HAlg 34 open-ended sem 2 final review

(1a) $\theta = \frac{5\pi}{8}$ → degs.

$\frac{5\pi}{8}\cdot\frac{180^\circ}{\pi} = \frac{225}{2}^\circ = 112.5^\circ$

(1b) $\theta = 175^\circ$ → rad

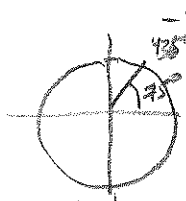
$175^\circ \cdot \frac{\pi}{180^\circ} = \frac{175}{180} = \boxed{\frac{35}{36}\pi}$

(2) $\csc\theta = \frac{25}{24}$, $\cos\theta = ?$

$\frac{1}{\sin\theta} = \frac{25}{24}$, $\sin\theta = \frac{24}{25} = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp} = \boxed{\frac{7}{25}}$

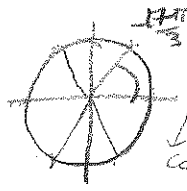
(3a) ref ∠ for $\theta = 435^\circ$



to nearest x-axis

$\boxed{\theta' = 75^\circ}$

(3b) $\theta = \frac{-17\pi}{3}$



$\boxed{\theta' = \frac{\pi}{3}}$

(4) find θ, $0^\circ \le \theta \le 360^\circ$, nearest deg

(a) $\tan\theta = 3.8958$

$\theta = \tan^{-1}(3.8958) = \boxed{76^\circ}, \boxed{256^\circ}$

(b) $\sin\theta = .2654$

$\theta = \sin^{-1}(.2654) = \boxed{15^\circ}, \boxed{165^\circ}$

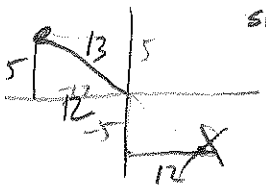
$\sec\theta = 2.0913$

$\cos\theta = \frac{1}{2.0913}$

$\theta = \cos^{-1}\left(\frac{1}{2.0913}\right) = \boxed{61^\circ}, \boxed{299^\circ}$

(5) $\cot\theta = \frac{-12}{5}$, $\cos\theta < 0$, find $\csc\theta = ?$

$\tan\theta = \frac{-5}{12} = \frac{y}{x}$

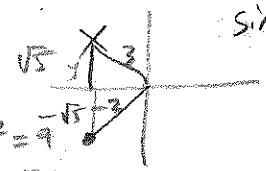


$\sin\theta = \frac{y}{r} = \frac{5}{13}$

$\csc\theta = \frac{1}{\sin\theta} = \boxed{\frac{13}{5}}$

(6) $\sec\theta = \frac{-3}{2}$ $\cot\theta > 0$ $\sin\theta = ?$

$\cos\theta = \frac{-2}{3} = \frac{x}{r}$



$4 + y^2 = 9$

$y^2 = 5$

$y = \sqrt{5}$

$\sin\theta = \frac{y}{r} = \boxed{\frac{-\sqrt{5}}{3}}$

(7)

$\sin\theta = \frac{opp}{hyp} = \frac{h}{100}$

$h = 100\sin 57^\circ = \boxed{83.9\text{ ft}}$

(8)

$\tan\theta = \frac{75}{625}$

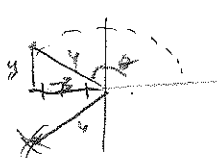
$\theta = \tan^{-1}\left(\frac{75}{625}\right) = \boxed{6.8^\circ}$

(9) Find exact value

(a) $\csc\left[\cos^{-1}\left(\frac{-3}{4}\right)\right]$

θ

$\cos\theta = \frac{-3}{4} = \frac{x}{r}$



$(3)^2 + y^2 = 4^2$

$9 + y^2 = 16$

$y^2 = 7$

$y = \sqrt{7}$

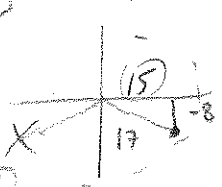
$\sin\theta = \frac{y}{r} = \frac{\sqrt{7}}{4}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{4}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}} = \boxed{\frac{4\sqrt{7}}{7}}$

(b) $\tan\left[\sin^{-1}\left(\frac{8}{17}\right)\right]$

θ

$\sin\theta = \frac{8}{17} = \frac{y}{r}$



$\cos\theta = \frac{15}{17}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{8/17}{15/17} = \boxed{\frac{-8}{15}}$

(10) Find period in degrees

$f(x) = \cos\left(\frac{1}{4}x\right)$

(a) $0 \le \frac{1}{4}x \le 2\pi$

$0 \le x \le 8\pi$

$8\pi \frac{180^\circ}{\pi} = \boxed{1440^\circ}$

(b) $f(x) = \tan(6x)$

$0 \le 6x \le \pi$

$0 \le x \le \frac{\pi}{6}$

$\frac{\pi}{6} \frac{180^\circ}{\pi} = \boxed{30^\circ}$

(11) period $= \frac{2\pi}{3}$ sine

$0 \le x \le \frac{2\pi}{3}$

$0 \le 3x \le 2\pi$

$\boxed{-3\sin 3x}$

(12) period $= 4\pi$ cosine

$0 \le x \le 4\pi$

$0 \le \frac{x}{2} \le 2\pi$

$\boxed{2\cos\frac{x}{2}}$

(13) graph $f(x) = -3\cos\left(\frac{\pi}{2}x + \pi\right) + 1$

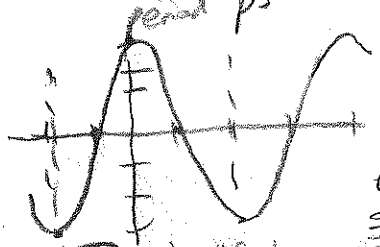
$0 \le \frac{\pi}{2}x + \pi \le 2\pi$

$-\pi \le \frac{\pi}{2}x \le 2\pi - \pi$

$-2\pi \le \pi x \le 4\pi - 2\pi$

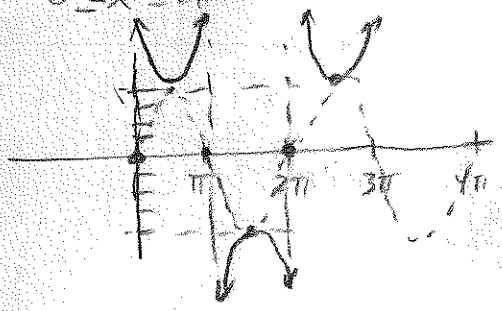
$-2 \le x \le 2$

cosine = cup



(14) graph $f(x) = 4\csc x$ $4\sin x$

$0 \le x \le 2\pi$



(15) simplify:

$\tan^2 x - \sin^2 x \tan^2 x$

$\tan^2 x(1 - \sin^2 x)$

$\tan^2 x \cos^2 x$

$\frac{\sin^2 x}{\cos^2 x} \frac{\cos^2 x}{1}$

$\boxed{\sin^2 x}$

(16) simplify:

$\frac{\sec^2 x - 1}{\sec x - 1}(\sec x + 1)$

$\frac{\tan^2 x(\sec x + 1)}{\sec^2 x - 1}$

$\frac{\tan^2 x(\sec x + 1)}{\tan^2 x}$

$\boxed{\sec x + 1}$

$\tan^2 + 1 = \sec^2$

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

(17) $\frac{1}{1+\cos x} + \frac{1}{1-\cos x}$

$\frac{1-\cos x}{(1+\cos x)(1-\cos x)} + \frac{1+\cos x}{(1+\cos x)(1-\cos x)}$

$\frac{1-\cos x + 1 + \cos x}{1 - \cos^2 x}$

$\frac{2}{\sin^2 x}$

$\boxed{2\csc^2 x}$

(18) $\cos x \sec x - \frac{\cos x}{\sec x}$

$\frac{\cos x \sec^2 x}{\sec x} - \frac{\cos x}{\sec x}$

$\frac{\cos x \sec^2 x - \cos x}{\sec x}$

$\frac{\cos x(\sec^2 x - 1)}{\sec x}$

$\frac{\cos x \tan^2 x}{\frac{1}{\cos x}}$

$\cos^2 x \tan^2 x$

$\cos^2 x \frac{\sin^2 x}{\cos^2 x}$

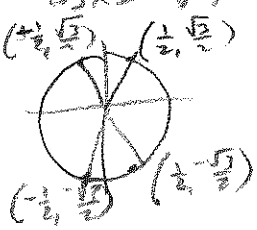
$\boxed{\sin^2 x}$

(19) $S = [0, 2\pi)$ $4\cos^2 x - 1 = 0$

$4\cos^2 x = 1$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm\sqrt{\frac{1}{4}} = \pm\frac{\sqrt{1}}{\sqrt{4}} = \pm\frac{1}{2}$



$\boxed{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

(0, 2π) $2\sin x\cos x = \sqrt{2}\sin x$

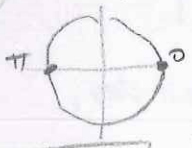
$\sin x\cos x - \sqrt{2}\sin x = 0$

$\sin x(2\cos x - \sqrt{2}) = 0$

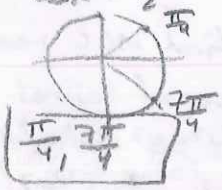
$\sin x = 0$ $\quad 2\cos x - \sqrt{2} = 0$

$2\cos x = \sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2}$



$0, \pi$



$\frac{\pi}{4}, \frac{7\pi}{4}$

(21) Solve: $\sec^2 x - \sec x = 2$

$\sec^2 x - \sec x - 2 = 0$

$u = \sec x$

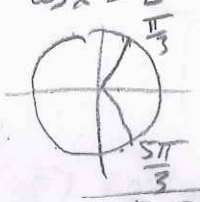
$u^2 - u - 2 = 0$

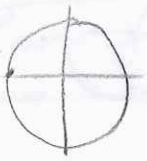
$(u-2)(u+1) = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x = 2$ $\quad \sec x = -1$

$\cos x = \frac{1}{2}$ $\quad \cos x = -1$





$\frac{\pi}{3}, \frac{5\pi}{3}, \pi$

(22) Solve: $[0, 2\pi)$ $2\sin^2 x - 5\sin x + 2 = 0$

$u = \sin x$

$2u^2 - 5u + 2 = 0$

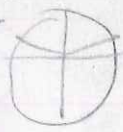
m	9
4	-5
	-1 -4

$(2u-1)(u-2) = 0$

$(\sin x - \frac{1}{2})(\sin x - 2) = 0$

$2\sin x = 1$ $\quad \sin x = 2$

$\sin x = \frac{1}{2}$ $\quad$ not possible



$\frac{\pi}{6}, \frac{5\pi}{6}$

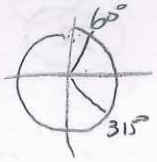
(23) $\cos 255°$ (use $255° = 315° - 60°$)

$\cos(255°) = \cos(315° - 60°)$ $\quad u = 315°$, $v = 60°$

$= \cos 315°\cos 60° + \sin 315°\sin 60°$

$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{-\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$



(24) $\sin 105°$ (use $105° = 150° - 45°$)

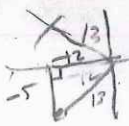
$\sin 105° = \sin(150° - 45°) = \sin 150°\cos 45° - \cos 150°\sin 45°$ $\quad u = 150°$, $v = 45°$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{-\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

