(1) $\sec x \cos\left(\frac{\pi}{2}-x\right)$

$\sec x \cdot \sin x$

$\frac{1}{\cos x} \cdot \frac{\sin x}{1}$

$\frac{\sin x}{\cos x}$

$\boxed{\tan x}$

(2) $\dfrac{\csc x}{\tan x + \cot x}$

$$\dfrac{\frac{1}{\sin x}\left(\frac{\cos x}{\cos x}\right)}{\left(\frac{\sin x}{\sin x}\right)\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\left(\frac{\cos x}{\cos x}\right)}$$

$$\dfrac{\frac{\cos x}{\sin x \cos x}}{\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}}$$

$$\dfrac{\cos x}{\sin^2 x + \cos^2 x}$$

$\frac{\cos x}{1}$

$\boxed{\cos x}$

(3) $\dfrac{\tan x}{\csc x} + \dfrac{\sin x}{\tan x} = 1$

$$\frac{\tan^2 x}{\csc x \tan x} + \frac{\sin x \csc x}{\tan x \csc x}$$

$$\frac{\tan^2 x + \sin x \csc x}{\csc x \tan x}$$

$$\frac{\tan^2 x + 1}{\csc x \tan x}$$

$$\frac{\sec^2 x}{\csc x \tan x}$$

$$\dfrac{\frac{1}{\cos^2 x}}{\frac{1}{\sin x}\,\frac{\sin x}{\cos x}}$$

$$\dfrac{\frac{1}{\cos^2 x}}{\frac{1}{\cos x}} = \frac{1}{\cos^2 x} \times \frac{\cos x}{1}$$

$$\frac{\cos x}{\cos^2 x} = \frac{1}{\cos x}$$

$\boxed{\sec x}$

(4) $\dfrac{\sin^2 x}{\sec^2 x - 1}$

$\dfrac{\sin^2 x}{\tan^2 x}$

$\sin^2 x \cot^2 x$

$\frac{\sin^2 x}{1}\,\frac{\cos^2 x}{\sin^2 x}$

$\boxed{\cos^2 x}$

(5) $\dfrac{1}{\cot\theta} + \dfrac{1}{\tan\theta}$

$\dfrac{\tan\theta}{\tan\theta\cot\theta} + \dfrac{\cot\theta}{\tan\theta\cot\theta}$

$\dfrac{\tan\theta + \cot\theta}{\tan\theta\cot\theta}$

$\dfrac{\tan\theta + \cot\theta}{1}$

$\boxed{\tan\theta + \cot\theta}$

(6) $\sec^2 x \csc^2 x - \sec^2 x - \csc^2 x + 1$

(by grouping)

$\sec^2 x(\csc^2 x - 1) - 1(\csc^2 x - 1)$

$(\csc^2 x - 1)(\sec^2 x - 1)$

$\cot^2 x \cdot \tan^2 x$

$\frac{1}{\tan^2 x}\,\frac{\tan^2 x}{1}$

$\boxed{1}$

(7) $\cot^4 x + 2\cot^2 x + 1$

Substitute $u = \cot^2 x$

$u^2 + 2u + 1$

$(u+1)(u+1)$

$(\cot^2 x + 1)(\cot^2 x + 1)$

$\csc^2 x \cdot \csc^2 x$

$\boxed{\csc^4 x}$

(8) $\dfrac{\cos^2 x}{1 - \sin x}$

$\dfrac{\cos^2 x\,(1+\sin x)}{(1-\sin x)(1+\sin x)}$

$\dfrac{\cos^2 x(1+\sin x)}{1 - \sin^2 x}$

$\dfrac{\cos^2 x(1+\sin x)}{\cos^2 x}$

$\boxed{1+\sin x}$

(9) $\sqrt{9-x^2}$

Substitute $x = 3\cos\theta$

$\sqrt{9-(3\cos\theta)^2}$

$\sqrt{9-9\cos^2\theta}$

$\sqrt{9(1-\cos^2\theta)}$

$\sqrt{9\sin^2\theta}$

$\sqrt{9}\sqrt{\sin^2\theta}$

$\boxed{3\sin\theta}$

(10) $\frac{\sec x-\cos x}{\tan x} = \sin x$

$\frac{\sec x}{\tan x} - \frac{\cos x}{\tan x} = \sin x$

$\frac{\sec x}{1}\frac{1}{\tan x} - \frac{\cos x}{1}\frac{1}{\tan x} = \sin x$

$\frac{1}{\cos x}\frac{\cos x}{\sin x} - \frac{\cos x}{1}\frac{\cos x}{\sin x} = \sin x$

