}$

Pairings – tree diagram

At a snack bar a combo meal consists of: a main item of pizza (P) or a burger (B), a side item of chips (C) or fries (F), and a drink which is regular (R) or diet (D). How many different combo meals can you get?



← each is a meal
ex. P, C, D

$\boxed{8 \text{ meals}}$

main   side   drink

possibilities → $\underline{2} \times \underline{2} \times \underline{2} = \boxed{8 \text{ meals}}$

Fundamental Counting Principle

"Let E_1 and E_2 be two events. The first event E_1 can occur in m_1 different ways. After E_1 has occurred, E_2 can occur in m_2 different ways. The number of ways that the two events can occur is: m_1m_2."

Practice: Students must select 1 of 2 math courses, 1 of 3 science courses, and 1 of 5 social studies courses. How many different class groupings are possible?

$\underbrace{2}_{\text{math}} \cdot \underbrace{3}_{\text{science}} \cdot \underbrace{5}_{\text{SS}} = \boxed{30}$

Example: 8 pieces of paper on which ~~is~~ are written the numbers 1 through 8 are put in a box. One piece of paper is drawn out and the number recorded. The paper is replaced in the box. Another piece of paper is drawn out and recorded, replaced and a paper is drawn out and recorded a third time, forming a 3 digit number. How many different 3 digit numbers are possible?

$\underline{8} \cdot \underline{8} \cdot \underline{8} = \boxed{512}$

Example: Same as above, except that once drawn out, a piece of paper is not replaced in the box.

$\underline{8} \cdot \underline{7} \cdot \underline{6} = \boxed{336}$

Practice: How many different 7-digit telephone numbers are possible if the 1st digit cannot be a zero or a one?

$\underline{8}\ \underline{10}\ \underline{10} - \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = \boxed{8{,}000{,}000}$

8 horses run in a race. In how many different ways can these horses come in 1st, 2nd and 3rd place?

$$\underset{1^{st}}{\underline{8}} \cdot \underset{2^{nd}}{\underline{7}} \cdot \underset{3^{rd}}{\underline{6}} = \boxed{336}$$

100 students are in 8th grade at a school. In how many ways can a student body president, vice president, and secretary be chosen from these 100 students?

$$\underset{P}{\underline{100}} \cdot \underset{VP}{\underline{99}} \cdot \underset{Sec}{\underline{98}} = 970{,}200$$

Permutation – A permutation of n different elements is an ordering of elements with one element first, another second, etc. ORDER MATTERS.

Can compute permutations using 'boxes' (as above) or using the permutation formula:

Number of permutations of n elements taken r at a time is:

$$_nP_r = \frac{n!}{(n-r)!}$$

Horses example: Select 3 horses from 8, order matters

$$_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 8 \cdot 7 \cdot 6 = \boxed{336}$$

What if some elements are identical? Example: In how many distinguishable ways can the letters BANANA be written?

not: $\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 720$

each is counted

$B A_1 N_1 A_2 N_2 A_3$

$B A_3 N_2 A_1 N_1 A_2$

$B A_2 N_1 A_1 N_2 A_3$

but not distinguishable.

Number of distinguishable permutations of n objects is:

$$\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdot \ldots \cdot n_k!}$$

BANANA

1 B: $n_1 = 1$

3 As: $n_2 = 3$

2 Ns: $n_3 = 2$

$n = 6$ total letters

$$\frac{6!}{1! \cdot 3! \cdot 2!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{1 \cdot \cancel{3 \cdot 2 \cdot 1} \cdot 2 \cdot 1} = \frac{6 \cdot 5 \cdot 4}{2} = \boxed{60}$$

What if order does not matter?

Example: 100 students are in 8th grade in a school. In how many ways can 3 students be chosen to form a student council?

not $\underline{100}\ \underline{99}\ \underline{98}$

$$\frac{970{,}200}{6} = 161{,}700$$

why?

Jill, Rob, Megan
Jill, Megan, Rob
Rob, Jill, Megan
Rob, Megan, Jill
Megan, Jill, Rob
Megan, Rob, Jill

all the same student council

6 ways these choices can be rearranged

Combination – A combintation is a subset of n elements taken r at a time, where ORDER DOESN'T MATTER.

