HAlg34 sem 1 final review (open ended)

1a $21x^4 - 14x^3 + 35x^2 + 7x$

$\boxed{7x(3x^3 - 2x^2 + 5x + 1)}$

(grouping doesn't work)

1b $64x^2 - 49y^2$

$(8x)^2 - (7y)^2$

$a^2 - b^2$

$\boxed{(8x+7y)(8x-7y)}$

1c $100x^2 - 60xy + 9y^2$

$(10x)^2 \qquad (3y)^2$

$a \qquad b$

$2ab$ would be $2(10x)(3y)$

$60xy$ ✓

$a^2 - 2ab + b^2$ pattern

$= (a-b)^2$

$\boxed{(10x-3y)^2}$

1d $x^2 - 5x - 24$

$\boxed{(x-8)(x+3)}$

m	A
-24	-5
$(-8)(3)$	$-8+3$

1e $2x^2 + 11x + 15$

$\frac{(2x+6)}{2}\frac{(2x+5)}{1}$

$\boxed{(x+3)(2x+5)}$

m	A
30	11
$(6)(5)$	$6+5$

1f $xy - 3y - 4x + 12$

$(xy - 3y) + (-4x + 12)$

$y(x-3) - 4(x-3)$

$\boxed{(x-3)(y-4)}$

2 $15x^2 - 17x = 4$

$15x^2 - 17x - 4 = 0$

$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(15)(-4)}}{2(15)}$

$x = \frac{17 \pm \sqrt{529}}{30}$

$x = \frac{17 \pm 23}{30} = \frac{40}{30}, \frac{-6}{30}$

$\boxed{\left(\frac{4}{3}\right), \left(\frac{-1}{5}\right)}$

3 $4x^2 + 20x + 23 = 0$

$x = \frac{-20 \pm \sqrt{(20)^2 - 4(4)(23)}}{2(4)}$

$x = \frac{-20 \pm \sqrt{32}}{8}$

$x = \frac{-20 \pm \sqrt{16}\sqrt{2}}{8}$

$x = \frac{-20 \pm 4\sqrt{2}}{8}$

$x = \frac{4(-5 \pm \sqrt{2})}{4(2)} = \boxed{\frac{-5 \pm \sqrt{2}}{2}}$

4 $(4x-1)^2 = 20$

$\sqrt{(4x-1)^2} = \pm\sqrt{20}$

$4x - 1 = \pm\sqrt{20}$

$4x - 1 = \pm\sqrt{4}\sqrt{5} = \pm 2\sqrt{5}$

$4x = 1 \pm 2\sqrt{5}$

$x = \boxed{\frac{1 \pm 2\sqrt{5}}{4}}$

5a $\sqrt{3-4x} = 2x$

$(\sqrt{3-4x})^2 = (2x)^2$

$3 - 4x = 4x^2$

$4x^2 + 4x - 3 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-3)}}{2(4)}$

$x = \frac{-4 \pm \sqrt{64}}{8}$

$x = \frac{-4 \pm 8}{8}, \frac{4}{8}\left(\frac{1}{2}\right), \frac{-12}{8}\left(\frac{-3}{2}\right)$

checking:

$\sqrt{3-4\left(\frac{1}{2}\right)} = 2\left(\frac{1}{2}\right)$

$\sqrt{3-2} = 1$

$1 = 1$ ✓

$\boxed{\frac{1}{2}}$

$\sqrt{3-4\left(\frac{-3}{2}\right)} = 2\left(\frac{-3}{2}\right)$

$\sqrt{3+6} = -3$

$3 = -3$ ✗

no

(5b) $\sqrt{x-2} - x = -8$

$\sqrt{x-2} = x-8$

$(\sqrt{x-2})^2 = (x-8)^2$

$x-2 = x^2-16x+64$

$x^2-17x+66=0$

$x = \frac{-(-17)\pm\sqrt{(17)^2-4(1)(66)}}{2(1)}$

$x = \frac{17\pm\sqrt{25}}{2}, \quad x = \frac{17\pm5}{2}$

11, 6

$\sqrt{11-2}-11=-8$ ✓ $\quad 3-11=-8$

$\sqrt{6-2}-6=-8$ $\quad 2-6=-8$ $\quad -4=-8$ X

11

(6) $\frac{4x}{x+1} + \frac{6x}{x-4} = 10$ list: $x+1$, $x-4$

$\frac{4x(x+1)(x-4)}{x+1} + \frac{6x(x+1)(x-4)}{x-4} = \frac{10(x+1)(x-4)}{1}$

$4x(x-4)+6x(x+1)=10(x+1)(x-4)$

$4x^2-16x+6x^2+6x=10(x^2-3x-4)$

$4x^2-16x+6x^2+6x=10x^2-30x-40$

$-10x=-30x-40$

$20x=-40$

$x=-2$

(7) $\frac{8}{x^2-16} + \frac{3}{4} = \frac{1}{x-4}$ list: $x-4$, $x+4$, 4

$\frac{8}{(x+4)(x-4)} + \frac{3}{4} = \frac{1}{x-4}$

$\frac{4\cdot8(x+4)(x-4)}{(x+4)(x-4)} + \frac{3(x+4)(x-4)\cdot4}{4} = \frac{4(x+4)(x-4)\cdot4}{x-4}$

