Worksheet 9.1-9.3

Name Key

Date Period

$a_n = a_1 + (n-1)d$

$a_n = dn + c, \quad c = a_1 - d$ $\qquad a_n = a_1 r^{n-1}$

$S = \frac{n}{2}(a_1 + a_n)$ $\qquad S = a_1\left(\frac{1-r^n}{1-r}\right)$ $\qquad S = \frac{a_1}{1-r}$

1: Write the first five terms of the sequence where $a_1 = 3$ and $a_{n+1} = a_n(n+1)$.

$a_1 = 3$

$a_2 = a_1(1+1) = 3(2) = 6$

$a_3 = a_2(2+1) = 6(3) = 18$

$a_4 = a_3(3+1) = 18(4) = 72$

$a_5 = a_4(4+1) = 72(5) = 360$

$\boxed{3, 6, 18, 72, 360}$

2. What is the most apparent nth term of this sequence (assume that n begins with 1):

a) $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$ $\qquad$ $n = 1\ 2\ 3\ 4$; $2\ 4\ 6\ 8$

$\boxed{a_n = \frac{1}{2n-1}}$

b) $\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \ldots$ $\qquad$ $n = 1\ 2\ 3\ 4$

$\boxed{a_n = (-1)^{n+1}\frac{n^2}{n+1}}$

3. Use **sigma notation** to write the given sum (assume n begins with 1):

$\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots + \frac{64}{729}$

$r = \frac{2}{3}$ $\quad r = \frac{2}{3}$

$a_n = \frac{2}{3}\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^n$

$\frac{64}{729} = \left(\frac{2}{3}\right)^n$

$n = 6$

$\boxed{\sum_{n=1}^{6}\left(\frac{2}{3}\right)^n}$

4. Write in expanded form and then find the sum:

a) $\sum_{n=1}^{4}\frac{2}{n} = \boxed{2 + 1 + \frac{2}{3} + \frac{1}{2}}$

not arithm or geo.

$\frac{12}{6} + \frac{6}{6} + \frac{4}{6} + \frac{3}{6}$

$= \boxed{\frac{25}{6}}$

b) $\sum_{x=1}^{5}x(x-1)(x-2) = \boxed{0 + 0 + 6 + 24 + 60}$

$= \boxed{90}$

5. The addends are terms of an arithmetic sequence. Find each sum.

a) 36 terms of $21\sqrt{2} + 18\sqrt{2} + 15\sqrt{2} + \ldots$

$-3\sqrt{2}$ $\quad -3\sqrt{2}$

arith, $d = -3\sqrt{2}$

$S = \frac{1}{2}n(a_1 + a_n)$

$a_n = a_1 + (n-1)d$

$a_{36} = 21\sqrt{2} + (35)(-3\sqrt{2}) = -84\sqrt{2}$

$S = \frac{1}{2}(36)(21\sqrt{2} - 84\sqrt{2})$

$\boxed{S = -1134\sqrt{2}}$

b) $\ln 2 + \ln 4 + \ln 8 + \ldots + \ln 1024$ $\qquad$ $n=1$ $\quad$ $n=10$

$d = \ln 4 - \ln 2$

$d = \ln\frac{4}{2} = \ln 2$

$a_n = a_1 + (n-1)d$

$\ln 1024 = \ln 2 + (n-1)\ln 2$

$\frac{\ln 1024}{\ln 2} = 1 + n - 1 = n = 10$

$S = \frac{1}{2}n(\ln 2 + \ln 1024)$

$S = \frac{1}{2}(10)(\ln 2 + \ln 1024) = \boxed{5\ln 2048}$

$= \boxed{\ln(2048)^5}$

$= \boxed{38.123}$

6. A theater has 43 seats in the back row, 41 in the next, 39 in the next, and so on for 14 rows. How many seats are in the theater?

