Honors Algebra 3-4, P5 (day1) Notes – Interval notation, Solving Inequalities

Properties of inequalities / Solving linear inequalities:

- Transitive: $a < b$ and $b < c \Rightarrow a < c$ example: $2<4$ & $4<12 \Rightarrow 2<12$
- Addition: $a < b$ and $c < d \Rightarrow a + c < b + d$ example: $2<3$, $+\ 4<7$, $6<10$
- Addition of constant: $a < b \Rightarrow a + c < b + c$ example: $2<4 \Rightarrow 2+1<4+1$ $(3<5)$
- Multiplying by a constant: for $c > 0$, $a < b \Rightarrow ac < bc$; $c < 0$, $a < b \Rightarrow ac > bc$ examples: $2x<4 \Rightarrow x<2$; $-2x<4 \Rightarrow x>-2$

Main thing to remember: if you multiply or divide by a negative number, you need to switch the sign.

Solving linear inequalities: perform algebraic operations, but keep in mind:

- If multiplying or dividing, switch direction of inequality.
- For double inequalities, perform operations on all 3 terms.

Examples: Solve $5x - 7 > 3x + 9$

$$\begin{aligned} &+7 \quad\quad +7 \\ &5x > 3x + 16 \\ &-3x \quad -3x \\ &2x > 16 \\ &\tfrac{2x}{2} > \tfrac{16}{2} \\ &\boxed{x > 8} \end{aligned}$$

Solve: $-3 \le 6x - 1 < 3$

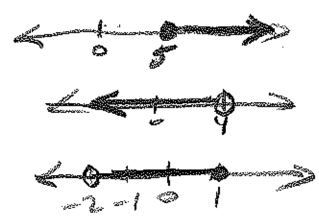
$$\begin{aligned} &+1 \quad\quad +1 \quad +1 \\ &-2 \le 6x < 3 \\ &\tfrac{-2}{6} \le \tfrac{6x}{6} < \tfrac{3}{6} \\ &-\tfrac{1}{3} \le x < \tfrac{1}{2} \end{aligned}$$

also $-3 \le -6x - 1 < 3$

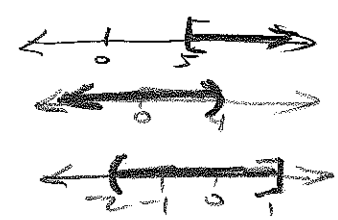
Interval notation:

	Equation form	interval notation
Examples:	$x \ge 5$	$[5, \infty)$
	$x < 4$	$(-\infty, 4)$
	$-2 < x \le 1$	$(-2, 1]$

old number line



book number line



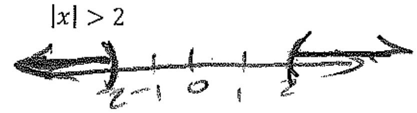
Sketching inequalities with absolute values:

For < think 'inside', for > think 'outside' ; or $|x|$ = 'distance' from some midpt

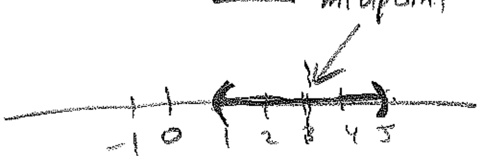
Examples:

$|x| < 2$

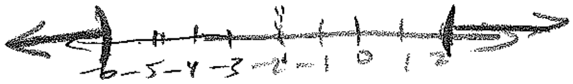
$|x| > 2$



$|x - 3| < 2$ 'midpoint'



$|x + 2| > 4$



Solving inequalities with absolute values - Use different methods to solve > and < cases:

For < case, make a double inequality: $|x-3| < 2$

$$-2 < x-3 < 2$$
$$+3 \quad +3 \quad +3$$
$$\boxed{1 < x < 5}$$

For > case, make two separate inequalities: $|x+2| > 4$

$$x+2 > 4$$
$$-2 \quad -2$$
$$\boxed{x > 2}$$

$$-(x+2) > 4$$
$$x+2 < -4$$
$$-2 \quad -2$$
$$\boxed{x < -6}$$

add linear case first

Honors Algebra 3-4, P5 (day2) Notes – Solving polynomial and rational inequalities

