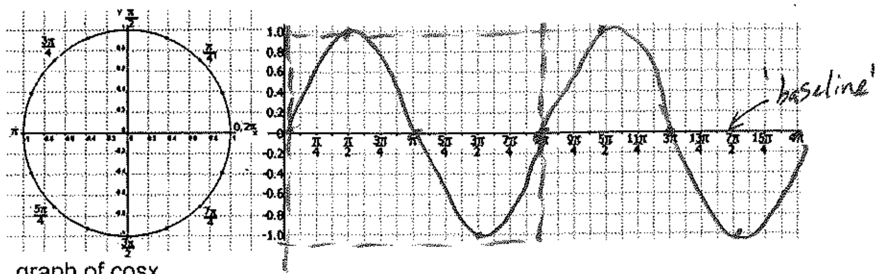
Precalculus – Lesson Notes: Chapter 4 Trigonometry Fundamentals

4.5 day 1: Graphs of Sine and Cosine Functions

Activity: graph sinx



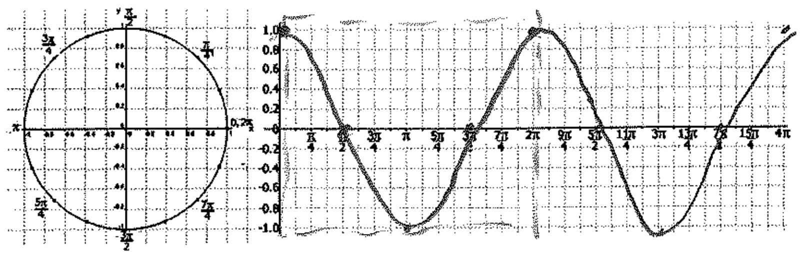
Things to note: 'Snake'

Period: 2π

Domain / Range: D: $(-\infty, \infty)$ R: $[-1, 1]$

Keypoints: baseline: start, middle, end
max: 1/4 period
min: 3/4 period

graph of cosx



Things to note: 'Cup'

Period: 2π

Domain / Range: D: $(-\infty, \infty)$ R: $[-1, 1]$

Keypoints:
baseline: 1/4, 3/4 period
max: start, end
min: middle

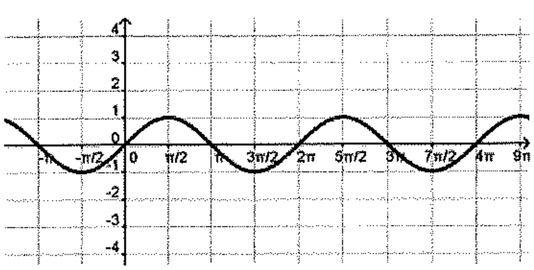
Modifications to basic equation: (sin as an example, cos similar)

$$y = d + a\sin(bx - c)$$ *where a, b, c, d are constants*

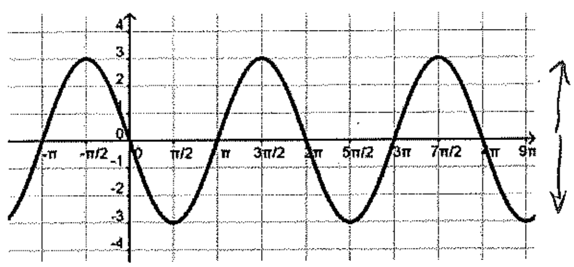
a => affects amplitude (vertical stretch)
b => affects period (horizontal stretch)
c => affects phase shift (horizontal shift)
d => affects vertical shift

Amplitude: Amplitude = $|a|$ (is always positive)