$d = -2 \quad a_1 = 43 \qquad a_{14} = 43-(14-1)2 = 17$

$a_n = a_1 + (n-1)d \qquad S = \frac{1}{2}(14)(43+17)$

$a_n = 43-(n-1)2 \qquad S = \boxed{420 \text{ seats}}$

7. The first three terms of a geometric sequence are –16, –8, –4,…. Write a rule that defines the sequence.

$\times\frac{1}{2} \quad r = \frac{1}{2} \quad a_1 = -16$

$a_n = a_1 r^{n-1}$

$\boxed{a_n = -16\left(\frac{1}{2}\right)^{n-1}}$

8. Find the fourth term of the geometric sequence whose first term is 4 and whose sixth term is $-\frac{243}{8}$.

$a_n = a_1 r^{n-1} \qquad a_4 = 4\left(-\frac{3}{2}\right)^{4-1}$

$\frac{-243}{8} = 4r^{6-1} \qquad = 4 \cdot -\frac{27}{8} = \boxed{\frac{-27}{2}}$

$\frac{-243}{8} = 4r^5$

$\frac{-243}{32} = r^5 \qquad r = \sqrt[5]{\frac{-243}{32}} = \frac{-3}{2}$

9. What is the common ratio of a geometric sequence whose second term is $\frac{2}{3}$ and whose sixth term is 54?

$a_2 = \frac{2}{3} \qquad \frac{a_6 = a_1 r^5}{a_2 = a_1 r^1} = \frac{54}{\frac{2}{3}}$

$a_6 = 54$

$r^4 = \frac{162}{2} = 81 \qquad \boxed{r=3}$

$r = (81)^{1/4} = 3$

10. Find the sum of the first twelve terms of the geometric sequence whose first three terms are 240, -120, 60,…. (Use a formula).

$r = -\frac{1}{2}$

$S = a_1\left(\frac{1-r^n}{1-r}\right) = 240\left(\frac{1-(-\frac{1}{2})^{12}}{1+\frac{1}{2}}\right) = \boxed{159.96}$

11. Find the sum: $\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1}$ converges $\quad S = \frac{a_1}{1-r} = \frac{2}{1-\frac{2}{3}} = \frac{2}{\frac{1}{3}} = \boxed{6}$

Worksheet 9.1-9.3

Name________________________

Date________________ Period__________

$a_n = a_1 + (n-1)d$

$a_n = dn + c$, $c = a_1 - d$

$S = \frac{n}{2}(a_1 + a_n)$

$a_n = a_1 r^{n-1}$

$S = a_1\left(\frac{1-r^n}{1-r}\right)$

$S = \frac{a_1}{1-r}$

1. Write the first five terms of the sequence where $a_1 = 3$ and $a_{n+1} = a_n(n+1)$.

2. What is the most apparent nth term of this sequence (assume that n begins with 1):

   a) $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots$

   b) $\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \ldots$

3. Use **sigma notation** to write the given sum (assume n begins with 1):

   $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots + \frac{64}{729}$

4. Write in expanded form and then find the sum:

   a) $\sum_{n=1}^{4} \frac{2}{n}$

   b) $\sum_{x=1}^{5} x(x-1)(x-2)$

5. **The addends are terms of an arithmetic sequence. Find each sum.**

   a) 36 terms of $21\sqrt{2} + 18\sqrt{2} + 15\sqrt{2} + \ldots$

   b) $\ln 2 + \ln 4 + \ln 8 + \ldots + \ln 1024$

6. A theater has 43 seats in the back row, 41 in the next, 39 in the next, and so on for 14 rows. How many seats are in the theater?

7. The first three terms of a geometric sequence are $-16, -8, -4, \ldots$. Write a rule that defines the sequence.

8. Find the fourth term of the geometric sequence whose first term is 4 and whose sixth term is $-\frac{243}{8}$.

9. What is the common ratio of a geometric sequence whose second term is $\frac{2}{3}$ and whose sixth term is 54?

10. Find the sum of the first twelve terms of the geometric sequence whose first three terms are 240, -120, 60,…. (Use a formula).