$32+3(x+4)(x-4)=4(x+4)$

$32+3(x^2-16)=4x+16$

$32+3x^2-48=4x+16$

$3x^2-4x-32=0$

m	a
−96	−4
(8)(−12)	−12+8

$\frac{(3x-12)}{3}\frac{(3x+8)}{1}$

$(x-4)(3x+8)=0$

$x\neq4$ (denom zero) $\quad 3x=-8$, $x=-\frac{8}{3}$

(8) $\frac{1}{2}|2x+4|<7$

$|2x+4|<14$ ← inside case, double inequality

$-14<2x+4<14$

$-4 \quad -4 \quad -4$

$\frac{-18}{2}<\frac{2x}{2}<\frac{10}{2}$

$-9<x<5$

$(-9,5)$

(9) $\frac{1}{3}|2x-5|\geq17$

$|2x-5|\geq51$ ← outside case, 2 inequalities

$2x-5\leq-51$ $\quad +5 \quad +5$

$\frac{2x}{2}\leq\frac{-46}{2}$

$x\leq-23$

$2x-5\geq51$ $\quad +5 \quad +5$

$\frac{2x}{2}\geq\frac{56}{2}$

$x\geq28$

$(-\infty,-23]\cup[28,\infty)$

(10a) $\left(\frac{16}{625}\right)^{-\frac{3}{4}}$

$\left(\frac{625}{16}\right)^{3/4}$

$\left(\left(\frac{625}{16}\right)^{1/4}\right)^3$

$\left(\frac{\sqrt[4]{625}}{\sqrt[4]{16}}\right)^3$

$\left(\frac{5}{2}\right)^3$

$\frac{125}{8}$

(10b) $(5y^3)^{-4}(15x^4y^2)^3$

$\frac{(15x^4y^2)^3}{(5y^3)^4}$

$\frac{15^3x^{12}y^6}{5^4y^{12}}$

$\frac{3375x^{12}}{625y^6}$

$\frac{27x^{12}}{5y^6}$

$\frac{3375}{625}$ reduce w/ calc. MATH ▸ FRAC

(11a) $h(x)=3x^4-2x^3+x^2-2$

$h(3)=3(3)^4-2(3)^3+(3)^2-2$

$=\boxed{196}$

—or— synthetic division (placeholder)

3	3	−2	1	0	−2
		9	21	66	198
	3	7	22	66	196

(11b) $h(-2)$

synthetic division

−2	3	−2	1	0	−2
		−6	16	−34	68
	3	−8	17	−34	66 ✓

(12) $f(x)=\frac{1}{4}(x-1)$

$g(x)=4x+1$

$f(g(x))$

$f(4x+1)=\frac{1}{4}([4x+1]-1)$

$=\frac{1}{4}(4x)$

$=\boxed{x}$

(13) $f(x)=3x^2-2x+1$

$g(x)=x-4$

$f(g(-4))$ $\quad g(-4)=(-4)-4=-8$

$f(-8)=3(-8)^2-2(-8)+1$

$=3\cdot 64+16+1$

$=\boxed{209}$

(14) $f(x)=2x+5$

$g(x)=x^2-3x+1$

(a) $(f+g)(x)=f(x)+g(x)$

$=2x+5+x^2-3x+1$

$=\boxed{x^2-x+6}$

(b) $(g-f)(x)=g(x)-f(x)$

$=(x^2-3x+1)-(2x+5)$

$=x^2-3x+1-2x-5$

$=\boxed{x^2-5x-4}$

(15) Domain:

(a) $\frac{2x+5}{x^2-9}$

$x^2-9\neq 0$

$x^2\neq 9$

$x\neq\pm 3$

$\boxed{\mathbb{R},\ x\neq 3,-3}$

(b) $\sqrt{7x-35}$

$7x-35\geq 0$

$7x\geq 35$

$x\geq 5$

$\boxed{[5,\infty)}$

(c) $\frac{2}{\sqrt{7-x}}$ (no $\sqrt{-}$ and no zero in denom)

$7-x>0$

$-x>-7$

$x<7$

$\boxed{(-\infty,7)}$

(16) $f(x)=x^3+x^2-4x-4\leq 0$

plug into calculator, look for x intervals where $f(x)$ is negative (or equal to zero so square brackets)

$\boxed{(-\infty,-2]\cup[-1,2]}$

(17) $\sqrt{x}\rightarrow -4\sqrt{x+2}-5$

left 2
down 5
vertical flip (reflect about x-axis)
vertical stretch (4x bigger in y direction)

(18) $|x|\rightarrow -\frac{1}{8}|x-3|+7$

right shift 3
shift up 7
vertical flip (reflect about x-axis)
vertical shrink ($\frac{1}{8}$ as tall)

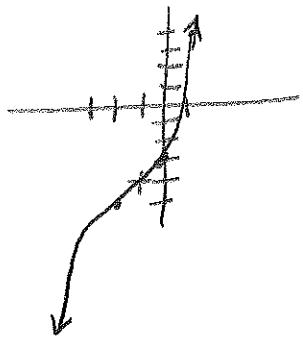
(19) graph is quadratic
- 'origin' at $(-2,-1)$
- flipped vertical $(-)$
- over 1 up 1, over 2 up 4 (squared)

