Chapter 8 Review Name Key

1. Use any method to solve (show matrices and thoroughly explain!)

$x+y-z+2w=7$
$3x-2y+5z-2w=-1$
$2x+3y-4z+w=0$
$-x-y+z-w=-3$

$$\begin{bmatrix} 1 & 1 & -1 & 2 \\ 3 & -2 & 5 & -2 \\ 2 & 3 & -4 & 1 \\ -1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \\ -3 \end{bmatrix}$$

$AX=B$

so $X=A^{-1}B$

by calc:

$X=A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 4 \end{bmatrix}$$

2. Use the following matrices:

$$A=\begin{bmatrix} 3 & -4 & 5 & 1 \\ 2 & 6 & -7 & 5 \\ -1 & 0 & -3 & 0 \end{bmatrix} \quad B=\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 5 & -5 & 7 \\ 6 & -1 & 0 & -4 \end{bmatrix} \quad C=\begin{bmatrix} 1 & 0 & 6 \\ 3 & 4 & 1 \\ -2 & -1 & -5 \\ 5 & 2 & 0 \end{bmatrix}$$

manually a. $3A-5B = \begin{bmatrix} 4 & -32 & 25 & -12 \\ 6 & -7 & 4 & -20 \\ -33 & 5 & -9 & 20 \end{bmatrix}$

manually b. AB not possible 3×4 3×4

c. AC $= \begin{bmatrix} -14 & -19 & -11 \\ 59 & 41 & 53 \\ 5 & 3 & 9 \end{bmatrix}$

d. CA

$= \begin{bmatrix} -3 & -4 & -13 & 1 \\ 16 & 12 & -16 & 23 \\ -3 & 2 & 12 & -7 \\ 19 & -8 & 11 & 15 \end{bmatrix}$

3. Solve for x, y, and z given A = B:

$$A=\begin{bmatrix} x+y & 1 & 3 \\ 2 & z & 5 \end{bmatrix} \text{ and } B=\begin{bmatrix} 4 & 1 & 3 \\ x-y & 2 & 5 \end{bmatrix}$$

$x+y=4$
$x-y=2$
$2x=6$
$x=3$
$3+y=4$
$y=1$

$z=2$

$\boxed{(3,1,2)}$

4. Solve the system using Cramer's Rule (you must show matrices/determinants):

$$2x - y + z = 5$$
$$3x + 2y + 5z = 18$$
$$x - y + 4z = 5$$

$$\underbrace{\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 5 \\ 18 \\ 5 \end{bmatrix}}_{=B}$$

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 4 \end{vmatrix} = 28 \qquad |A_x| = \begin{vmatrix} 5 & -1 & 1 \\ 18 & 2 & 5 \\ 5 & -1 & 4 \end{vmatrix} = 84 \qquad |A_y| = \begin{vmatrix} 2 & 5 & 1 \\ 3 & 18 & 5 \\ 1 & 5 & 4 \end{vmatrix} = 56$$

$$|A_z| = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 2 & 18 \\ 1 & -1 & 5 \end{vmatrix} = 28$$

$$x = \frac{|A_x|}{|A|} = \frac{84}{28} = 3 \qquad y = \frac{|A_y|}{|A|} = \frac{56}{28} = 2 \qquad z = \frac{|A_z|}{|A|} = \frac{28}{28} = 1$$

$$\boxed{(3, 2, 1)}$$

manual 5. Use the determinant to decide if the points (-1, -1), (2, -3), and (-2, 2) are collinear. (Show all work!)

$$\begin{vmatrix} -1 & -1 & 1 \\ 2 & -3 & 1 \\ -2 & 2 & 1 \end{vmatrix} = -1\begin{vmatrix} -3 & 1 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ -2 & 2 \end{vmatrix}$$
$$= -1(-3-2) + 1(2+2) + 1(4-6)$$
$$= -1(-5) + 1(4) + 1(-2) = 5 + 4 - 2 = 7 \quad \boxed{\text{not collinear}}$$

manual 6. Find the value(s) of x such that the triangle with vertices (0, 2), (3, 4), and (x, -4) has an area of 10. (Show all work!)

$$\begin{vmatrix} 0 & 2 & 1 \\ 3 & 4 & 1 \\ x & -4 & 1 \end{vmatrix} = -2\begin{vmatrix} 3 & 1 \\ x & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 4 \\ x & -4 \end{vmatrix} = -2(3-x) + 1(-12-4x)$$
$$= -6 + 2x - 12 - 4x$$
$$= -2x - 18 =$$
$$\pm\tfrac{1}{2}(-2x-18) = 10$$
$$-2x - 18 = \pm 20 \qquad -2x = 18 \pm 20 \qquad \boxed{x = -19 \text{ or } 1}$$

manual 7. Find the equation of the line that has the points (1, 3) and (4, 9). (Show all work!)

$$\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 4 & 9 & 1 \end{vmatrix} = x\begin{vmatrix} 3 & 1 \\ 9 & 1 \end{vmatrix} - y\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 4 & 9 \end{vmatrix}$$
$$= x(3-9) - y(1-4) + 1(9-12) \qquad -6x + 3y - 3 = 0$$
$$= x(-6) - y(-3) + (-3)$$

$$\boxed{\begin{matrix} -6x + 3y = 3 \\ \text{or} \\ -2x + y = 1 \end{matrix}}$$

manual 8. Find A^{-1} given $A = \begin{bmatrix} 1 & 1 & 0 \\ 3 & 1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$ (no decimals!)

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 1 & 0 \\ -1 & 1 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_1+R_3]{-3R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 2 & -3 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 2 & -3 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right] \xrightarrow{-\frac{1}{2}} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right]$$

$$\xrightarrow[R_3+R_2]{-R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right] \xrightarrow{-R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{2} & -\frac{1}{2} & -1 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 1 \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right]$$

$-\frac{4}{2}$

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a. 3A – 5B

b. AB

c. AC

d. CA

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