Honors Algebra 3-4 Name Key

5.4-5.5 Worksheet Period ______

Show all supporting work. All answers must be **exact**, but you may use calculator to check.

<u>Sum and difference formulas:</u>

(Hint: $105° = 60° + 45°$)

1. $\sin 105° = \sin(60+45) = \sin 60 \cos 45 + \cos 60 \sin 45$
$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\cos 105° = \cos 60 \cos 45 - \sin 60 \sin 45$
$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$

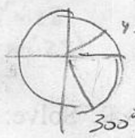
$\tan 105° = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} = \frac{(\sqrt{3}+1)(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = -2-\sqrt{3}$

$\tan 60 = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$\tan 45 = 1$

(Hint: $\cancel{225}° = 300° - 45°$) 255

2. $\sin \cancel{225}° = \sin 300 \cos 45 - \cos 300 \sin 45$
$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6}-\sqrt{2}}{4}$



$\cos \cancel{225}° = \cos 300 \cos 45 + \sin 300 \sin 45$
$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$

$\tan \cancel{225}° = \frac{\tan 300 - \tan 45}{1 + \tan 300 \tan 45} = \frac{-\sqrt{3}-1}{1-\sqrt{3}}$

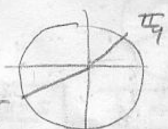
$\tan 300° = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

$\tan 45° = 1$

3. $\cos 32° \cos 15° - \sin 32° \sin 15° = \cos(32° + 15°) = \cos(47°)$

4. $\frac{\tan 212° - \tan 84°}{1 + \tan 212° \tan 84°} = \tan(212-84) = \tan 128°$

5. (Hint: $\frac{17\pi}{12} = \frac{7\pi}{6} + \frac{\pi}{4}$)



$\sin \frac{17\pi}{12} = \sin \frac{7\pi}{6} \cos \frac{\pi}{4} + \cos \frac{7\pi}{6} \sin \frac{\pi}{4}$
$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2}-\sqrt{6}}{4}$

$\tan \frac{7\pi}{6} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\tan \frac{\pi}{4} = 1$

$\cos \frac{17\pi}{12} = \cos \frac{7\pi}{6} \cos \frac{\pi}{4} - \sin \frac{7\pi}{6} \sin \frac{\pi}{4}$
$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6}+\sqrt{2}}{4}$

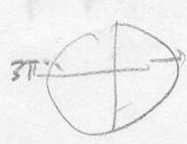
$\tan \frac{17\pi}{12} = \frac{\tan \frac{7\pi}{6} + \tan \frac{\pi}{4}}{1 - \tan \frac{7\pi}{6} \tan \frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{2} + 1}{1 - \frac{\sqrt{3}}{2}(1)} = \boxed{\frac{\sqrt{3}+2}{2-\sqrt{3}}}$

or continuing:

$\frac{(\sqrt{3}+2)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{2\sqrt{3}+3+4+2\sqrt{3}}{4-3} = \frac{4\sqrt{3}+7}{1} = \boxed{4\sqrt{3}+7}$

6. $\sin 5\theta \cos 4\theta - \cos 5\theta \sin 4\theta = \sin(5\theta - 4\theta) = \boxed{\sin \theta}$

7. Verify: $\sin(3\pi - x) = \sin x$



$\sin 3\pi \cos x - \cos 3\pi \sin x = \sin x$

$(0)\cos x - (-1)\sin x = \sin x$

$\sin x = \sin x$ ✓

Double Angles:

8. Solve: $\cos 2x + \sin x = 0$

$1 - 2\sin^2 x + \sin x = 0$

$-2\sin^2 x + \sin x + 1 = 0$

$2\sin^2 x - \sin x - 1 = 0$

$\frac{(2\sin x - 2)(2\sin x + 1)}{2 \quad 1}$

$(\sin x - 1)(2\sin x + 1) = 0$

$\sin x - 1 = 0 \qquad 2\sin x + 1 = 0$

$\sin x = 1 \qquad \sin x = -\frac{1}{2}$

$\boxed{\frac{\pi}{2}} \qquad \boxed{\frac{7\pi}{6}, \frac{11\pi}{6}}$

9. Solve: $\sin 2x + \cos x = 0$

$2\sin x \cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0 \qquad \boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2} \qquad \boxed{\frac{7\pi}{6}, \frac{11\pi}{6}}$

10. Given: $\cos\theta = -\frac{2}{3}, \frac{x}{r}$ and $\frac{\pi}{2} < x < \pi$

$y^2 + 4 = 9$

$y^2 = 5$

$y = \sqrt{5}$

$\sin\theta = \frac{\sqrt{5}}{3}$

Find: $\sin 2\theta = 2\sin\theta\cos\theta = 2\frac{\sqrt{5}}{3}\left(-\frac{2}{3}\right) = -\frac{4\sqrt{5}}{9}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{4}{9} - \frac{5}{9} = -\frac{1}{9}$

Half Angles:

(Hint: $165° = \frac{330°}{2}$)

11. $\sin 165° = \pm\sqrt{\frac{1-\cos 330°}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$

$\cos 165° = \pm\sqrt{\frac{1+\cos 330°}{2}} = -\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}}$

(Hint: $22.5° = \frac{45°}{2}$)

12. $\sin 22.5° = \pm\sqrt{\frac{1-\cos 45°}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$

$\cos 22.5° = \pm\sqrt{\frac{1+\cos 45°}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$

$= \frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}}$

$= \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

(Hint: $\frac{\pi}{12} = \frac{\left(\frac{\pi}{6}\right)}{2}$)

13. $\sin\frac{\pi}{12} = \pm\sqrt{\frac{1-\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$

$\cos\frac{\pi}{12} = \pm\sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$

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1. (Hint: $105° = 60° + 45°$)

   $\sin 105° =$

   $\cos 105° =$

   $\tan 105° =$

2. (Hint: $225° = 300° - 45°$)

   $\sin 225° =$

   $\cos 225° =$

   $\tan 225° =$

3. $\cos 32° \cos 15° - \sin 32° \sin 15° =$

4. $\dfrac{\tan 212° - \tan 84°}{1 + \tan 212° \tan 84°} =$

5. (Hint: $\dfrac{17\pi}{12} = \dfrac{7\pi}{6} + \dfrac{\pi}{4}$)

   $\sin \dfrac{17\pi}{12} =$

   $\cos \dfrac{17\pi}{12} =$

   $\tan \dfrac{17\pi}{12} =$

6. $\sin 5\theta \cos 4\theta - \cos 5\theta \sin 4\theta =$

7. Verify: $\sin(3\pi - x) = \sin x$

Double Angles:

8. Solve: $\cos 2x + \sin x = 0$

9. Solve: $\sin 2x + \cos x = 0$

10. Given: $\cos\theta = -\dfrac{2}{3}, \quad and \quad \dfrac{\pi}{2} < x < \pi$

Find: $\sin 2\theta$

$\cos 2\theta$

Half Angles:

(Hint: $165° = \dfrac{330°}{2}$)

11. $\sin 165° =$

$\cos 165° =$

(Hint: $22.5° = \dfrac{45°}{2}$)

12. $\sin 22.5° =$

$\cos 22.5° =$

(Hint: $\dfrac{\pi}{12} = \dfrac{\left(\dfrac{\pi}{6}\right)}{2}$)

13. $\sin\dfrac{\pi}{12} =$

$\cos\dfrac{\pi}{12} =$