Number of combinations of n elements taken r at a time is:

$${}_nC_r = \frac{n!}{(n-r)!r!}$$ ← Same as binomial coefficient can use Pascal's Δ to compute

show easier for each c/p

Example: 100 students are in 8th grade in a school. In how many ways can 3 students be chosen to form a student council?

$${}_{100}C_3 = \frac{100!}{(100-3)!\,3!} = \frac{100!}{97!\cdot 3!} = \frac{100\cdot 99\cdot 98\cdot 97\cdot 96\cdots 1}{97\cdots 1\cdot 3\cdot 2} = \frac{100\cdot 99\cdot 98}{3\cdot 2} = \boxed{161{,}700}$$

number of ways the 3 chosen are re-arranged

P6 start

Practice: In how many ways can 3 letters be chosen from the letters A, B, C, D, E if order of the letters does not matter?

$${}_5C_3 = \frac{5!}{(5-3)!\,3!} = \frac{5!}{2!\,3!} = \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 3\cdot 2\cdot 1} = \frac{5\cdot 4}{2} = \boxed{10}$$

More complex examples: ↓↓ 2nd day if needed.

A shipment of 25 television sets contains 3 defective units. In how many ways can a vending company purchase 4 of these units and receive (a) all good units, (b) 2 good units, (c) at least 2 good units? 22 good, 3 bad units

(a) good · bad

$${}_{22}C_4 \cdot {}_3C_0$$

$$\frac{22!}{18!\,4!} \cdot \frac{3!}{3!\,0!}$$

$$7315 \cdot 1 = \boxed{7315}$$

(b) good · bad

$${}_{22}C_2 \cdot {}_3C_2$$

$$\frac{22!}{20!\,2!} \cdot \frac{3!}{1!\,2!}$$

$$231 \cdot 3 = \boxed{693}$$

(c) 2 good + 3 good + 4 good

$$693 + {}_{22}C_3 \cdot {}_3C_1 + 7315$$

$$\frac{22!}{19!\,3!} \cdot \frac{3!}{2!\,1!}$$

$$1540 \cdot 3 = 4620$$

$$693 + 4620 + 7315 = \boxed{12628}$$

5 cards are selected from an ordinary deck of 52 playing cards. In how many ways can you get a full house? (3 of a kind and two of another, e.g. 8-8-8-5-5).

possible cards to have 3 of · # ways to get 3 of this card · # possible cards to have 2 of · # ways to get 2 of this card

$${}_{13}C_1 \cdot {}_4C_3 \cdot {}_{12}C_1 \cdot {}_4C_2$$

$$\frac{13!}{12!\,1!}(13) \cdot \frac{4!}{1!\,3!}(4) \cdot \frac{12!}{11!\,1!}(12) \cdot \frac{4!}{2!\cdot 2!}(6) = \boxed{3744}$$

In how many ways can 5 girls and 3 boys walk through a doorway single file?

8 · 7 · 6 · 5 · 4 · 3 · 2 · 1 = $\boxed{40{,}320}$
(1st, 2nd, …, 8th)

What if girls must enter before boys?

5 4 3 2 1 3 2 1 = $\boxed{720}$

Three couples have reserved seats in one row at a concert. In how many ways can they be seated?

6 5 4 3 2 1 = $\boxed{720}$

What if couples wish to sit together?

6 1 4 1 2 1 = $\boxed{48}$

HAlg3-4, 9.7 day 1 Notes – Probability

If you roll a 6-sided, fair die, what is the probability that you will roll a 4?

Possible outcomes = $\{1, 2, 3, 4, 5, 6\}$

Desired outcomes 4

Probability = $\frac{1}{6}$

What is the probability that you roll an even number? $\{1, 2, 3, 4, 5, 6\}$ $P(\text{even}) = \frac{2}{6} = \boxed{\frac{1}{2}}$

Terms:

Any happening whose result is uncertain is called an **experiment.**
Possible results of the experiment are **outcomes.**
The set of all possible outcomes is called the **sample space.**
Any subcollection of a sample space is called an **event.**

Probability of an Event

$$P(E) = \frac{n(E)}{n(S)} = \frac{\textit{number of desired outcomes}}{\textit{total number of outcomes}}$$