basic shape $y=x^2$

$$\boxed{y=-(x+2)^2-1}$$

(20) $f(x)=(x+1)^3-4$

origin $(-1,-4)$ x^3 shape



(21a) $f(x)=16x^5-8x^3+4x+2$

$f(-x)=16(-x)^5-8(-x)^3+4(-x)+2$

$f(-x)=-16x^5+8x^3-4x+2$

$f(-x)=-(16x^5-8x^3+4x-2)$

$\boxed{\text{neither}}$

(21b) $f(x)=x^5-7x^2-23x$

$f(-x)=(-x)^5-7(-x)^2-23(-x)$

$f(-x)=-x^5-7x^2+23x$

$f(-x)=-(x^5+7x^2-23x)$

$\boxed{\text{neither}}$

(21c) $f(x)=6x^4-3x^2+1$

$f(-x)=6(-x)^4-3(-x)^2+1$

$f(-x)=6x^4-3x^2+1$

$f(-x)=f(x)$

$\boxed{\text{even}}$

(22) $g(x)=\frac{2x+3}{6}$

$y=\frac{2x+3}{6} \longmapsto x=\frac{2y+3}{6}$

$6x=2y+3$

$2y+3=6x$

$2y=6x-3$

$y=3x-\frac{3}{2}$

$\boxed{g^{-1}(x)=3x-\frac{3}{2}}$

(23) $f(x)=8x^3-1$

$y=8x^3-1$

$x=8y^3-1$

$8y^3-1=x$

$8y^3=x+1$

$y^3=\frac{x+1}{8}$

$y=\sqrt[3]{\frac{x+1}{8}}=\frac{\sqrt[3]{x+1}}{\sqrt[3]{8}}$

$y=\frac{1}{2}\sqrt[3]{x+1}$

$\boxed{f^{-1}(x)=\frac{1}{2}\sqrt[3]{x+1}}$

(24) x-int: $(y=0)$

$f(x)=\frac{6x^2-13x+5}{x^2-9}$

$0=\frac{6x^2-13x+5}{x^2-9}$

fraction = zero when numerator = zero

$6x^2-13x+5=0$

$(6x-10)(6x-3)$

$\frac{2}{2}\quad\frac{3}{3}$

$(3x-5)(2x-1)=0$

$3x-5=0 \quad 2x-1=0$

$3x=5 \quad 2x=1$

$x=\frac{5}{3} \quad x=\frac{1}{2}$

m	A
30	-13
$-10,-3$	$-10-3$

$\boxed{\left(\frac{5}{3},0\right)\left(\frac{1}{2},0\right)}$

(25) $5x^4+x^3-3x^2+5 \div x-1$

placeholder; zero when $x=1$

1	5	1	-3	0	5
		5	6	3	3
	5	6	3	3	8

$\boxed{5x^3+6x^2+3x+3+\frac{8}{x-1}}$

(26) $f(x)=\frac{2x^2}{3x^2+1}$ $n=2$ $m=2$

$n=m$

horiz. asymptote: $\boxed{y=\frac{2}{3}}$

(no slant asymptote)

(27) $f(x)=\frac{2x^2-3x+3}{x+2}$ $n=2$ $m=1$

$n>m$ (no horiz. asymptote)

greater by 1, slant asymptote exists.

$$x+2\overline{)\,2x^2-3x+3}\qquad 2x-7$$

$-(2x^2+4x)$

$-7x+3$

$-(-7x-14)$

17

Slant asymptote: $\boxed{y=2x-7}$

(28a) $f(x)=\frac{4}{\sqrt{x^2-9}}$

vertical asymptotes where there are "holes" in the domain

where: $x^2-9=0$

$x^2=9$

$x=\pm 3$

asymptotes are lines:

$\boxed{x=3,\ x=-3}$

(28b) $g(x)=\frac{x+4}{3x-5}$

VA. where denom $=0$

$3x-5=0$

$3x=5$

$\boxed{x=\frac{5}{3}}$

(29a) $(4+3i)+(8-12i)$

$\boxed{12-9i}$

(29b) $(4+3i)-(8-12i)$

$4+3i-8+12i$

$\boxed{-4+15i}$

(30a) $(2+3i)(4-2i)$

FOIL:

$8-4i+12i-6i^2$ $(i^2=-1)$

$8+8i-6(-1)$

$\boxed{14+8i}$

(30b) $\frac{(2+3i)}{(4-2i)}$

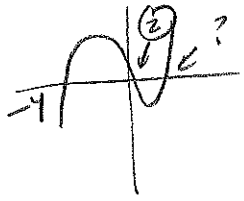
$\frac{(2+3i)(4+2i)}{(4-2i)(4+2i)}$ ← complex conjugate

$\frac{8+4i+12i+6i^2}{16+8i-8i-4i^2}$ $(i^2=-1)$

$\frac{8+16i-6}{16+4}=\frac{2+16i}{20}=\boxed{\frac{1}{10}+\frac{4}{5}i}$

(31) real zeros: $f(x)=2x^3-3x^2-30x+56$

graph w/ calculator:



Synth. div. to verify and factor:

$2\overline{)\,2\ -3\ -30\ 56}$

$\quad\ \ 4\ \ 2\ -56$

$2\ \ 1\ -28\ \ 0$ ✓

$-4\overline{)\,2\ \ 1\ -28}$

$\quad\ -8\ \ 28$

$2\ -7\ \ 0$ ✓

$2x-7$

$(x-2)(x+4)(2x-7)=0$

$x=2\quad x=-4$

$2x-7=0$

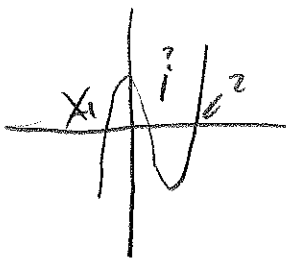
$2x=7$

$x=\frac{7}{2}$

$\boxed{2,-4,\frac{7}{2}}$

(32) real zeros: $f(x) = 10x^3 - 15x^2 - 16x + 12$

(same as 31) graph, synth division:



-1 | 10 −15 −16 12
−10 25 −9
10 −25 9 3 ✗

-1 not a zero, try 2

2 | 10 −15 −16 12
20 10 −12
10 5 −6 | 0 ✓

$(x-2)(10x^2+5x-6)$

quadratic formula

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(10)(-6)}}{2(10)}$

$x = \frac{-5 \pm \sqrt{265}}{20}$

zeros: $2, \frac{-5 \pm \sqrt{265}}{20}$

(33) zeros @ 0, −3, 2

a factor for each zero:

$f(x) = (x-0)(x+3)(x-2)$

$f(x) = x(x+3)(x-2)$

$f(x) = x(x^2 + x - 6)$

$f(x) = x^3 + x^2 - 6x$

(34) 4th deg poly, zeros @ $1, -1, 4i$

4th deg, 4 zeros, complex zeros occur in pairs so $-4i$ also

$f(x) = (x-1)(x+1)(x-4i)(x+4i)$

$f(x) = (x^2-1)(x^2-16i^2)$

$f(x) = (x^2-1)(x^2+16)$

$f(x) = x^4 + 15x^2 - 16$

(35a) zeros from graph:

−1 (multiplicity 2)

1 (multiplicity 2)

$f(x) = A(x-1)(x-1)(x+1)(x+1)$

$f(x) = A(x-1)^2(x+1)^2$

↑ could have multiplier here

plug in a point (0, 2)