$\frac{1-\cos^2 x}{\sin x} = \sin x$

$\frac{\sin^2 x}{\sin x} = \sin x$

$\boxed{\sin x = \sin x}$ ✓

(11) $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$

$\frac{\csc x}{1}\frac{1}{\sin x} - \frac{\cot x}{1}\frac{1}{\tan x} = 1$

$\frac{1}{\sin x}\frac{1}{\sin x} - \frac{\cos x}{\sin x}\frac{\cos x}{\sin x} = 1$

$\frac{1-\cos^2 x}{\sin^2 x} = 1$

$\frac{\sin^2 x}{\sin^2 x} = 1$

$\boxed{1=1}$ ✓

(12) $\frac{1+\tan x}{\sin x} - \sec x = \csc x$

$\frac{1}{\sin x} + \frac{\tan x}{\sin x} - \sec x = \csc x$

$\csc x + \frac{\tan x}{1}\frac{1}{\sin x} - \sec x = \csc x$

$\csc x + \frac{\sin x}{\cos x}\frac{1}{\sin x} - \sec x = \csc x$

$\csc x + \frac{1}{\cos x} - \sec x = \csc x$

$\csc x + \underbrace{\sec x - \sec x}_{0} = \csc x$

$\boxed{\csc x = \csc x}$ ✓

(13) $\sin(\frac{\pi}{2}-x)\underbrace{\cos(-x)}_{\text{even function}} = \cos^2 x$

$\cos x \cdot \cos x = \cos^2 x$

$\boxed{\cos^2 x = \cos^2 x}$ ✓

(14) $\frac{\cos x}{1-\sin^2 x} = \sec x$

$\frac{\cos x}{\cos^2 x} = \sec x$

$\frac{1}{\cos x} = \sec x$

$\boxed{\sec x = \sec x}$ ✓

(15) $$1+\frac{1}{\csc^2x-1}=\sec^2x$$

$$\frac{(\csc^2x-1)1}{(\csc^2x-1)1}+\frac{1}{\csc^2x-1}=\sec^2x$$

$$\frac{\csc^2-1+1}{\csc^2x-1}=\sec^2x$$

$$\frac{\csc^2x}{\csc^2x-1}=\sec^2x$$

$$\frac{\csc^2x}{\cot^2x}=\sec^2x$$

$$\frac{\csc^2x}{1}\,\frac{1}{\cot^2x}=\sec^2x$$

$$\frac{1}{\sin^2x}\cdot\frac{\sin^2x}{\cos^2x}=\sec^2x$$

$$\frac{1}{\cos^2x}=\sec^2x$$

$$\boxed{\sec^2x=\sec^2x}$$ ✓

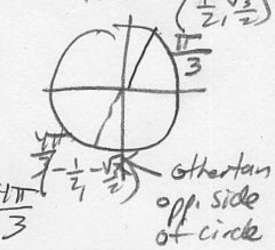
(16) Solve $\underbrace{\tan 3t}_{\theta}=\sqrt{3}$ $\qquad \theta=3t$

Sub: θ

$\tan\theta=\sqrt{3}$

$\tan\theta=\frac{\sqrt{3}/2}{1/2}$ ← divide by 2 trick

$\frac{\sin\theta}{\cos\theta}=\frac{\sqrt{3}/2}{1/2}$



other tan opp. side of circle

$\theta=\frac{\pi}{3}$ or $\frac{4\pi}{3}$

$3t=\frac{\pi}{3}$ or $3t=\frac{4\pi}{3}$

$\boxed{t=\frac{\pi}{9}}$ or $\boxed{t=\frac{4\pi}{9}}$

(technically also $\frac{7\pi}{9}$ & one other $[0,2\pi)$ applies to t, not θ)

