# Honors Algebra 3-4 'Big Picture'

**Functions**

- Definition and properties:
  - Mapping, Domain and Range
  - Exactly 1 output for every input
  - Vertical Line Test
  - Increasing/decreasing
  - Even/odd: graphically, algebraically.
- Common function shapes: constant, line, absolute value, square root, $x^2$, $x^3$, piecewise defined, step/greatest integer.
- Combining functions:
  - arithmetic: (f+g)(x)=f(x)+g(x), (fg)(x)=f(x)*g(x)
  - composition: fog(x)=f(g(x))
- Transformations: shifting, reflecting, stretch/shrink
- Inverse of a function:
  - 'undo' each other.
  - Domain/Range swap
  - Sketch reflect over y=x line
  - Procedure to find: swap x and y, resolve for y
  - f has inverse if 1:1, Horizontal Line Test

**Toolbox**

- Exponent/Radical rules
- Factoring:
  - Patterns
  - Trinomial procedures
  - By Grouping
- Completing the square
- Sketching inequalities:
  - simple cases: 'inside', 'outside'
  - higher order: critical numbers, regions, test points.
- Complex numbers:
  - definition ( $i=\sqrt{-1}$ , $i^2=-1$ )
  - add/subtract/multiply
  - complex conjugate to divide
  - plotting on complex plane

**Polynomials**

- Degree, $f(x)=a_4x^4+a_3x^3+a_2x^2+a_zx+a_0$
- Quadratic functions:
  - Parabolic shape
  - Complete square to std form to sketch: $y=a(x-h)^2+k$
  - min, max has real-world application (max height of ball, min cost of manufacturing, etc.)
- Higher degree polynomials:
  - LH, RH behavior (leading coefficient test)
  - Find zeros by factoring, other means
  - Multiplicity of zeros / sketching
- Finding Zeros:
  - Descartes' Rule (# even and odd real zeros)
  - Rational Zero Test (list possible rational real zeros)
  - Polynomial division / synthetic division to test zeros (remainder theorem – if remainder is zero, is a factor/zero)
  - Quadratic equation sometimes gives complex zeros
  - Fundamental Theorem of Algebra: #zeros = degree
- Factor Theorem: f(k) = remainder when doing synthetic division by k

**Rational Functions**

- Ratio of polynomials
- Vertical asymptotes where denom = 0
- Horizontal asymptote:
  - y=0 if n<m,
  - y=a/b (leading coefficients) if n=m
  - no HA if n>m
- Slant asymptote if n=m+1, find by polynomial division
- Sketch from zero crossings and asymptotes

**Exponential/Logarithmic Functions**

- $y=a^x$ , sketching
- inverse of exponential is $y=\log_z x$ (sketch reflected over y=x)
- Compound interest:
  - Monthly, annually: $A=P\left(1+\frac{r}{n}\right)^{nt}$
  - Continuously: $A=Pe^{rt}$
- Definition of e, e=2.7182818....
- Log properties:
  - log(uv)=logu+logv, log(u/v)=logu-logv
  - $\log u^v = v\log u$
- Change of base: $\log_b x=(\log_{10}x)/(\log_{10}b)$
- Solving exp/log equations...strategies:
  - Combine logs to get single log on a side.
  - Rewrite log equations in exponential form.
  - 'Undo' $a^x$ by taking $\log_a$ of a side.
  - Get matching bases or matching logs on both sides and use 1:1 property
- Exponential Decay: $Q=Ce^{kt}$ C is initial quantity.
  - Radioactive decay: half life is time (t) when quantity is half the initial quantity.

**Conic Sections**

- Parabolas:
  - $(x-h)^2=4p(y-k)$ or
  - $(y-k)^2=4p(x-h$
  - $(x-h)^2$ like $y=x^2$, $(y-k)^2$ 'other one'
  - (h,k)=vertex
  - p=dist. vertex to focus and vertex to directrix
  - Complete the square to get standard form equation
- Ellipses:
  - $$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$
  - $$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$
  - equation has + sign
  - a is always bigger and a is under term of major axis
  - (h,k)=center
  - $c^2=a^2-b^2$
  - a=dist. center to vertex
  - b=dist. center to point on minor axis
  - c=dist. center to focus
  - eccentricity e=c/a
  - Complete the square to get standard form equation
- Hyperbolas:
  - $$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$
  - $$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$
  - equation has - sign
  - a is not always bigger, a always under first term, first term is transverse axis
  - (h,k)=center
  - $c^2=a^2+b^2$
  - a=dist. center to vertex
  - b=dist. to 'other side of box'
  - c=dist. center to focus
  - eccentricity e=c/a
  - asymptotes through corners of box:

    $(y-k)=\pm\frac{b}{a}(x-h)$ or $(y-k)=\pm\frac{a}{b}(x-h)$

    (look at box to see which)
  - Complete the square to get standard form equation
- Given foci, vertices, start with sketch, then fill in std form equation.
- Which conic from general equation: $Ax^2+Cy^2+Dx+Ey+F=0$:
  - 2 squared terms, same coefficient = circle
  - 2 squared terms, same sign, different = ellipse
  - 2 squared terms, opposite signs = hyperbola
  - 1 squared term = parabola