$2 = A(0-1)^2(0+1)^2$

$2 = A(1)(1)$

$f(x) = 2(x-1)^2(x+1)^2$

(35b) zeros from graph:

−4 (mult 1)

1 (mult 1)

3 (mult 1)

$f(x) = A(x+4)(x-1)(x-3)$

goes through point (0, −3)

$-3 = A(0+4)(0-1)(0-3)$

$-3 = A(4)(-1)(-3)$

$-3 = A(12)$

$-\frac{1}{4} = A$

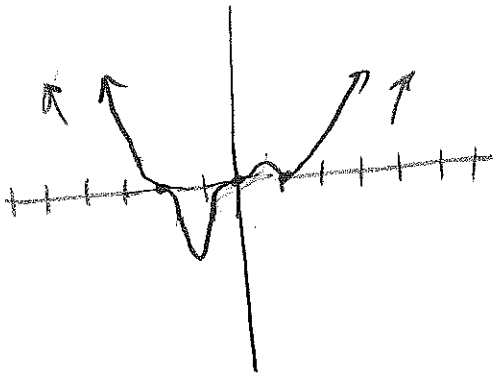
$f(x) = -\frac{1}{4}(x+4)(x-1)(x-3)$

(36) Sketch $f(x) = x^5(x+2)^3(x-1)^2$

zeros @ 0 (m5), −2 (m3), 1 (m2)

$f(x) = x^{10} + \cdots$

Rt ↑ Lft ↑



(37) $f(x)=x^3-4x^2-4x+16$

factor to find zeros:

$(x^3-4x^2)+(-4x+16)$

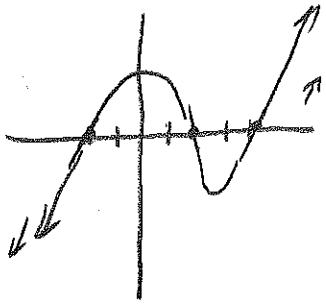
$x^2(x-4)-4(x-4)$

$(x-4)(x^2-4)=0$

$x=4 \quad x^2-4=0$

$x^2=4$

$x=\pm 2$



(38a) $a^x=b$ $\boxed{\log_a b=x}$

(38b) $\log_c t=a$ $\boxed{c^a=t}$

(39a) $\log_6 11=\frac{\log_{10}11}{\log_{10}6}=\boxed{1.34}$

(39b) $\log_8 3=\frac{\log_{10}3}{\log_{10}8}=\boxed{0.53}$

(39c) $\log_a x=\boxed{\frac{\log_{10}x}{\log_{10}a}}$

(40) $2\log 4+\log 3-\log 2$

$\log 4^2+\log 3-\log 2$

$\log\left(\frac{4^2\cdot 3}{2}\right)$

$\log\left(\frac{16\cdot 3}{2}\right)$

$\boxed{\log 24}$

(41) $\frac{1}{4}\left[2\log(x+3)-4\log x-\log(x^2-1)\right]$

$\frac{1}{4}\left[\log(x+3)^2-\log x^4-\log(x^2-1)\right]$

$\frac{1}{4}\log\left(\frac{(x+3)^2}{x^4(x^2-1)}\right)$

$\log\left(\frac{(x+3)^2}{x^4(x^2-1)}\right)^{1/4}$

$\boxed{\log\left(\sqrt[4]{\frac{(x+3)^2}{x^4(x^2-1)}}\right)}$

(42) $\log c+\log a+\log b+\log i+\log n$

$\log(c\cdot a\cdot b\cdot i\cdot n)$

$\log$ cabin ☺

(43) $\log\left(\frac{x(x+2)^2}{(x-4)^3(x-5)}\right)^4$

$4\log\left(\frac{x(x+2)^2}{(x-4)^3(x-5)}\right)$

$4\left[\log x+\log(x+2)^2-\log(x-4)^3-\log(x-5)\right]$

$\boxed{4\left[\log x+2\log(x+2)-3\log(x-4)-\log(x-5)\right]}$

(44) $8^x=9876$

$\log_8 9876=x$

$x=\log_8 9876=\boxed{\frac{\log 9876}{\log 8}}$

(45) $7^x=139$

$\log_7(7^x)=\log_7(139)$

$x=\log_7(139)=\frac{\log_{10}139}{\log_{10}7}=\boxed{2.54}$

(46) $5^{5x-3}=625$

$5^{5x-3}=5^4$

$5x-3=4$

$5x=7$

$\boxed{x=\frac{7}{5}}$

(47) $8^{3x}=32^{x+1}$

$(2^3)^{3x}=(2^5)^{x+1}$

$2^{9x}=2^{5x+5}$

$9x=5x+5$

$4x=5$

$\boxed{x=\frac{5}{4}}$

(48) $\log_{10} 10^{5x+2} = 87$

$10 = 10$ base

$\log_{10} 10^{5x+2} = 87$

$5x+2 = 87$

$5x = 85$

$\boxed{x = 17}$

(49) $\ln e^{4x-7} = 21$

$4x-7 = 21$

$4x = 28$

$\boxed{x = 7}$

(50) $\log_3(x^2-9) - \log_3(x+3) = 1$

$\log_3\left(\frac{x^2-9}{x+3}\right) = 1$

$3^1 = \frac{x^2-9}{x+3}$

$\frac{(x^2-9)}{(x+3)} \cdot \frac{3}{1}$

$x^2-9 = 3(x+3)$

$x^2-9 = 3x+9$

$x^2-3x-18 = 0$

$(x-6)(x+3) = 0$

$\boxed{x=6}$ $x \neq -3$ (log of negative)

(51) $\log(x+60) = \log(x+5) + \log(x)$

$\log(x+60) = \log(x(x+5))$

$x+60 = x(x+5)$

$x+60 = x^2+5x$

$x^2+4x-60 = 0$

$(x+10)(x-6) = 0$

$x \neq -10$ (log of negative) $\boxed{x=6}$

(52) \$15000 at 3.8% continuous compounding

years to triple?