(17) Solve $\sec^2x=\sec x+2$

$\sec^2x-\sec x-2=0$

substitute: $u=\sec x$

$u^2-u-2=0$

$(u-2)(u+1)=0$

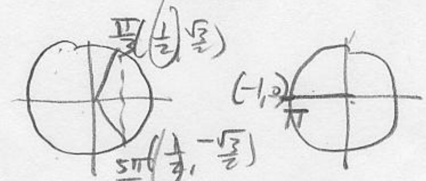
$(\sec x-2)(\sec x+1)=0$

$\sec x-2=0 \qquad \sec x+1=0$

$\sec x=2 \qquad \sec x=-1$

$\frac{1}{\cos x}=2 \qquad \frac{1}{\cos x}=-1$

$\cos x=\frac{1}{2} \qquad \cos x=-1$



$$\boxed{x=\frac{\pi}{3},\ \frac{5\pi}{3},\ \pi}$$

(18) Solve $\cot^2x-\tan^2x=0$

$$\frac{1}{\tan^2x}-\frac{\tan^2x}{1}=0$$

$$\frac{1}{\tan^2x}-\frac{\tan^4x}{\tan^2x}=0$$

$$\frac{1-\tan^4x}{\tan^2x}=0$$ fraction $=0$ when numerator $=0$

$1-\tan^4=0$

(a^2-b^2)

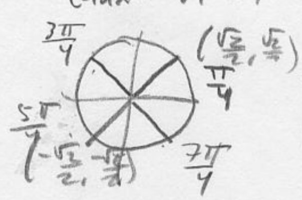
$(1+\tan^2x)(1-\tan^2x)=0$

$1+\tan^2x=0 \qquad 1-\tan^2x=0$

$\tan^2x=-1 \qquad \tan^2x=1$

$\tan x=\pm\sqrt{-1} \qquad \tan x=\pm\sqrt{1}=\pm1$

(no sol'n)



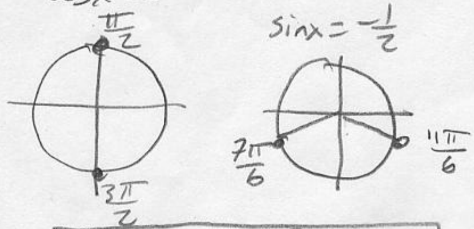
$$\boxed{x=\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}}$$

(19) Solve $2\sin x\cos x + \cos x = 0$

$(\cos x)(2\sin x + 1) = 0$

$\cos x = 0 \qquad 2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$



$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

(20) $\sin x = \frac{3}{10}$, $\cos x = \frac{-\sqrt{91}}{10}$ find $\tan x$

$$\tan x = \frac{\sin x}{\cos x} = \frac{3/10}{-\sqrt{91}/10} = \frac{3}{-\sqrt{91}}$$

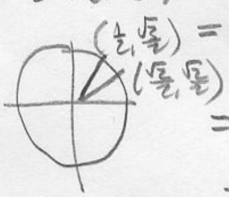
$$\tan x = \frac{3}{-\sqrt{91}} \cdot \frac{\sqrt{91}}{\sqrt{91}} = \boxed{\frac{-3\sqrt{91}}{91}}$$

(21) Eval $\sin 105°$ $(105° = 60° + 45°)$

$\sin(105°) = \sin(60° + 45°)$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\sin(105°) = \sin 60° \cos 45° + \cos 60° \sin 45°$



$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

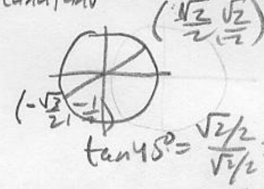
$$= \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