11. Find the sum: $\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^{n-1}$

Honors Algebra 3-4 Name Key

9.1-9.3 Review #2 Period ______

$a_n = a_1 + (n-1)d$ $\quad a_n = a_1(r)^{n-1}$

$S = \frac{n}{2}(a_1 + a_n)$ $\quad S = a_1\left(\frac{1-r^n}{1-r}\right)$ $\quad S = \frac{a_1}{1-r}$

1. Write the first five terms of the sequence where $a_1 = 3$ and $a_{n+1} = 2a_n(3n-1)$

$a_2 = 2(3)[3(2)-1] = 6(5) = 30$

$a_3 = 2(30)[3(3)-1] = 60(8) = 480$

$a_4 = 2(480)[3(4)-1] = 960(11) = 10560$

$a_5 = 2(10560)[3(5)-1] = 21120(14) = 295680$

$\boxed{3, 30, 480, 10560, 295680}$

2. Find a formula for the nth term of this sequence: $1, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{24}{5}, 20...$

(hint: make the first term a fraction and note a pattern in the denominators)

$\frac{1}{1}\ \frac{1}{2}\ \frac{2}{3}\ \frac{6}{4}\ \frac{24}{5}\ \frac{120}{6}$

n: 1 2 3 4 5 6

n! 1 2 6 24 120

(n-1)! 1 1 2 6 24 120

$\boxed{a_n = \frac{(n-1)!}{n}}$

3. Use sigma notation to write the given sum: $3+1-1-3-5$ arithmetic

$\overset{-2}{\to}\ \overset{-2}{\to}\ \overset{-2}{\to}$

(don't compute the sum – write using a $\sum$ and assume n begins with 1)

$a_n = 3-2(n-1)$ $\quad S = \boxed{\sum_{n=1}^{5} 3-2(n-1)}$ or $\boxed{\sum_{n=1}^{5} 5-2n}$

4. Find the sums:

(a) $\sum_{n=1}^{4} \frac{3n^2}{n+1}$ use calculator

$\frac{3(1)^2}{(1)+1} + \frac{3(2)^2}{(2)+1} + \frac{3(3)^2}{(3)+1} + \frac{3(4)^2}{(4)+1}$

$\frac{3}{2} + \frac{12}{3} + \frac{27}{4} + \frac{48}{5} = \boxed{\frac{437}{20}}$

(b) $\sum_{n=0}^{\infty} 10\left(\frac{1}{2}\right)^n$ infinite geometric w/ $|r|<1$