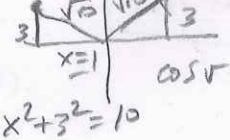
(25) $\cos u = \frac{-12}{13}$ $\quad \pi < u < \frac{3\pi}{2}$ $\quad \csc v = \frac{\sqrt{10}}{3}$, $\frac{\pi}{2} < v < \pi$ $\quad$ find $\cos(u+v)$

$\cos u = \frac{x}{r} = \frac{-12}{13}$



$\sin u = \frac{y}{r} = \frac{-5}{13}$

$\sin v = \frac{3}{\sqrt{10}} = \frac{y}{r}$



$x = 1$

$\cos v = \frac{x}{r} = \frac{-1}{\sqrt{10}}$

$x^2 + 3^2 = 10$

$x^2 = 1$

$x = 1$

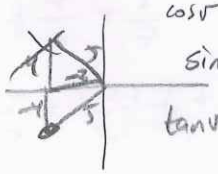
$\cos(u+v) = \cos u\cos v - \sin u\sin v$

$= \frac{-12}{13}\cdot\frac{-1}{\sqrt{10}} - \frac{-5}{13}\cdot\frac{3}{\sqrt{10}}$

$= \frac{12+15}{13\sqrt{10}} = \boxed{\frac{27\sqrt{10}}{130}}$

(26) $\cot u = \frac{2}{5}$ $\quad 0 < u < \frac{\pi}{2}$ $\quad \cos v = \frac{-3}{5}$ $\quad \pi < v < \frac{3\pi}{2}$ $\quad$ find $\tan(u+v)$

$\tan u = \frac{5}{2}$



$\cos v = \frac{x}{r} = \frac{-3}{5}$

$\sin v = \frac{y}{r} = \frac{-4}{5}$

$\tan v = \frac{\sin v}{\cos v} = \frac{-4/5}{-3/5} = 4/3$

$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u\tan v}$

$= \frac{\left(\frac{5}{2} + \frac{4}{3}\right)6}{\left(1 - \frac{5}{2}\cdot\frac{4}{3}\right)6} = \frac{15+8}{6-20}$

$= \boxed{\frac{23}{-14}}$

(27) Δ area, sides 5, 9, 10

Huron's formula

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2} = \frac{5+9+10}{2} = 12$

$A = \sqrt{12(12-5)(12-9)(12-10)}$

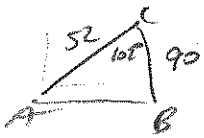
$= \sqrt{12 \cdot 7 \cdot 3 \cdot 2}$

$= \sqrt{4}\sqrt{3}\sqrt{7}\sqrt{3}\sqrt{2}$

$= 2 \cdot 3\sqrt{14}$

$= 6\sqrt{14} \approx 22.4$

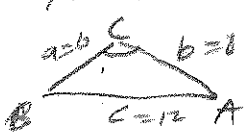
(28) ΔABC, BC = 90, AC = 52, m∠C = 102°, A



$A = \frac{1}{2}ab\sin C = \frac{1}{2}(90)(52)\sin 102°$

$A = 2288.9\ u^2$

(29) Δ a = 6, b = 8, c = 12 Find m∠C nearest deg,

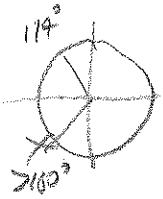


law of cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

$12^2 = 6^2 + 8^2 - 2(6)(8)\cos C$

$\cos C = -.458$



$C = 117°$

(30) Δ A = 115°, b = 15, c = 10 find a nearest 10th

law of cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 15^2 + 10^2 - 2(15)(10)\cos 115°$

$a = 21.3$

(31)

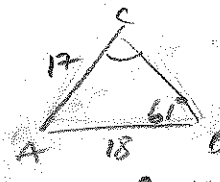
law of sines

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{x}{\sin 23°} = \frac{5.2}{\sin 44°}$

$x = 2.9$ in

(32) solve Δ: B = 61°, c = 18, b = 17



$\frac{18}{\sin C} = \frac{17}{\sin 61°}$

$\sin C = \frac{18\sin 61°}{17}$

$\sin C = .92607$

$C = 67.8°$ or $112.2°$

2 triangles:

$C = 67.8°$, $A = 51.2°$

$\frac{a}{\sin 51.2°} = \frac{17}{\sin 61°}$

$a = 15.1$

$C = 112.2°$, $A = 6.8°$

$\frac{a}{\sin 6.8°} = \frac{17}{\sin 61°}$

$a = 2.3$

(33) rewrite in trig form, deg & rad.

(a) -3

$r = \sqrt{a^2+b^2} = 3$

$\theta = 180°, \pi$

$-3 = 3(\cos 180° + i\sin 180°)$

$= 3(\cos\pi + i\sin\pi)$

(b) $-2 - 2i\sqrt{3}$

$r = \sqrt{(-2)^2 + (-2\sqrt{3})^2}$

$= \sqrt{4 + 4\cdot 3}$

$= \sqrt{16}$

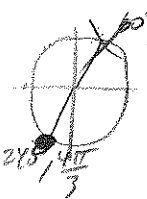
$= 4$

$\tan\theta = \frac{b}{a} = \frac{-2\sqrt{3}}{-2} = \frac{\sqrt{3}/2}{1/2}$, $\theta = 240°, \frac{4\pi}{3}$

$-2 - 2\sqrt{3}i =$

$4(\cos 240° + i\sin 240°)$

$4(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3})$



Pb

Rewrite to eval $[2(\cos 300^\circ + i\sin 300^\circ)]^6$

$= 2^6[\cos 6\cdot 300^\circ + i\sin 6\cdot 300^\circ]$

$= 64[\cos 1800^\circ + i\sin 1800^\circ)$

$\boxed{64(\cos 0^\circ + i\sin 0^\circ]}$

(35) Determine cube roots of $-27i$

$-27i = 27[\cos 270^\circ + i\sin 270^\circ]$

$\theta = 270^\circ$, $-27i$

1^{st} root $= \sqrt[n]{r}\left[\cos\frac{\theta}{n} + i\sin\frac{\theta}{n}\right]$