(22) Eval. $\tan(165°)$ $(165° = 210° - 45°)$

$\tan(165°) = \tan(210° - 45°)$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$\tan 165° = \frac{\tan 210° - \tan 45°}{1 + \tan 210° \tan 45°}$$



$$\tan 45° = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$$

$$\tan 210° = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\tan 165° = \frac{\frac{1}{\sqrt{3}} - 1}{1 + \frac{1}{\sqrt{3}}(1)}$$

$$= \frac{\left(\frac{1}{\sqrt{3}} - 1\right)\sqrt{3}}{\left(1 + \frac{1}{\sqrt{3}}\right)\sqrt{3}}$$

$$= \boxed{\frac{1 - \sqrt{3}}{\sqrt{3} + 1}}$$

or continue to simplify

$$\frac{(1-\sqrt{3})(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$\frac{\sqrt{3} - 1 - 3 + \sqrt{3}}{3 - 1}$$

$$\frac{2\sqrt{3} - 4}{2}$$

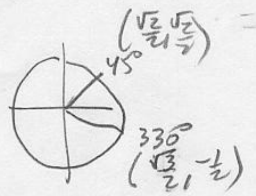
$$\boxed{\sqrt{3} - 2}$$

(23) Eval. $\cos 285°$ $(285° = 330° - 45°)$

$\cos 285° = \cos(330° - 45°)$

$= \cos(u - v) = \cos u \cos v + \sin u \sin v$

$\cos 285° = \cos 330° \cos 45° + \sin 330° \sin 45°$



$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

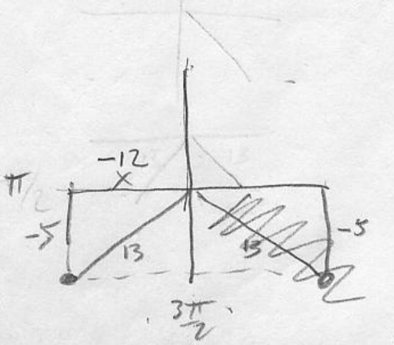
(24) simplify $\sin 8x \cos 2x + \cos 8x \sin 2x = \sin(8x+2x) = \boxed{\sin(10x)}$

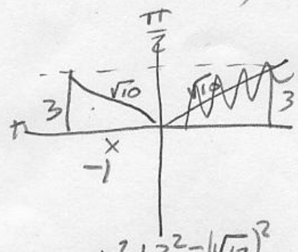
$\sin u \cos v + \cos u \sin v = \sin(u+v)$

(25) $\sin u = \frac{-5}{13} \left(\frac{y}{r}\right)$ $\quad \csc v = \frac{\sqrt{10}}{3}$ $\quad$ find $\cos(u-v)$

$\pi < u < \frac{3\pi}{2}$ $\quad \frac{\pi}{2} < v < \pi$

$\sin v = \frac{3}{\sqrt{10}} \left(\frac{y}{r}\right)$





$x^2 + 5^2 = 13^2$

$x^2 + 25 = 169$

$x^2 = 144$

$x \overset{(-)}{=} 12$

(or by triples)

so $\cos u = \frac{x}{r}$

$\cos u = \frac{-12}{13}$

$x^2 + 3^2 = (\sqrt{10})^2$

$x^2 + 9 = 10$

$x^2 = 1$

$x \overset{(-)}{=} 1$

so $\cos v = \frac{x}{r}$

$\cos v = \frac{-1}{\sqrt{10}}$

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

$= \left(\frac{-12}{13}\right)\left(\frac{-1}{\sqrt{10}}\right) + \left(\frac{-5}{13}\right)\left(\frac{3}{\sqrt{10}}\right)$

$= \frac{12 - 15}{13\sqrt{10}}$

$= \frac{-3}{13\sqrt{10}} \frac{(\sqrt{10})}{(\sqrt{10})}$

$= \boxed{\frac{-3\sqrt{10}}{130}}$

(26) solve $\cos 2x + \sin x = 0$

$(\cos 2x = 1 - 2\sin^2 x)$

$1 - 2\sin^2 x + \sin x = 0$

$-2\sin^2 x + \sin x + 1 = 0$

multiply by -1

$2\sin^2 x - \sin x - 1 = 0$

substitute $u = \sin x$

$2u^2 - u - 1 = 0$

$\left(\frac{2u - 2}{2}\right)\left(\frac{2u + 1}{1}\right) = 0$ $\quad$ M: -2 $\quad$ A: -1 $\quad -2+1$