converges to $S = \frac{a_1}{1-r} = \frac{10}{1-\frac{1}{2}} = \boxed{20}$

5. Given $a_3 = -2$, $a_6 = -11$ for an ***arithmetic sequence*** find:

(a) d

$a_6 = a_1 + d(6-1) = -11$

$-a_3 = -(a_1 + d(3-1)) = -(-2)$

$5d$

$-2d$

$3d = -9$

$\boxed{d = -3}$

(b) a_1

$a_3 = a_1 + d(3-1)$

$-2 = a_1 - 3(2)$

$-2 = a_1 - 6$

$+6 \quad +6$

$\boxed{4 = a_1}$

(c) a_n

$\boxed{a_n = 4 - 3(n-1)}$

(d) a_{100}

$a_{100} = 4 - 3(100-1)$

$= 4 - 3(99)$

$= \boxed{-293}$

6. Given $a_2 = 8,\ a_5 = 64$ for an ***geometric sequence*** find:

(a) r (b) a_1 (c) a_n (d) a_8

(a) $\frac{a_5}{a_2} = \frac{a_1 r^{5-1}}{a_1 r^{2-1}} = \frac{64}{8}$

$= \frac{r^4}{r^1} = r^3 = 8$

so $\boxed{r=2}$

(b) $a_2 = a_1 r^{2-1}$

$8 = a_1 (2)^1$

$\boxed{a_1 = 4}$

(c) $a_n = a_1 (r)^{n-1}$

$\boxed{a_n = 4(2)^{n-1}}$

(d) $a_8 = 4(2)^{8-1}$

$a_8 = 4(128) = \boxed{512}$

7. Find the sum of the first 40 terms of the sequence which begins: 1, 5, 9, 13, 17...

$+4\ +4\ +4\ +4$ arithmetic $d=4$

$a_1 = 1$

$a_n = 1 + 4(n-1)$

so $a_{40} = 1 + 4(40-1)$

$= 157$

$S = \frac{n}{2}(a_1 + a_n)$

$S = \frac{40}{2}(1 + 157)$

$\boxed{S = 3160}$

8. Simplify each ratio fully:

(a) $\frac{15!}{8!4!}$

$\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot \cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}\ 4 \cdot 3 \cdot 2 \cdot 1}$

$\frac{15 \cdot \cancel{14}^{7} \cdot 13 \cdot \cancel{12} \cdot 11 \cdot 10 \cdot 9}{\cancel{4 \cdot 3 \cdot 2} \cdot 1} = 15 \cdot 7 \cdot 13 \cdot 11 \cdot 10 \cdot 9$

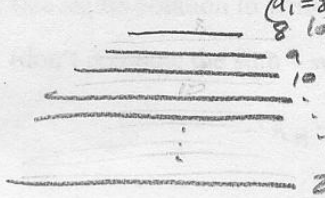
$= \boxed{1351350}$

(b) $\frac{(n+2)!}{(n-1)!} = \frac{(n+2)(n+1)(n)\cancel{(n-1)(n-2)\ldots 1}}{\cancel{(n-1)(n-2)\ldots 1}}$

$= \boxed{(n+2)(n+1)(n)}$

or $(n^2+3n+2)(n) = \boxed{n^3+3n^2+2n}$

9. Logs are stacked in a pile. Each row of logs has one less log than the row below it. The top row has 8 logs and the bottom row has 20 logs. How many total logs are in the pile?



arithmetic

$a_1 = 8$

number of rows

$(20-8)+1$

$n = 13$ rows

$S = \frac{n}{2}(a_1 + a_n)$

$S = \frac{13}{2}(8+20)$

$\boxed{S = 182 \text{ logs}}$

10. A city of 150,000 people is growing at a rate of 5% per year. The city's population can be modeled using a geometric sequence.

(a) Write a formula for the population, P, of the city versus t if P=150,000 when t=1.

(b) Use this formula to find the population when t=10.

(a) $P = P_0 (r)^t$

$\boxed{P = 150{,}000\,(1.05)^t}$

(b) $P = 150{,}000\,(1.05)^{10} = \boxed{244{,}334}$

Honors Algebra 3-4 Name____________________

9.1-9.3 Review #2 Period ______

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$S = \frac{n}{2}(a_1 + a_n)$ $\qquad$ $S = a_1\left(\frac{1-r^n}{1-r}\right)$ $\qquad$ $S = \frac{a_1}{1-r}$

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3. Use sigma notation to write the given sum: $3+1-1-3-5$
 (don't compute the sum – write using a $\sum$ and assume n begins with 1)

4. Find the sums:
 (a) $\sum_{n=1}^{4} \frac{3n^2}{n+1}$ $\qquad$ (b) $\sum_{n=0}^{\infty} 10\left(\frac{1}{2}\right)^n$

5. Given $a_3 = -2$, $a_6 = -11$ for an ***arithmetic sequence*** find:
 (a) d $\qquad$ (b) a_1 $\qquad$ (c) a_n $\qquad$ (d) a_{100}

6. Given $a_2 = 8,\ a_5 = 64$ for an ***geometric sequence*** find:

(a) r (b) a_1 (c) a_n (d) a_8

7. Find the sum of the first 40 terms of the sequence which begins: $1, 5, 9, 13, 17...$

8. Simplify each ratio fully:

(a) $\dfrac{15!}{8!4!}$ (b) $\dfrac{(n+2)!}{(n-1)!}$

9. Logs are stacked in a pile. Each row of logs has one less log than the row below it. The top row has 8 logs and the bottom row has 20 logs. How many total logs are in the pile?

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