$= \sqrt[3]{27}\left[\cos\frac{270}{3} + i\sin\frac{270}{3}\right]$

$= \boxed{3(\cos 90^\circ + i\sin 90^\circ)}$

spacing $= \frac{360}{n} = 120$

$\boxed{2^{nd}\text{ root} = 3(\cos 210^\circ + i\sin 210^\circ)}$

$\boxed{3^{rd}\text{ root} = 3(\cos 330^\circ + i\sin 330^\circ)}$

(36) explicit formula for (arithmetic) $2, 5, 8, 11, 14$ ($a_1 = 2$; $+3\ +3\ +3\ +3 = d$)

$a_n = a_1 + (n-1)d$

$\boxed{a_n = 2 + (n-1)3}$

or

$a_n = 2 + 3n - 3$

$\boxed{a_n = 3n - 1}$

(37) formula for arithmetic seq. with $a_1 = 100$, $d = -8$

$\boxed{a_n = 100 - 8(n-1)}$

or

$a_n = 100 - 8n + 8$

$\boxed{a_n = 108 - 8n}$

(38) 83^{rd} term of arithmetic seq. $a_1 = 1.8$, $d = 2.4$

$a_n = 1.8 + 2.4(n-1)$

$a_{83} = 1.8 + 2.4(83-1) = \boxed{198.6}$

(39) Sum of 1st 21 terms of arithmetic seq.

$a_n = 2n + 5$ (start $n = 1$)

$a_1 = 2(1) + 5 = 7$

$a_2 = 2(2) + 5 = 9$ $\Rightarrow d = 2$

$a_{21} = 2(21) + 5 = 47$

$S_n = \frac{n}{2}(a_1 + a_n)$

$= \frac{21}{2}(7 + 47)$

$= \boxed{567}$

(40) Find Sum

(a) $\sum_{n=0}^{50}(1000 - 5n) = 1000 + \underbrace{995}_{\text{arithm.}}$

$d = -5$

first term $= 1000$

$a_{50} = 750$

arithm. $S_n = \frac{n}{2}(a_1 + a_n)$

$= \frac{51}{2}(1000 + 750)$

$= \boxed{44{,}625}$

(41) Evaluate

(a) $\sum_{n=1}^{\infty} 4\left(\frac{7}{3}\right)^{n-1}$ (r)

geometric seq. w/ $|r| > 1$

$\boxed{\text{diverges, Sum does not exist}}$

(b) $\sum_{n=0}^{\infty} 2\left(\frac{-2}{3}\right)^n$

geometric, $|r| < 1$, converges

$S = \frac{a_1}{1-r}$

$= \frac{2}{1-\left(-\frac{2}{3}\right)}$

$= \frac{2}{1+\frac{2}{3}} = \frac{2}{\frac{5}{3}}$

$= \frac{2}{1}\cdot\frac{3}{5} = \boxed{\frac{6}{5}}$

(b) $\sum_{n=2}^{5}\frac{4}{n+2} = \frac{4}{4} + \frac{4}{5} + \frac{4}{6} + \frac{4}{7}$

$\frac{4\cdot 5\cdot 6\cdot 7}{4\cdot 5\cdot 6\cdot 7} + \frac{4\cdot 4\cdot 6\cdot 7}{4\cdot 5\cdot 6\cdot 7} + \frac{4\cdot 4\cdot 5\cdot 7}{4\cdot 5\cdot 6\cdot 7} + \frac{4\cdot 4\cdot 5\cdot 6}{4\cdot 5\cdot 6\cdot 7}$

$\frac{840}{840} + \frac{672}{840} + \frac{560}{840} + \frac{480}{840} = \frac{2552}{840} = \frac{1276}{420}$

$= \frac{639}{210}$ $\boxed{\frac{319}{105}}$

(42) Find coefficient of:

(a) x^2y^3, exp. of $(2x-3y)^5$

term: 1 2 3 4 5

$\binom{5}{0}(2x)^5(-3y)^0 \quad \binom{5}{1}(2x)^4(-3y)^1 \quad \binom{5}{3}(2x)^2(-3y)^3$

$\binom{5}{3}(2x)^2(-3y)^3$

$10(4x^2)(-27y^3)$

$-1080x^2y^3$

$\boxed{-1080}$

(43) 1 card from deck

(a) $P(\text{red}) = \boxed{\frac{1}{2}}$

(b) $P(\text{King}) = \frac{4}{52} = \boxed{\frac{1}{13}}$

Pascal's triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

(44) pw 4 letters, 3 digits

order matters:

26 26 26 26 10 10 10

$= \boxed{456,976,000}$

(45) pw 5 letters, 2 digits,
letters O, I not used.

24 24 24 24 24 10 10

$= \boxed{79,626,240}$

(46) bag: 10 Q, 7 D, 5 N, 3 coin selected w/o replacement,
AND = multiply $P(3Q) = ?$

$P(3\text{quarters}) = P(1^{st}Q) \cdot P(2^{nd}Q) \cdot P(3^{rd}Q)$

$= \frac{10}{22} \cdot \frac{9}{21} \cdot \frac{8}{20} = \frac{720}{9240} = \boxed{\frac{6}{77}}$