Probability is a number between 0 and 1 (usually expressed as a fraction or decimal):

impossible (cannot occur) $\longrightarrow 0 \le P(E) \le 1 \longleftarrow$ certain (must occur)

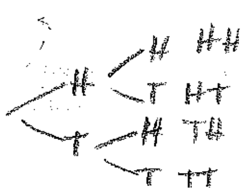
Probability of the complement of an Event (Probability of an event <u>not</u> occurring):

$$P(E') = 1 - P(E)$$

ex: $P(\text{rain}) = 0.6$ (60%) then $P(\text{not rain}) = 1 - 0.6 = 0.4$ (40%)

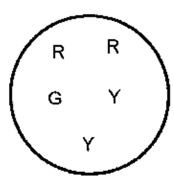
Examples: Find the sample space

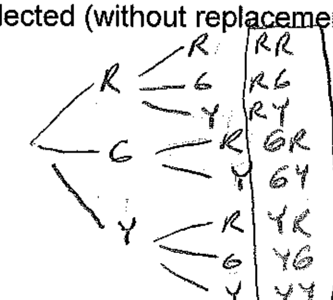
Two coins are tossed



$\{HH, HT, TH, TT\}$

2 marbles are selected (without replacement)





If two coins are tossed, what is the probability that both land heads up?

$\{HH, HT, TH, TT\}$ $P(HH) = \boxed{\frac{1}{4}}$

If a card is drawn from a standard deck of cards, what is the probability that it is an ace?

$P(\text{ace}) = \frac{4}{52} = \boxed{\frac{1}{13}}$

4 aces out of 52 cards

Two 6-sided dice are tossed. What is the probability that the total of the two dice is 7?

possible outcomes: $\underset{1^{st}\text{die}}{6} \cdot \underset{2^{nd}\text{die}}{6} = 36$ desired, sum = 7 how many ways?

36 total outcomes

1st die: 1 2 3 4 5 6
2nd die: 6 5 4 3 2 1

6 desired outcomes

$P(\text{sum is } 7) = \frac{6}{36} = \boxed{\frac{1}{6}}$ $(\approx .1666)$ (16.72)

In a state lottery, a player chooses 6 different numbers from 1 to 40. If these 6 numbers match the 6 winning numbers (order does not matter), the player wins. What is the probability of winning if a single ticket is purchased? combination

total possible winning numbers: $_{40}C_6 = \frac{40!}{(40-6)!\,6!} = \frac{40\cdot39\cdot38\cdot37\cdot36\cdot35\cdots1}{34\cdot33\cdots\,6\cdot5\cdot4\cdot3\cdot2\cdot1} = \frac{40\cdot39\cdot38\cdot37\cdot36\cdot35}{6\cdot5\cdot4\cdot3\cdot2\cdot1}$

$\therefore\ _{40}C_6 = 3{,}838{,}380$ $\quad P(\text{win}) = \boxed{\frac{1}{3{,}838{,}380}}$

Mutually Exclusive Events

Two events from the same sample space, A and B, are mutually exclusive if they have no outcomes in common.

Probability of either of mutually exclusive events occurring: $P(A \overset{\text{'or'}}{\cup} B) = P(A) \overset{\text{add}}{+} P(B)$

Example: The personnel department of a company has compiled data on employee's number of years of service, shown in the table. In an employee is chosen at random, what is the probability that the employee has 9 or fewer years of service?

Yrs of Service	Number Employees
0-4	157
5-9	89
10-14	74
15-19	63
20-24	42
25-29	38
30-34	37
35-39	21
40-44	8
	529

9 or fewer

$P(0-4) = \frac{157}{529}$

$P(5-9) = \frac{89}{529}$

$P(0-9) = P(0-4) + P(5-9)$

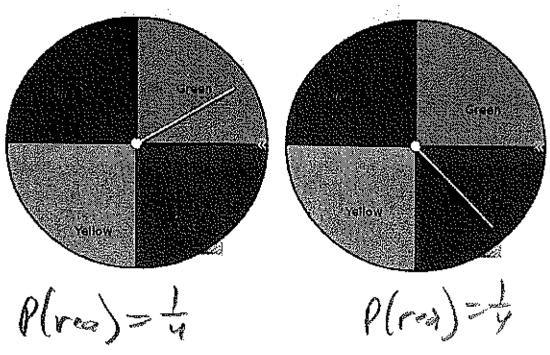
$= \frac{157}{529} + \frac{89}{529} = \frac{246}{529}$

≈ 0.47 (47%)

Independent Events – Two events are independent if the occurrence of one has no effect on the occurrence of the other.