t	A
0	15000
t	45000

$A = Pe^{rt}$

$A = 15000e^{.038t}$

$45000 = 15000e^{.038t}$

$3 = e^{.038t}$

$\ln 3 = \ln(e^{.038t})$

$\ln 3 = .038t$

$t = \frac{\ln 3}{.038} = \boxed{28.9 \text{ years}}$

(53) Find initial amt $r = .06$

daily ($n = 365$) 15000 @ 8 yrs

t	A
0	P
8	15000

$A = P\left(1+\frac{.06}{365}\right)^{365t}$

$15000 = P\left(1+\frac{.06}{365}\right)^{365(8)}$

$\frac{15000}{\left(1+\frac{.06}{365}\right)^{365(8)}} = \frac{P\left(1+\frac{.06}{365}\right)^{365(8)}}{\left(1+\frac{.06}{365}\right)^{365(8)}}$

$P = \frac{15000}{\left(1+\frac{.06}{365}\right)^{365(8)}} = \boxed{\$9282.12}$

(54) continuous, triple in 18 yrs

t	A
0	P
18	3P

$A = Pe^{rt}$

$3P = Pe^{r(18)}$

$3 = e^{r(18)}$

$\ln(3) = r(18)$

$r = \frac{\ln(3)}{18} = .06103$

$\boxed{6.1\%}$

(55) a. $25(x-3)^2+25(y+4)^2=100$

$25x^2+\dots+25y^2+\dots=100$ **circle**

b. $4(x-3)^2+25(y+4)^2=100$

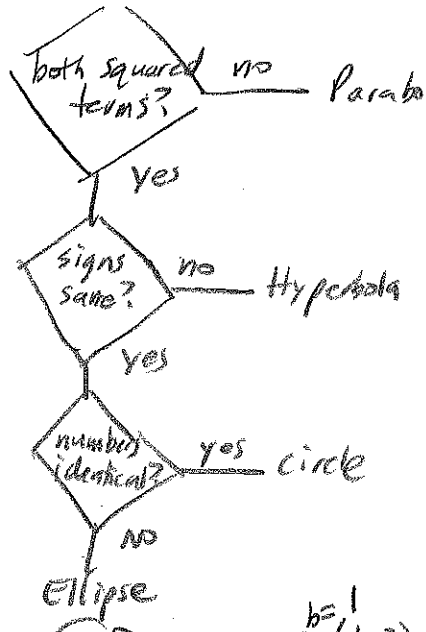
$4x^2+\dots+25y^2\dots$ **Ellipse**

c. $4(x-3)^2-25(y+4)^2=100$

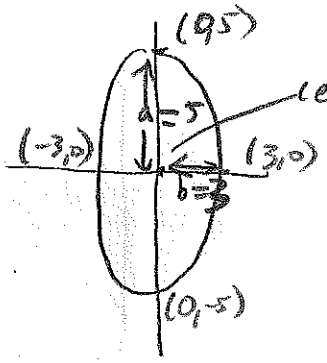
$4x^2+\dots-25y^2\dots$ **Hyperbola**

d. $4(x-3)+25(y+4)^2=10$

$4x\dots\dots 25y^2+\dots$ **Parabola**



(56a)



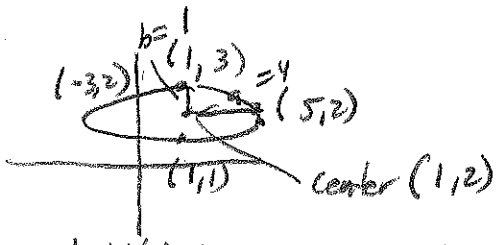
up/down y-major axis:

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

$$\frac{(x-0)^2}{9}+\frac{(y-0)^2}{25}=1$$

$$\boxed{\frac{x^2}{9}+\frac{y^2}{25}=1}$$

(56b)



left/right x-major axis:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$\boxed{\frac{(x-1)^2}{16}+\frac{(y-2)^2}{1}=1}$$

(57) $x^2-4x+8y+12=0$

$(x^2-4x+\underline{4})=-8y-12+\underline{4}$

$(x-2)^2=-8y-8$

$\boxed{(x-2)^2=-8(y+1)}$

$(x-h)^2=4p(y-k)$

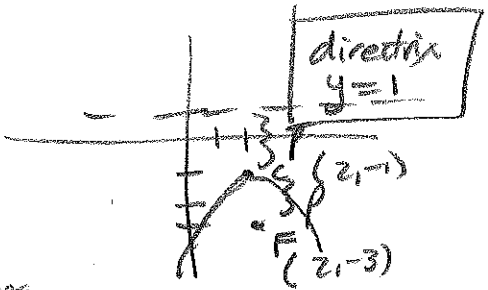
$\boxed{\text{center}=(2,-1)}$

$4p=-8$

$p=-2$

x^2 type, p neg ∩

$\boxed{\text{Focus}:(2,-3)}$



(58) $9x^2+4y^2-54x+40y+37=0$

$(9x^2-54x\quad)+(4y^2+40y\quad)=-37$

$9(x^2-6x+\underline{9})+4(y^2+10y+\underline{25})=-37+\underline{81}\ +\underline{100}$

$\frac{9(x-3)^2}{144}+\frac{4(y+5)^2}{144}=\frac{144}{144}$

$$\frac{(x-3)^2}{16}+\frac{(y+5)^2}{36}=1$$

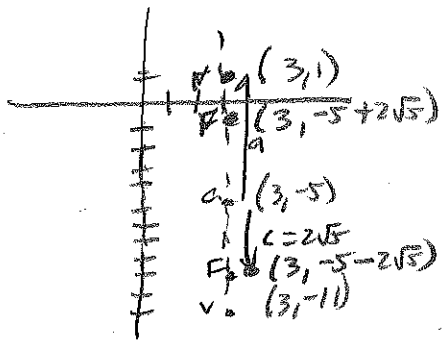
big number under y

y-major axis

Center: $(3,-5)$

Vertices: $(3,1)(3,-11)$

Foci: $(3,-5\pm2\sqrt{5})$



$\frac{(x-h)^2}{b^2}+\frac{(y+5)^2}{a^2}=1$

$b=4 \qquad a=6$

$c^2=a^2-b^2$

$c^2=36-16$

$c^2=20$

$c=\sqrt{20}=\sqrt{4}\sqrt{5}=2\sqrt{5}\approx 4.5$

(59) $9x^2-y^2-36x-6y+18=0$

$(9x^2-36x\quad)+(-y^2-6y\quad)=-18$

$9(x^2-4x+\underline{4})-(y^2+6y+\underline{9})=-18+\underline{36}\ -\underline{9}$