(47) bag: 10 Q, 7 D, 5 N. 3 select w/o replace.
$P(1 \text{ of each})$

total # ways to select 3 coins: $_{22}C_3 = 1540$

# ways to choose a Q	# ways to choose a D	# ways to choose a N
$_{10}C_1$	$_7C_1$	$_5C_1$

$10 \cdot 7 \cdot 5 = 350$ $\quad P = \frac{350}{1540} = \boxed{\frac{5}{22}}$

(48) 12 people, 4 job opportunities
5 women (7 men) How many ways to hire

(a) random selection

$_{12}C_4 = \boxed{495}$

(b) exactly 2 women selected

$_5C_2 \cdot {}_7C_2 = 10 \cdot 21 = \boxed{210}$

(49) graph

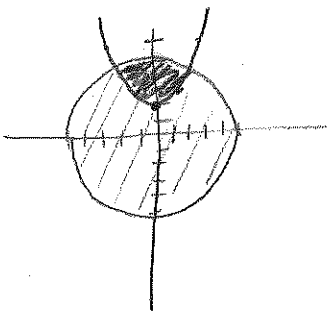
(a) $\begin{cases} x^2+y^2 \le 25 \\ y \ge x^2+2 \end{cases}$

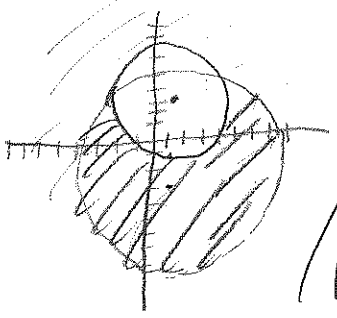
(b) $\begin{cases} (x-1)^2+(y-3)^2 \ge 16 \\ (x-1)^2+(y+3)^2 \le 36 \end{cases}$

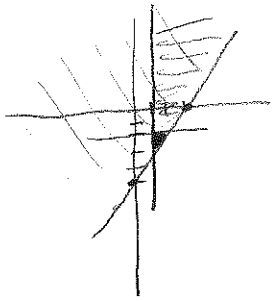
(50) graph

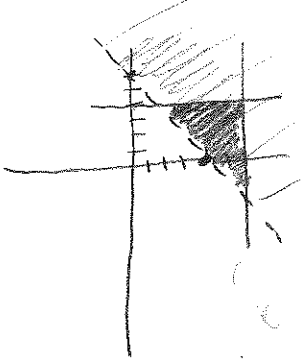
(a) $\begin{cases} 5x-3y \le 15 \\ x \ge 1 \\ y \le -2 \end{cases}$

(b) $\begin{cases} 3x+2y > 12 \\ x \le 6 \\ y \le 4 \end{cases}$









/p6

initial inv. $5000
$C = \$21.60$
ellipse = $34.10
first break even
$C = 5000 + 21.6x$
$R = 34.10x$
$34.1x = 5000 + 21.6x$
$12.5x = 5000$
$\boxed{x = 400}$

(52) Solve $\begin{cases} x+y=4 \\ x^2+y^2=4x \end{cases}$

$y = 4 - x$
$x^2 + (4-x)^2 = 4x$
$x^2 + 16 - 8x + x^2 = 4x$
$2x^2 - 12x + 16 = 0$
$x^2 - 6x + 8 = 0$
$(x-4)(x-2) = 0$
$x = 4 \quad x = 2$
$y = 0 \quad y = 4-2$
$\boxed{(4,0) \quad (2,2)}$

valid?

(53) Solve $\begin{cases} x^2+y^2=169 \\ x^2-8y=104 \end{cases}$

$x^2 = 169 - y^2$
$169 - y^2 - 8y = 104$
$-y^2 - 8y + 65 = 0$
$y^2 + 8y - 65 = 0$
$y = \frac{-8 \pm \sqrt{8^2 - 4(1)(-65)}}{2(1)}$
$y = \frac{-8 \pm \sqrt{324}}{2} = \frac{-8 \pm 18}{2}$
$y = 5$ or -13
$x^2 = 169 - (5)^2 \qquad x^2 = 169 - (-13)^2$
$x^2 = 144 \qquad x^2 = 0$
$x = \pm 12 \qquad x = 0$
$\boxed{(0,-13) \quad (12,5), \quad (-12,5)}$

(54) Solve $\begin{cases} 2x+3y+3z=3 \\ 6x-6y+12z=13 \\ 12x+9y-z=2 \end{cases}$

$\begin{bmatrix} 2 & 3 & 3 \\ 6 & -6 & 12 \\ 12 & 9 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 2 \end{bmatrix}$

$A \quad X = B$

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} .5 \\ -.13333 \\ 1 \end{bmatrix}$

$\boxed{\left(\frac{1}{2}, -\frac{1}{3}, 1\right)}$

(55) Solve $\begin{cases} -x+2y=1.5 \\ 2x-4y=3 \end{cases}$

$-x + 2y = 1.5$
$x = 2y - 1.5$
Substituting:
$2(2y-1.5) - 4y = 3$
$4y - 3 - 4y = 3$
$-3 = 3$
$\boxed{\text{No solution}}$

(56) Solve $2x+y-7z=4$
$2x-y-z=0$

Non-square
Gaussian elim.

$\left[\begin{array}{ccc|c} 2 & 1 & -7 & 4 \\ 2 & -1 & -1 & 0 \end{array}\right]$

$-R_1 + R_2 \left[\begin{array}{ccc|c} 2 & 1 & -7 & 4 \\ 0 & -2 & 6 & -4 \end{array}\right]$

$-2y + 6z = -4$
say $z = a$ (a real number)
$-2y + 6a = -4$
$-2y = -6a - 4$
$y = 3a + 2$
$2x + (3a+2) - 7(a) = 4$
$2x + 3a + 2 - 7a = 4$
$2x = 4a + 2$
$x = 2a + 1$
$\boxed{(2a+1, 3a+2, a)}$

use rref on calc. to simplify

(57) $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 6 & -3 & 2 \\ -4 & 1 & -2 \end{bmatrix}$

(a) $A+B = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 6 & -3 & 2 \\ -4 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 & -2 & 3 \\ -5 & 0 & 3 \end{bmatrix}$