Probability of both independent events occurring: $P(A \overset{\text{'and'}}{\text{ and }} B) = P(A) \overset{\text{multiply}}{\bullet} P(B)$

Example: If two 4-color spinners below are spun, what is the probability that the both spinners will land on red?



$P(\text{red}) = \frac{1}{4}$ $\quad P(\text{red}) = \frac{1}{4}$

$P(\text{both red}) = \frac{1}{4} \cdot \frac{1}{4} = \boxed{\frac{1}{16}}$

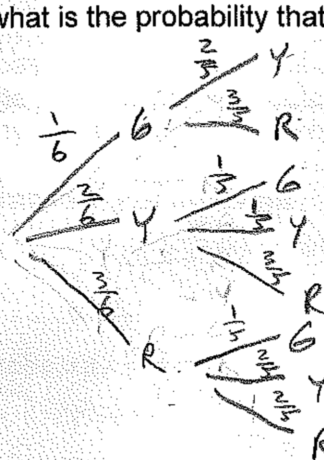
Example: In 1997, 58% of the population of the U.S. were 30 years old or older. Suppose that in a survey, ten people were chosen at random from the population. What is the probability that all ten were 30 years or older?

$(.58)(.58)(.58)\cdots = (.58)^{10} = .0043 \quad (0.43\%)$

1st person, 2nd, 3rd

Example: A bag contains: 1 green marble, 2 yellow marbles and 3 red marbles. If a marble is drawn out and the color recorded, then a second marble is drawn out (without replacement) and the color recorded, what is the probability that at least 1 red marble is drawn?

G Y Y R R R



$\frac{1}{6}\cdot\frac{2}{5} = \frac{2}{30}$

$\frac{1}{6}\cdot\frac{3}{5} = \frac{3}{30}$ ✓

$\frac{2}{6}\cdot\frac{1}{5} = \frac{2}{30}$

$\frac{2}{6}\cdot\frac{1}{5} = \frac{2}{30}$

$\frac{2}{6}\cdot\frac{3}{5} = \frac{6}{30}$ ✓

$\frac{3}{6}\cdot\frac{1}{5} = \frac{3}{30}$ ✓

$\frac{3}{6}\cdot\frac{2}{5} = \frac{6}{30}$ ✓

$\frac{3}{6}\cdot\frac{2}{5} = \frac{6}{30}$ ✓

note: all outcomes add to $\frac{30}{30} = 1$

$\frac{3}{30} + \frac{6}{30} + \frac{3}{30} + \frac{6}{30} + \frac{6}{30} = \frac{24}{30} = \boxed{\frac{4}{5}}$

$P(\text{at least 1 red}) = 1 - P(\text{no red})$

$= 1 - [P(YG) + P(GY) + P(YY)]$

$= 1 - \left[\frac{2}{6}\cdot\frac{1}{5} + \frac{1}{6}\cdot\frac{2}{5} + \frac{2}{6}\cdot\frac{1}{5}\right]$

$= 1 - \left[\frac{2}{30} + \frac{2}{30} + \frac{2}{30}\right] = 1 - \frac{6}{30} = 1 - \frac{1}{5}$

$= \boxed{\frac{4}{5}}$

Example: If 5 cards are drawn from a standard deck of 52 playing cards, what is the probability that the 5 cards make a full house?