$\frac{9(x-2)^2}{9}-\frac{(y+3)^2}{9}=\frac{9}{9}$

$$\frac{(x-2)^2}{1}-\frac{(y+3)^2}{9}=1$$

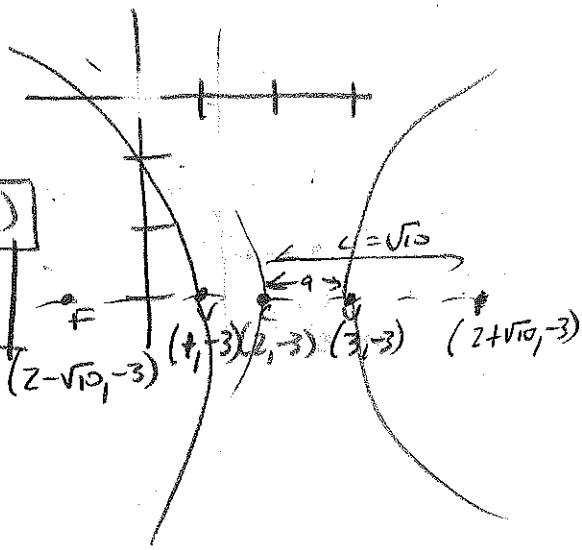
x term 1st

x-major axis

Center: $(2,-3)$

Vertices: $(3,-3)(1,-3)$

Foci: $(2\pm\sqrt{10},-3)$



$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

$a=1 \qquad b=3$

$c^2=a^2+b^2$

$c^2=1+9$

$c^2=10$

$c=\sqrt{10}\approx 3.2$

(60) V: $(-2,-2)$ $(4,-2)$

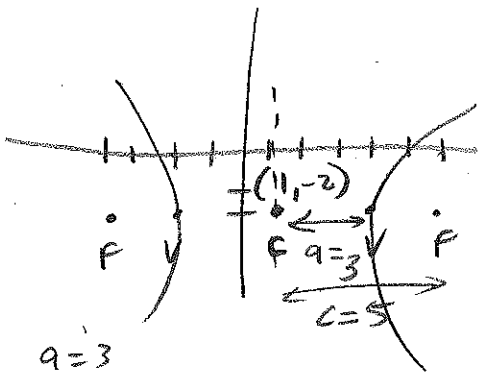
F: $(-4,-2)$ $(6,-2)$

center: $(1,-2)$

Hyperbola, x major axis

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\boxed{\frac{(x-1)^2}{9} - \frac{(y+2)^2}{16} = 1}$$



$a=3$

$k=5$

$c^2=a^2+b^2$

$25=9+b^2$

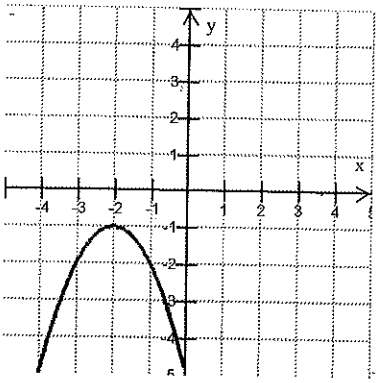
$b^2=16$

Name _____ Period _____

Honors Algebra 3-4 Fall Semester Open-Ended Review

1. Factor each of the following completely:
   a. $21x^4 - 14x^3 + 35x^2 + 7x$
   b. $64x^2 - 49y^2$
   c. $100x^2 - 60xy + 9y^2$
   d. $x^2 - 5x - 24$
   e. $2x^2 + 11x + 15$
   f. $xy - 3y - 4x + 12$
2. Solve for x: $15x^2 - 17x = 4$
3. Solve for x: $4x^2 + 20x = -23$
4. Solve for x: $(4x-1)^2 = 20$
5. Solve for x: a. $\sqrt{3-4x} = 2x$ b. $\sqrt{x-2} - x = -8$
6. Solve for x: $\frac{4x}{x+1} + \frac{6x}{x-4} = 10$.
7. Solve for x: $\frac{8}{x^2-16} + \frac{3}{4} = \frac{1}{x-4}$.
8. Solve for x: $\frac{1}{2}|2x+4| < 7$.
9. Solve for x: $\frac{1}{3}|2x-5| \geq 17$.
10. Simplify the following expressions: a. $\left(\frac{16}{625}\right)^{-\frac{3}{4}}$ b. $(5y^3)^{-4}(15x^4y^2)^3$
11. For $h(x) = 3x^4 - 2x^3 + x^2 - 2$, find a. $h(3)$ b. $h(-2)$
12. If $f(x) = \frac{1}{4}(x-1)$ and $g(x) = 4x+1$, find $f(g(x))$.
13. If $f(x) = 3x^2 - 2x + 1$ and $g(x) = x - 4$, find $f(g(-4))$.
14. Given $f(x) = 2x+5$ and $g(x) = x^2 - 3x + 1$, find each of the following
    a. $(f+g)(x)$.
    b. $(g-f)(x)$
15. Find the domain of the following: a. $\frac{2x+5}{x^2-9}$ b. $\sqrt{7x-35}$ c. $f(x) = \frac{2}{\sqrt{7-x}}$.
16. Find all the values of x such that $f(x) = x^3 + x^2 - 4x - 4 \leq 0$.
17. Describe the transformation of the graph of $f(x) = \sqrt{x}$ for the graph of $g(x) = -4\sqrt{x+2} - 5$.
18. Describe the transformation of the graph of $f(x) = |x|$ for the graph of $g(x) = -\frac{1}{8}|x-3| + 7$.