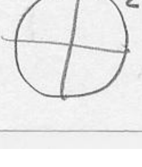
$(u-1)(2u+1) = 0$

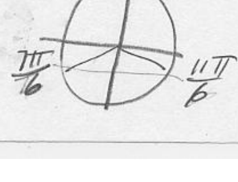
$(\sin x - 1)(2\sin x + 1) = 0$

$\boxed{x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

$\sin x - 1 = 0$ $\quad 2\sin x + 1 = 0$

$\sin x = 1$ $\quad \sin x = -\frac{1}{2}$





Honors Algebra 3-4
Ch. 5 Review Worksheet

Name______________________________
Period ______

(Do work on separate sheets of paper)

1. Simplify: $\sec x \cos\left(\frac{\pi}{2} - x\right)$

2. Simplify: $\frac{\csc x}{\tan x + \cot x}$

3. Perform the addition and simplify:

$$\frac{\tan x}{\csc x} + \frac{\sin x}{\tan x}$$

4. Simplify: $\frac{\sin^2 x}{\sec^2 x - 1}$

5. Simplify: $\frac{1}{\cot\theta} + \frac{1}{\tan\theta}$

6. Factor and simplify:

$$\sec^2 x \csc^2 x - \sec^2 x - \csc^2 x + 1$$

7. Factor and simplify: $\cot^4 x + 2\cot^2 x + 1$

8. Rewrite the expression so that it is not in fractional form: $\frac{\cos^2 x}{1 - \sin x}$

9. Use the substitution $x = 3\cos\theta$ to write the algebraic expression $\sqrt{9 - x^2}$ as a trigonometric expression involving θ, where $0 < \theta < \frac{\pi}{2}$

10. Verify the identity: $\frac{\sec x - \cos x}{\tan x} = \sin x$

11. Verify the identity: $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$

12. Verify the identity: $\dfrac{1+\tan x}{\sin x}-\sec x=\csc x$

13. Verify the identity: $\sin\left(\dfrac{\pi}{2}-x\right)\cos(-x)=\cos^2 x$

14. Verify the identity: $\dfrac{\cos x}{1-\sin^2 x}=\sec x$

15. Verify the identity: $1+\dfrac{1}{\csc^2 x-1}=\sec^2 x$

16. Find all the solutions in the interval $[0,2\pi)$: $\tan 3t=\sqrt{3}$

17. Find all the solutions in the interval $[0,2\pi)$: $\sec^2 x=\sec x+2$

18. Find all the solutions in the interval $[0,2\pi)$: $\cot^2 x-\tan^2 x=0$

19. Find all the solutions in the interval $[0,2\pi)$: $2\sin x\cos x+\cos x=0$

20. Given $\sin x=\dfrac{3}{10}$ *and* $\cos x=-\dfrac{\sqrt{91}}{10}$, find $\tan x$. (Draw the diagrams)

21. Evaluate: $\sin 105°$. (Use the fact that $105°=60°+45°$)

22. Evaluate: $\tan 165°$. (Use the fact that $165°=210°-45°$)

23. Evaluate: $\cos 285°$. (Use the fact that $285°=330°-45°$)

24. Simplify: $\sin 8x\cos 2x+\cos 8x\sin 2x$ (sum & difference formulas)

25. Given $\sin u=-\dfrac{5}{13}, \pi<u<\dfrac{3\pi}{2}$ *and* $\csc v=\dfrac{\sqrt{10}}{3}, \dfrac{\pi}{2}<v<\pi$, find $\cos(u-v)$.
(Draw the diagrams)

26. Find all solutions in the interval $[0,2\pi)$: $\cos 2x+\sin x=0$