8 8 8 3 3

ways to chose card to have 3 of · ways to get this card · ways to choose other card · ways to get that card

${}_{13}C_1 \cdot {}_4C_3 \cdot {}_{12}C_1 \cdot {}_4C_2$

$13 \cdot 4 \cdot 12 \cdot 6 = 3744$

total ways to choose 5 cards

${}_{52}C_5 = 2598960$

$P(\text{full house}) = \frac{3744}{2598960} = 0.00144$

(0.144%)

Example: What is the probability of tossing two 6-sided dice and getting a sum of at least 8?

sum: die 2 \ die 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(\text{at least } 8) = \frac{15}{36} = \boxed{\frac{5}{12}}$

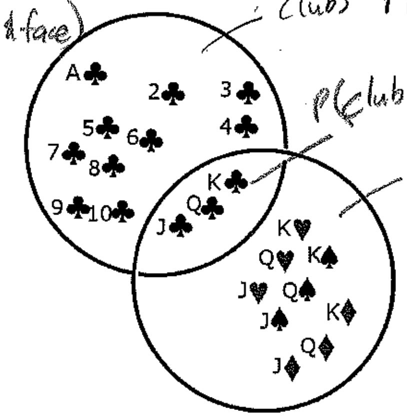
HAlg3-4, 9.7 day 2 Notes – Probability

Probability of a Union

Example: One card is selected from a standard deck. What is the probability that the card is either a club or a face card?

There are 13 clubs, and there are 3 face cards in each suit (12 face cards in all), but some cards are both clubs and face cards. To solve, use a **Venn diagram**:

$P(\text{club or facecard}) = P(\text{club}) + P(\text{face}) - P(\text{club \& face})$



Probability of a Union with overlap:

or — add — subtract overlap

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Probability of a Union without overlap (mutually exclusive):

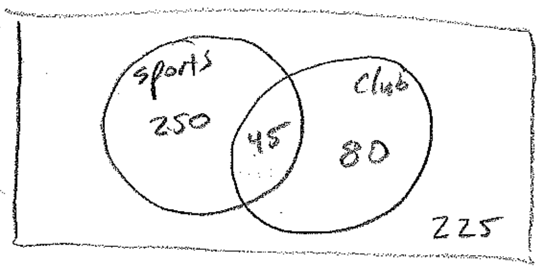
or — add

$$P(A \cup B) = P(A) + P(B)$$

Example: There are 600 seniors at a school. 250 seniors only play a sport and 80 seniors are only part of a club. 45 seniors participate in both a sport and a club.

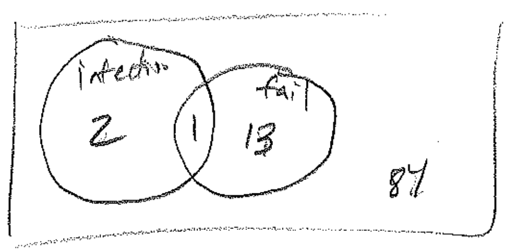
(a) How many seniors are in neither sports nor a club? $\boxed{225}$
(b) How many seniors play sports? $\boxed{295}$
(c) How many seniors are in clubs? $\boxed{125}$
(d) If a senior is selected at random, what is the probability that they will play a sport? $\boxed{\frac{295}{600}}$

$250 + 45 + 80 = 375$
$600 - 375 = 225$



Example: You have a torn tendon and are facing surgery to repair it. The orthopedic surgeon explains to you the risk involved. Infection occurs in 3% of such operations, the repair fails in 14%, and both infection and failure occur together in 1%. What percent of these operations both succeed and are free from infection?

$100 - 16 = 84\%$



$\boxed{84\%}$

Example: A manufacturer has determined that a certain machine averages one faulty unit for every 1000 that it produces. What is the probability that an order of 200 units will have one or more faulty units?

Could try $P(1 \text{ bad unit}) + P(2 \text{ bad units}) + P(3 \text{ bad units}) + \ldots$ too long

Instead, Find $P(\text{all good})$, then $P(\text{at least one bad}) = 1 - P(\text{all good})$

$P(\text{a unit is good}) = \frac{999}{1000}$

$P(200 \text{ units are all good}) = \left(\frac{999}{1000}\right)^{200} = .818649$

$P(\text{one or more bad units}) = 1 - .818649 = \boxed{.1814}$

Review of counting and probability strategies:

Simple counting – list entire sample space.

Multiple elements, pairings (combo meals, outfits of clothing) – use a tree diagram to 'see' the sample space.