19. Write the equation for the given graph.



20. Graph the following: $f(x)=(x+1)^3-4$

21. Is the following function even, odd or neither?

a. $f(x)=16x^5-8x^3+4x+2$ b. $f(x)=x^5-7x^2-23x$ c. $f(x)=6x^4-3x^2+1$

22. Find the inverse of: $g(x)=\dfrac{2x+3}{6}$.

23. Find the inverse of: $f(x)=8x^3-1$

24. Find the x-intercept(s) of: $f(x)=\dfrac{6x^2-13x+5}{x^2-9}$

25. Use synthetic division to divide: $(5x^4+x^3-3x^2+5)\div(x-1)$.
(Write your answer as a polynomial with a fractional remainder)

26. Identify the horizontal or slant asymptote(s) of: $f(x)=\dfrac{2x^2}{3x^2+1}$

27. Identify the horizontal or slant asymptote(s) of: $f(x)=\dfrac{2x^2-3x+3}{x+2}$

28. Find the vertical asymptote(s) for the following:

a. $f(x)=\dfrac{4}{\sqrt{x^2-9}}$. b. $g(x)=\dfrac{x+4}{3x-5}$.

29. Simplify the following and express your answers in standard form:

a. $(4+3i)+(8-12i)$ b. $(4+3i)-(8-12i)$

30. Simplify the following and express your answers in standard form: (no decimals)

a. $(2+3i)(4-2i)$ b. $\dfrac{2+3i}{4-2i}$

31. Find the real zeros of the function: $f(x)=2x^3-3x^2-30x+56$.

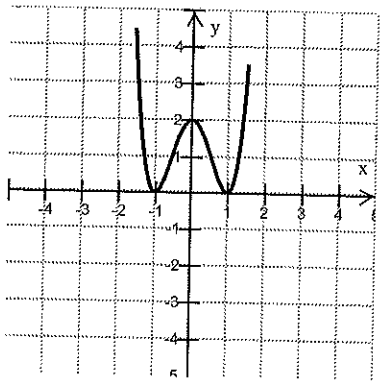
32. Find the real zeros of the function: $f(x)=10x^3-15x^2-16x+12$

33. Find a polynomial function that has the zeros 0, -3, and 2.

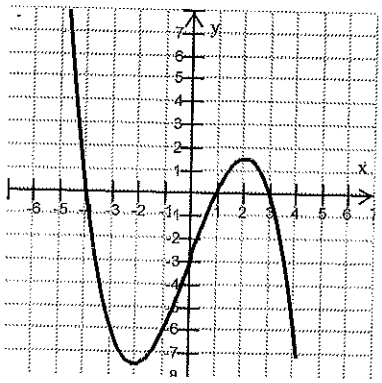
34. Find a 4th degree polynomial that has the following zeros: 1, -1, $4i$

35. Write the equations for the following:

a. 4th Degree Polynomial



b. 3rd Degree Polynomial



36. Sketch the graph of: $f(x)=x^5(x+2)^3(x-1)^2$.

37. Find the zeros and use them to sketch the graph of: $f(x)=x^3-4x^2-4x+16$

38. a. Write $a^x=b$ in logarithmic form. b. Write $\log_c t=a$ in exponential form.

39. Use the change of base formula to evaluate the following:
(to the nearest hundredth, when appropriate)
a. $\log_6 11$ b. $\log_8 3$ c. $\log_a x$

40. Rewrite 2log4 + log3 – log2 as a single logarithm.

41. Write the expression as the logarithm of a single quantity: $\frac{1}{4}[2\log(x+3)-4\log x-\log(x^2-1)]$

42. Write the expression as the logarithm of a single quantity: $\log c+\log a+\log b+\log i+\log n$

43. Write as a sum, difference, or multiple of logarithms: $\log\left(\frac{x(x+2)^2}{(x-4)^3(x-5)}\right)^4$

44. Solve $8^x=9876$ for x in common log form (Do not use a calculator).

45. Solve $7^x=139$ for x to the nearest hundredth.

46. Solve for x: $5^{5x-3}=625$

47. Solve for x: $8^{3x}=32^{x+1}$

48. Solve for x: $\log 10^{5x+2}=87$

49. Solve for x: $\ln e^{4x-7}=21$

50. Solve for x: $\log_3(x^2-9)-\log_3(x+3)=1$

51. Solve for x: $\log(x+60)=\log(x+5)+\log x$

52. Suppose you invest $15,000 at 3.8% interest compounded continuously. How many years will it take to triple your money?

53. Find the initial amount invested at 6% compounded daily if, after 8 years, it has grown to $15,000.

54. Determine the annual rate of interest compounded continuously for the sum of money in an account to triple in 18 years.

55. Identify the conic:

a. $25(x-3)^2+25(y+4)^2=100$

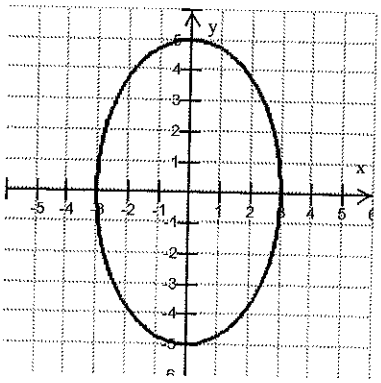
b. $4(x-3)^2+25(y+4)^2=100$

c. $4(x-3)^2-25(y+4)^2=100$

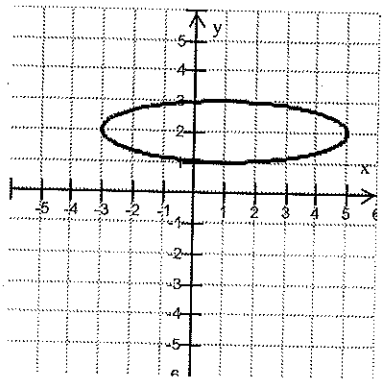
d. $4(x-3)+25(y+4)^2=100$

56. Write the equations that represent the graphs for the given ellipses.

a.



b.