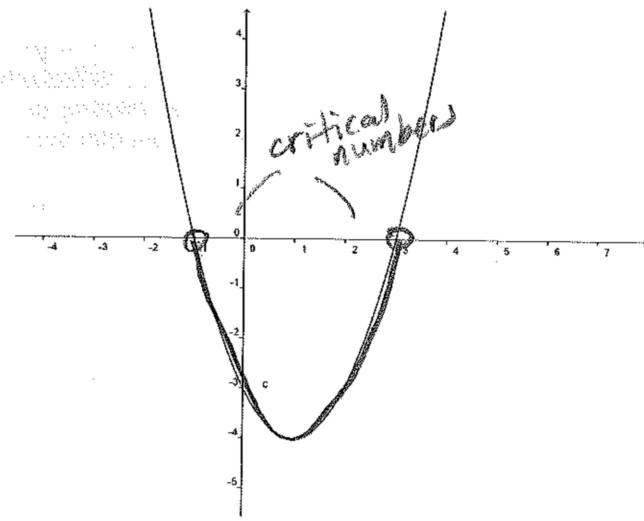
Solving polynomial inequalities means finding all the x values that make the equality true.

Graphically:

Solve: $x^2 - 2x - 3 < 0$

From the plot, it is clear that the expression is negative for x between -1 and 3.

These x-values, -1 and 3 are called 'critical numbers'. They represent where the expression changes sign.



Procedure for solving polynomial inequalities:

1. Make right side zero.
2. Factor left side to find critical numbers, and arrange them in ascending order.
3. Make a table that divides all possible x-values into intervals, dividing at the critical numbers.
4. For each interval, choose a test x-value.
5. Plug in the test x-value to determine if the left side is + or -.
6. Find the intervals that give the desired outcome and write in interval notation.

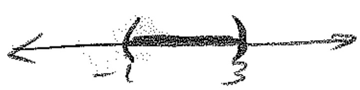
Example (same as above): Solve: $x^2 - 2x - 3 < 0$

$x^2 - 2x - 3 < 0$

$(x-3)(x+1) < 0$

crit #'s $-1, 3$

$(-1, 3)$



Interval	Test x-value	Test	Result
$(-\infty, -1)$	-2	$(-)(-)=+$	✗
$(-1, 3)$	0	$(-)(+)=-$	✓
$(3, \infty)$	4	$(+)(+)=+$	✗

Another example: Solve: $2x^2 + 5x \geq 12$

$2x^2 + 5x - 12 \geq 0$

$\frac{(2x+8)}{2}\frac{(2x-3)}{1}$ $\quad -24 \mid 5$, $8 \; -3$

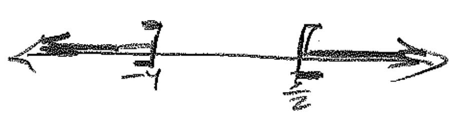
$(x+4)(2x-3) \geq 0$

$x = -4 \qquad 2x - 3 = 0 \qquad$ crit #'s: $-4, \frac{3}{2}$

$2x = 3$

$x = \frac{3}{2}$

$(-\infty, -4] \cup [\frac{3}{2}, \infty)$



Interval	Test x-value	Test	Result
$(-\infty, -4)$	-5	$(-)(-)=+$	✓
$(-4, \frac{3}{2})$	0	$(+)(-)=-$	✗
$(\frac{3}{2}, \infty)$	2	$(+)(+)=+$	✓

Solving rational inequalities: Procedure for solving is similar to polynomials, but with a couple of extra considerations. Rational inequalities are those that contain at least one fractional expression, for example: $\frac{x+12}{x+2} \geq 3$

Procedure for solving rational inequalities:

1. Make right side zero.
2. Make left side a single fraction (common denominator.)
3. Critical numbers are the x-values that make either numerator or denominator zero. (rest of procedure is the same as polynomial procedure)...
4. Make a table that divides all possible x-values into intervals, dividing at the critical numbers.
5. For each interval, choose a test x-value.
6. Plug in the test x-value to determine if the left side is + or -.
7. Find the intervals that give the desired outcome and write in interval notation.

Example: Solve: $\frac{x+12}{x+2} \geq 3$

$$\frac{x+12}{x+2} - 3 \geq 0$$

$$\frac{x+12}{x+2} - \frac{3(x+2)}{x+2} \geq 0$$

$$\frac{x+12-3x-6}{x+2} \geq 0$$

$$\frac{-2x+6}{x+2} \geq 0$$

crit #s: $-2x+6=0$, $-2x=-6$, $x=3$ $\quad$ $x+2=0$, $x=-2$ $\quad$ $-2, 3$

Interval	Test x-value	Test	Result
$(-\infty,-2)$	-3	$\frac{(+)}{(-)} = -$	
$(-2,3)$	0	$\frac{(+)}{(+)} = +$	✓
$(3,\infty)$	4	$\frac{(-)}{(+)} = -$	

$(-2, 3]$