'Selection' problems – choose some of a larger group (races, committees, order in a line):

- Order matters – Permutation: ${}_nP_r = \frac{n!}{(n-r)!}$ or use 'boxes' to compute.
- Order doesn't matter – Combination: ${}_nC_r = \frac{n!}{(n-r)!r!}$ or use Pascal's triangle to compute.
- Order matters, duplicates (BANANA) - $\frac{n!}{n_1! \cdot n_2! \cdot n_3! \cdot \ldots \cdot n_k!}$

Probability: $P(E) = \frac{n(E)}{n(S)} = \frac{\textit{number of desired outcomes}}{\textit{total number of outcomes}}$

Probability of either of two events occurring:
- Mutually exclusive (non-overlapping): $P(A \cup B) = P(A) + P(B)$
- Overlapping: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Use Venn Diagram)

Probability of both of two events occuring (independent events): $P(A \text{ and } B) = P(A) \bullet P(B)$

Complex cases: Use a tree diagram, ~~add~~ calculate probabilities for each branch, multiply to get each 'leaf' probability. Add the probabilities for the outcomes that are desired.

Probability of something not happening: $P(E') = 1 - P(E)$

— (in groups) — on board

#1. A committee of 3 is to be selected at random from a group of 4 boys and 5 girls. What is the probability that the committee selected will consist entirely of boys?

total # ways to choose 3 of 9

$_9C_3 = \frac{9!}{6!\,3!} = 84$

ways to choose boys · # ways to choose girls

$_4C_3 \quad _5C_0$

$4 \cdot 1 = 4$

$P = \frac{4}{84} = \boxed{\frac{1}{21}} = \boxed{.0476}$

3 #2. A bag contains: 4 red, 2 yellow and 3 blue marbles. A marble is taken out and its color recorded, then, without replacement, another marble is taken out and its color recorded. What is the probability that at least 1 blue marble was drawn out of the bag?

RRRR YY BBB

R $\frac{4}{9}$: R $\frac{3}{8}$ $\frac{12}{72}$; Y $\frac{2}{8}$ $\frac{8}{72}$; B $\frac{3}{8}$ $\frac{12}{72}$ ✓

Y $\frac{2}{9}$: R $\frac{4}{8}$ $8/72$; Y $\frac{1}{8}$ $2/72$; B $\frac{3}{8}$ $6/72$ ✓

B $\frac{3}{9}$: R $4/8$ $12/72$ ✓; Y $2/8$ $6/72$ ✓; B $2/8$ $6/72$ ✓

$\frac{12}{72} + \frac{6}{72} + \frac{12}{72} + \frac{6}{72} + \frac{6}{72} = \frac{42}{72} = \frac{21}{36} = \boxed{\frac{7}{12}}$

or $P(\text{at least 1 blue}) = 1 - P(\text{no blue})$

$= 1 - (P(RY) + P(YR) + P(RR) + P(YY))$

$= 1 - \left(\frac{4}{9}\cdot\frac{2}{8} + \frac{2}{9}\cdot\frac{4}{8} + \frac{4}{9}\cdot\frac{3}{8} + \frac{2}{9}\cdot\frac{1}{8}\right)$

$= 1 - \left(\frac{8}{72} + \frac{8}{72} + \frac{12}{72} + \frac{2}{72}\right) = \frac{72}{72} - \frac{30}{72} = \frac{42}{72}$

4 #3. A shipment of 20 CD players contains 4 defective units. A retail outlet has ordered 5 of these units, and will receive 5 at random from the shipment. What is the probability that:
(a) exactly 4 units are good?
(b) at least one unit is good?

16 good units
4 bad units

(a) # good · # bad

$_{16}C_4 \quad _4C_1$

$1820 \cdot 4$

7280 desired $_{20}C_5$ 15504 total

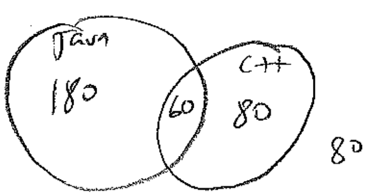
$P = \frac{7280}{15504} = \boxed{.4696}$

(b) complement = all bad

P(all bad) = 0 (there are only 4 bad units)

so, P(at least one good) = $\boxed{100\%}$

2 #4. ABC Tech employs 400 people, including 180 who can write only Java programs, 60 who can write both C++ and Java programs, and 80 who cannot write programs at all. If a random employee is selected, what is the probability that they can write C++ programs?



$\frac{140}{400} = \frac{7}{20}$ $\boxed{.35}$