57. Rewrite the equation of the parabola in standard form, then find the vertex, focus and directrix:
$x^2-4x+8y+12=0$

58. Rewrite the equation of the ellipse in standard form, then find the center, vertices and foci:
$9x^2+4y^2-54x+40y+37=0$

59. Rewrite the equation of the hyperbola in standard form, then find the center, vertices and foci:
$9x^2-y^2-36x-6y+18=0$

60. Find the equation of the hyperbola with vertices (-2,-2) and (4,-2), and foci (-4,-2) and (6,-2).

Honors Algebra 3-4 Fall Open-Ended Answer Key

1. a. $7x(3x^3-2x^2+5x+1)$

b. $(8x+7y)(8x-7y)$

c. $(10x-3y)^2$

d. $(x-8)(x+3)$

e. $(x+3)(2x+5)$

f. $(x-3)(y-4)$

2. $-\frac{1}{5}, \frac{4}{3}$ **3.** $\frac{-5\pm\sqrt{2}}{2}$ **4.** $\frac{1\pm2\sqrt{5}}{4}$

5. a. $\frac{1}{2}$ b. 11 **6.** $x = -2$ **7.** $x=-\frac{8}{3}$

8. (-9, 5) **9.** $(-\infty,-23]\cup[28,\infty)$ **10.** a. $\frac{125}{8}$ b. $\frac{27x^{12}}{5y^6}$

11. a. 196 b. 66 **12.** x **13.** 209

14. a. x^2-x+6 b. x^2-5x-4

15. a. All real numbers except ± 3 b. $[5,\infty)$ c. $(-\infty,7)$

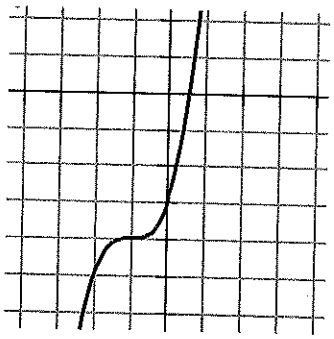
16. $(-\infty,-2]\cup[-1,2]$

17. Horizontal shift 2 to the left, vertical stretch, reflection over the x-axis, and down 5.

18. Horizontal shift 3 to the right, vertical shrink, reflection over the x-axis, and up 7.

19. $f(x)=-(x+2)^2-1$

20.



21. a. neither b. neither c. even

22. $g^{-1}(x)=3x-\frac{3}{2}$

23. $f(x)=\frac{1}{2}\sqrt[3]{x+1}$

24. $\left(\frac{5}{3},0\right) and \left(\frac{1}{2},0\right)$

25. $5x^3+6x^2+3x+3+\frac{8}{x-1}$

26. $y=\frac{2}{3}$

27. $y=2x-7$

28. a. $x=\pm 3$ b. $x=\frac{5}{3}$

29. a. $12-9i$ b. $-4+15i$

30. a. $14+8i$ b. $\frac{1}{10}+\frac{4}{5}i$

31. -4, 3.5, 2

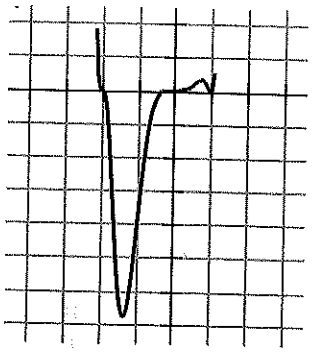
32. 2 and $\frac{-5\pm\sqrt{265}}{20}$

33. $f(x)=x^3+x^2-6x$

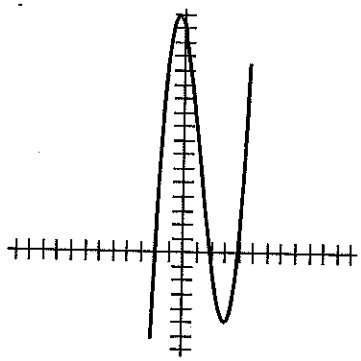
34. x^4+15x^2-16

35. a. $f(x)=2(x+1)^2(x-1)^2$ b. $f(x)=-\frac{1}{4}(x+4)(x-1)(x-3)$

36.



37. -2, 2, 4



38. a. $\log_a b=x$ b. $c^a=t$

39. a. 1.34 b. .53 c. $\frac{\log x}{\log a}$

40. $\log 24$ **41.** $\log 4\sqrt{\frac{(x+3)^2}{x^4(x^2-1)}}$ **42.** log cabin

43. $4[\log x+2\log(x+2)-3\log(x-4)-\log(x-5)]$

44. $\frac{\log 9876}{\log 8}$ **45.** 2.54 **46.** $\frac{7}{5}$ **47.** $\frac{5}{4}$

48. 17 **49.** 7 **50.** $x=6$ **51.** $x=6$

52. 28.91 years **53.** \$9,282.12 **54.** 6.1%

55. a. Circle b. Ellipse c. Hyperbola d. Parabola

56. a. $\frac{x^2}{9}+\frac{y^2}{25}=1$ b. $\frac{(x-1)^2}{16}+(y-2)^2=1$

57. $(x-2)^2=-8(y+1)$; Vertex: (2, -1); Focus: (2, -3); Directrix: $y=1$

58. $\frac{(x-3)^2}{16}+\frac{(y+5)^2}{36}=1$; Center: (3, -5); Vertices: (3, 1) and (3, -11): and Foci: $3,-5\pm2\sqrt{5}$

59. $(x-2)^2-\frac{(y+3)^2}{9}=1$; Center: (2, -3); Vertices: (3, -3) and (1, -3); Foci: $2\pm\sqrt{10},-3$ and equations of asymptotes: $y=-3\pm3(x-2)$

60. $\frac{(x-1)^2}{9}-\frac{(y+2)^2}{16}=1$