(b) $5A - 4B \quad \begin{bmatrix} 10 & 5 & 5 \\ -5 & -5 & 25 \end{bmatrix} - \begin{bmatrix} 24 & -12 & 8 \\ -16 & 4 & -8 \end{bmatrix} = \begin{bmatrix} -14 & 17 & -3 \\ 11 & -9 & 33 \end{bmatrix}$

(58) (a) $\begin{bmatrix} -5 & 4 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 9 & 37 \\ 48 & 54 \end{bmatrix}$

$-15 + 24$
$5 + 32$
$6 + 42$
$-2 + 56$

(b) $\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} -3 & 8 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$

3×2 2×2

(c) 2×2 3×2 not possible

(59) Find A^{-1}

(a) $\left[\begin{array}{cc|cc} -2 & 7 & 1 & 0 \\ -4 & 11 & 0 & 1 \end{array}\right]$

(not simple) use calc

$\begin{bmatrix} 1.83\overline{3} & -1.1666\overline{6} \\ .6666\overline{6} & -.33\overline{3} \end{bmatrix}$

use frac> feature of calc

$\begin{bmatrix} \frac{11}{6} & -\frac{7}{6} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ -4 & 4 & 3 \end{bmatrix}$ by calc

$A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 7 & -\frac{17}{2} & 1 \\ -8 & 10 & -1 \end{bmatrix}$

(60) Find det.

(a) $\begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{bmatrix}\begin{matrix} -3 & 2 \\ 4 & 5 \\ 2 & -3 \end{matrix}$

$10 + 24 + 8 = 72$

$-15 + 24 - 12 = -3$

det = both $= -3 - 72 = -75$

(b) $\begin{bmatrix} 1 & -1 & 8 & 4 \\ 2 & 6 & 0 & 4 \\ 2 & 0 & 2 & 6 \\ 0 & 2 & 8 & 0 \end{bmatrix}$

by calc $= 16$

Name_____ Period_____

Spring Semester Open-Ended Final Exam Review – Honors Algebra 3-4

1. a. Convert to degrees: $\theta = \frac{5\pi}{8}$ radians. b. Convert to radians: $\theta = 175°$, leave answer in terms of π.

2. If the cosecant of an acute angle is $\frac{25}{24}$, find the value of the cosine.

3. Find the reference angle for: a. $\theta = 435°$ b. $\theta = \frac{-17\pi}{3}$

4. Given the following, find θ, $0° \le \theta < 360°$, to the nearest degree:
   a. $\tan\theta = 3.8958$ b. $\sin\theta = .2654$ c. $\sec\theta = 2.0717$

5. Given $\cot\theta = \frac{-12}{5}$ and $\cos\theta < 0$, find $\csc\theta$.

6. Given $\sec\theta = \frac{-3}{2}$ and $\cot\theta > 0$, find $\sin\theta$.

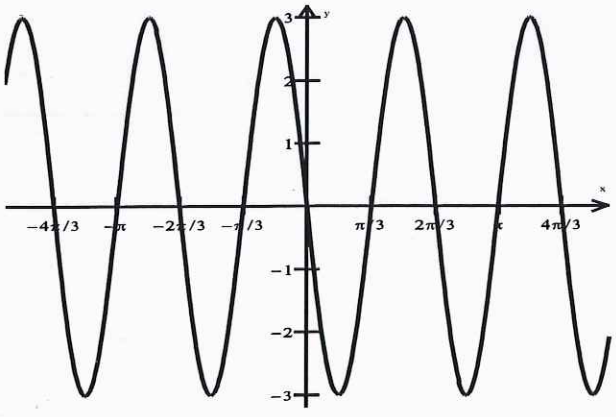
7. Two 100' guy wires are attached to the top of a telephone pole, one on each side. The angle of elevation of each wire with the ground is $57°$. How tall is the telephone pole to the nearest foot?

8. From a 75-foot observation tower on the coast, a Coast Guard officer sights a boat in difficulty. The boat is 625 feet from the base of the tower. Find the angle of depression from the top of the tower to the boat to the nearest degree

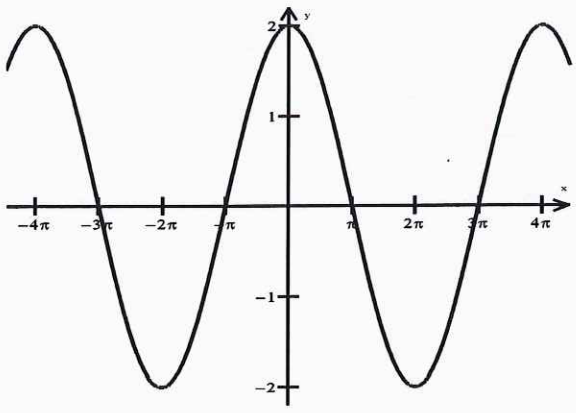
9. Find the exact value of the expression: a. $\csc\left[\cos^{-1}\left(\frac{-3}{4}\right)\right]$ b. $\tan\left[\sin^{-1}\left(\frac{-8}{17}\right)\right]$

10. Find the period in degrees: a. $f(x) = \cos\left(\frac{1}{4}x\right)$ b. $f(x) = \tan(6x)$

11. Write the sine equation for the following:



12. Write the cosine equation for the following:



13. Graph: $f(x) = -3\cos\left(\frac{\pi}{2}x + \pi\right) + 1$

14. Graph: $f(x) = 4\csc(x)$

15. Simplify: $\tan^2 x - \sin^2 x \tan^2 x$

16. Simplify: $\frac{\sec^2 x - 1}{\sec x - 1}$

17. Perform the addition and simplify: $\frac{1}{1+\cos x} + \frac{1}{1-\cos x}$

18. Perform the subtraction and simplify: $\cos x \sec x - \frac{\cos x}{\sec x}$

19. Find all solutions in the interval $[0, 2\pi)$: $4\cos^2 x - 1 = 0$

20. Find all solutions in the interval $[0, 2\pi)$: $2\sin x \cos x = \sqrt{2}\sin x$

21. Find all solutions in the interval $[0, 2\pi)$: $\sec^2 x - \sec x = 2$

22. Find all solutions in the interval $[0, 2\pi)$: $2\sin^2 x - 5\sin x + 2 = 0$

23. Evaluate: $\cos 255°$ (Use the fact that 255° = 315° - 60°).

24. Evaluate: $\sin 105°$ (Use the fact that 105° = 150°- 45°).

25. Given $\cos u = \frac{-12}{13}, \pi < u < \frac{3\pi}{2}$ and $\csc v = \frac{\sqrt{10}}{3}, \frac{\pi}{2} < v < \pi$, find $\cos(u+v)$.

26. Given $\cot u = \frac{2}{5}, 0 < u < \frac{\pi}{2}$ and $\cos v = \frac{-3}{5}, \pi < v < \frac{3\pi}{2}$, find $\tan(u+v)$.

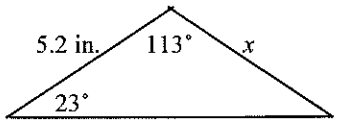
27. Find the area of the triangle to the nearest tenth, with sides of length, 5, 9, and 10.

28. In ΔABC, BC = 90, AC = 52 and $m\angle C = 102°$. What is the area of ΔABC to the nearest tenth?

29. Given a triangle with sides a = 6, b = 8, and c = 12, find $m\angle C$ to the nearest degree.

30. Given a triangle with $A = 115°$, $b = 15$, and $c = 10$, find a to the nearest tenth.

31. Solve for x, to the nearest tenth, in the given triangle.



32. Find all remaining sides and angles of the triangle with B = 61°, c = 18, and b = 17.
Find angles to the nearest degree and sides to the nearest tenth.

33. Rewrite the following in trigonometric form; express the answers in both degrees and radians:
a. -3 b. $-2-2i\sqrt{3}$

34. Use DeMoivre's theorem to evaluate: $[2(\cos 300° + i\sin 300°)]^6$; express your answer in degrees.

35. Use DeMoivre's theorem to find the cube roots of: $-27i$; express your answers in degrees.

36. Write an explicit formula for the arithmetic sequence: 2, 5, 8, 11, 14, ….

37. Write an explicit formula for the arithmetic sequence with a first term of 100 and a common difference of -8.

38. Find the 83rd term of the **arithmetic** sequence with $a_1 = 1.8$ *and* $d = 2.4$.
(Assume that n begins with 1)

39. Find the **sum** of the first 21 terms of the **arithmetic** sequence whose nth term is $a_n = 2n+5$
(Assume that n begins with 1).

40. Find the sum: a. $\sum_{n=0}^{50}(1000-5n)$ b. $\sum_{n=2}^{5}\frac{4}{n+2}$ (express the answer as a fraction)

41. Evaluate: a. $\sum_{n=1}^{\infty}4\left(\frac{7}{3}\right)^{n-1}$ b. $\sum_{n=0}^{\infty}2\left(\frac{-2}{3}\right)^{n}$

42. Find the coefficient of: a. x^2y^3 in the expansion of $(2x-3y)^5$ b. x^8y^4 in the expansion of $(x+5y)^{12}$.

43. A card is drawn at random from a standard deck of 52 playing cards. Find the probability that the card is:
a. red b. a king.

44. A password is comprised of 4 letters followed by 3 digits. How many passwords are possible?

45. A password is comprised of 5 letters followed by 2 digits. To avoid confusion between "O" and "zero" and "I" and "one", the letters "O" and "I" are not used. How many passwords are possible?

46. A bag contains 10 quarters, 7 dimes, and 5 nickels. If three coins are selected without replacement, what is the probability of selecting three quarters?

47. A bag contains 10 quarters, 7 dimes, and 5 nickels. If three coins are selected without replacement, what is the probability of selecting one of each coin?

48. An employer interviews 12 people for four opportunities in the company. Five of the 12 people are women. If all 12 are qualified, in how many ways can the employer fill the four positions if:
a. the selection is random? b. exactly two women are selected?

49. Graph the following:

a. $\begin{cases} x^2+y^2 \le 25 \\ y \ge x^2+2 \end{cases}$ b. $\begin{cases} (x-1)^2+(y-3)^2 \ge 16 \\ (x-1)^2+(y+3)^2 \le 36 \end{cases}$

50. Graph the system of inequalities.

a. $\begin{aligned} 5x-3y &\le 15 \\ x &\ge 1 \\ y &\le -2 \end{aligned}$ b. $\begin{aligned} 3x+2y &> 12 \\ x &\le 6 \\ y &\le 4 \end{aligned}$

51. A small business has an initial investment of \$5000. The unit cost of the product is \$21.60, and the selling price is \$34.10. Find the sales necessary to break even.

52. Solve the following system: $\begin{aligned} x+y &= 4 \\ x^2+y^2 &= 4x \end{aligned}$

53. Solve the following system: $\begin{aligned} x^2+y^2 &= 169 \\ x^2-8y &= 104 \end{aligned}$

54. Solve the system of linear equations: $\begin{aligned} 2x+3y+3z &= 3 \\ 6x+6y+12z &= 13 \\ 12x+9y-z &= 2 \end{aligned}$

55. Solve the system of linear equations: $\begin{aligned} -x+2y &= 1.5 \\ 2x-4y &= 3 \end{aligned}$

56. Solve the system of linear equations: $\begin{aligned} 2x+y-7z &= 4 \\ 2x-y-z &= 0 \end{aligned}$

57. Given: $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -3 & 2 \\ -4 & 1 & -2 \end{bmatrix}$ Find: a. $A + B$ b. $5A - 4B$

58. Find the product of matrices: a. $\begin{bmatrix} -5 & 4 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 6 & 8 \end{bmatrix}$ b. $\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}$ c. $\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}$

59. Given the following, Find A^{-1} a. $A = \begin{bmatrix} -2 & 7 \\ -4 & 11 \end{bmatrix}$ b. $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ -4 & 4 & 3 \end{bmatrix}$

60. Find the determinant of the following matrices: a. $\begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & -1 & 8 & 4 \\ 2 & 6 & 0 & 4 \\ 2 & 0 & 2 & 6 \\ 0 & 2 & 8 & 0 \end{bmatrix}$

Spring Open-Ended Answer Key

1. a. $112.5°$ b. $\frac{35\pi}{36}$

2. $\frac{7}{25}$

3. a. $75°$ b. $\frac{\pi}{3}$

4. a. $76°, 256°$ b. $15°, 165°$ c. $61°, 299°$

5. $\frac{13}{5}$

6. $\frac{-\sqrt{5}}{3}$

7. 84 feet

8. $7°$

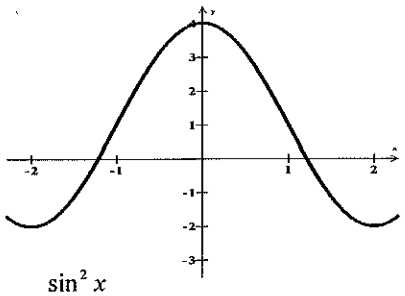
9. a. $\frac{4\sqrt{7}}{7}$ b. $\frac{-8}{15}$

10. a. $1440°$ b. $30°$

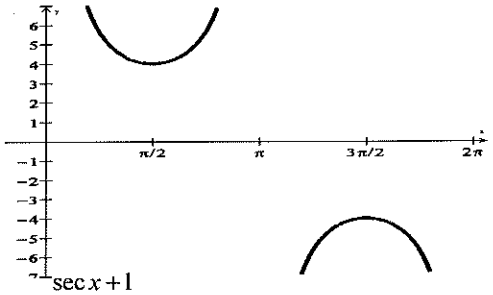
11. $f(x) = -3\sin(3x)$

12. $f(x) = 2\cos\left(\frac{1}{2}x\right)$

13.



14.



15. $\sin^2 x$

16. $\sec x + 1$

17. $2\csc^2 x$

18. $\sin^2 x$

19. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

20. $0, \pi, \frac{\pi}{4}, \frac{7\pi}{4}$

21. $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

22. $\frac{\pi}{6}, \frac{5\pi}{6}$

23. $\frac{\sqrt{2}-\sqrt{6}}{4}$

24. $\frac{\sqrt{2}+\sqrt{6}}{4}$

25. $\frac{27\sqrt{10}}{130}$

26. $\frac{-23}{14}$

27. 22.4

28. 2288.9

29. $117°$

30. 21.3

31. 2.9 inches

32.

Case 1	**Case 2**
$\angle C = 68°$	$\angle C = 112°$
$\angle A = 51°$	$\angle A = 7°$
$a = 15.1$	$a = 2.4$

33. a. $z = 3(\cos 180° + i\sin 180°)$
$z = 3(\cos\pi + i\sin\pi)$

34. $64(\cos 0° + i\sin 0°)$

$z = 4(\cos 240° + i \sin 240°)$

b. $z = 4\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$

35. $r_1 = 3(\cos 90° + i \sin 90°)$
$r_2 = 3(\cos 210° + i \sin 210°)$
$r_3 = 3(\cos 330° + i \sin 330°)$

36. $a_n = 2 + (n-1)3$ or $a_n = 3n - 1$

37. $a_n = 100 + (n-1)(-8)$ or $a_n = -8n + 108$

38. 198.6

39. 567

40. a. 44,625 b. $\frac{319}{105}$

41. a. Does not exist b. $\frac{6}{5}$

42. a. -1,080 b. 309,375

43. a. $\frac{1}{2}$ b. $\frac{1}{13}$

44. 456,976,000

45. 796,262,400

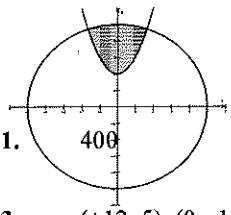
46. $\frac{6}{77}$

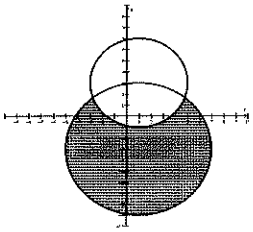
47. $\frac{5}{22}$

48. a. 495 b. 210

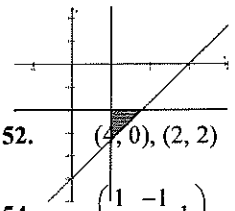
49. a. b.

50. a. b.

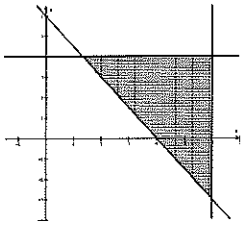




51. 400



52. (-1, 0), (2, 2)



53. $(\pm 12, 5), (0, -13)$

54. $\left(\frac{1}{2}, \frac{-1}{3}, 1\right)$

55. no solution

56. (2a+1, 3a+2, a)

57. a. $\begin{bmatrix} 8 & -2 & 3 \\ -5 & 0 & 3 \end{bmatrix}$ b. $\begin{bmatrix} -14 & 17 & -3 \\ 11 & -9 & 33 \end{bmatrix}$

58. a. $\begin{bmatrix} 9 & 37 \\ 48 & 54 \end{bmatrix}$ b. $\begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$ c. not possible

59. a. $\begin{bmatrix} \frac{11}{6} & \frac{-7}{6} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$ b. $\begin{bmatrix} 1 & -1 & 0 \\ 7 & \frac{-17}{2} & 1 \\ -8 & 10 & -1 \end{bmatrix}$

60. a. -75 b. 16