$y = \sin x$



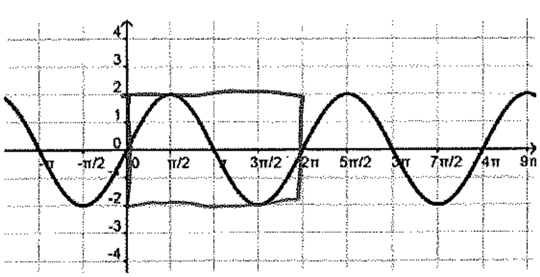
$y = -3\sin x$



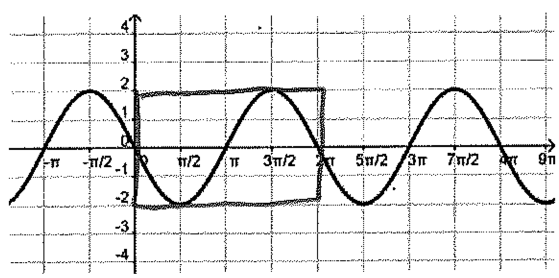
± 3 amplitude is $\boxed{3}$

If a is negative, curve is reflected over x-axis (flips vertically):

$y = 2\sin x$

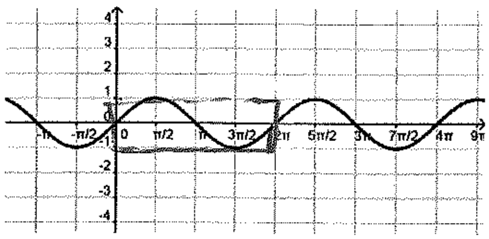


$y = -2\sin x$

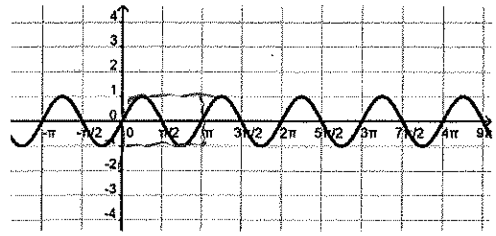


Period: period = how long it takes for a full cycle

For basic $y=\sin(x)$, period = 2π



For $y=\sin(2x)$:



inequality procedure:

$0 < 2x < 2\pi$

$\frac{0}{2} < \frac{2x}{2} < \frac{2\pi}{2}$

$0 < x < \pi$

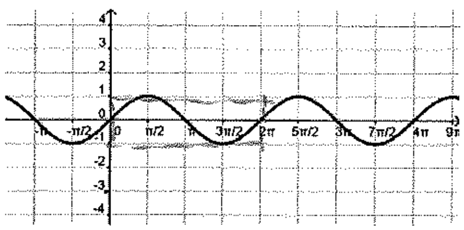
$\boxed{\text{period} = \pi}$

Procedure for finding the period of a sine or cosine equation:

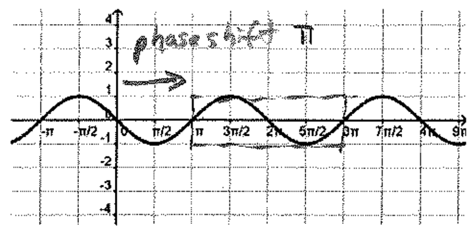
1) Put the argument (input) of the trig function in an equality from 0 to 2π
2) Solve the inequality for x.
3) Right side first number = period

Phase Shift: horizontal (x direction) shift

For basic $y=\sin(x)$, a period starts at 0.



For $y=\sin(x-\pi)$:



$0 < x-\pi < 2\pi$

$0 < x-\pi < 2\pi$
$+\pi \quad +\pi \quad +\pi$

$\pi < x < \underbrace{2\pi+\pi}_{\text{keep separate}}$

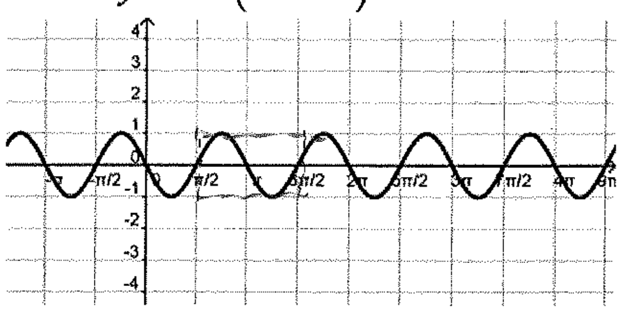
$\underbrace{\pi}_{\text{period starts at }\pi} < x < \underbrace{2\pi + \pi}_{\text{period ends at }3\pi}$

period, phase shift

Procedure for finding the phase shift of a sine or cosine equation:

1) Put the argument (input) of the trig function in an equality from 0 to 2π
2) Solve the inequality for x and keep the 2 terms on the right separate.
3) Left side = starting x of one period.
   Right side 1st number = period, 2nd number = phase shift
   Combined right side = ending x of one period.

$y=\sin(2x-\pi)$



$0 < 2x-\pi < 2\pi$
$+\pi \quad +\pi \quad +\pi$

$\frac{\pi}{2} < \frac{2x}{2} < \frac{2\pi}{2} + \frac{\pi}{2}$

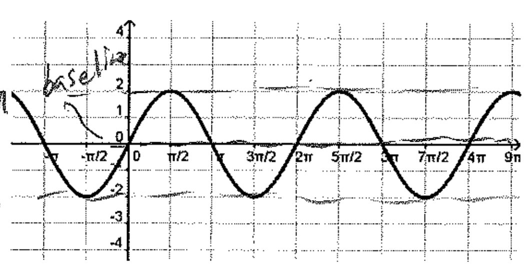
$\underbrace{\frac{\pi}{2}}_{\text{period start}} < x < \underbrace{\pi + \frac{\pi}{2}}_{\text{period ends at } \frac{3\pi}{2}}$

period $=\pi$, phase shift $=\frac{\pi}{2}$

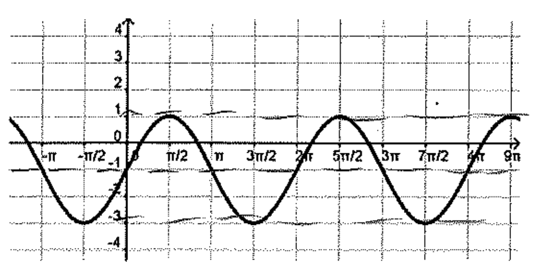
Vertical Shift: d just shifts graph up or down. When graphing, use d to find a new 'baseline'...

$y=2\sin(x)$

±2 from baseline



$y=2\sin(x)-1$

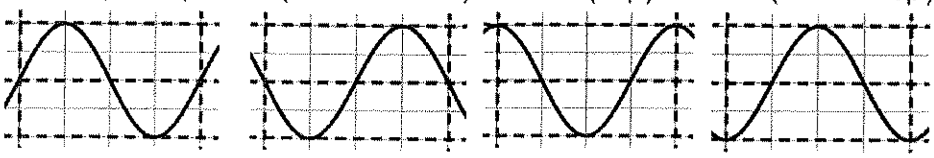


±2 from baseline

baseline

Procedure for sketching by hand:

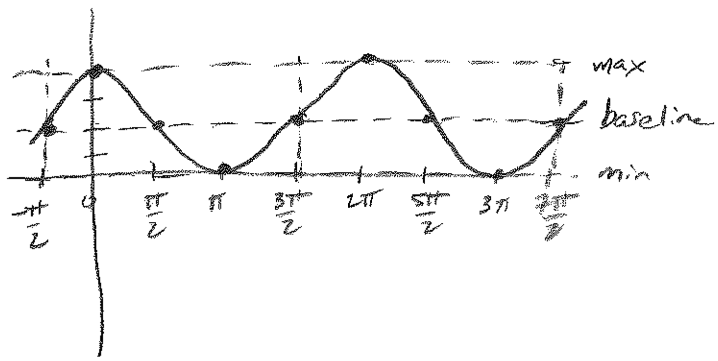
1) Is there a vertical shift? Draw the baseline.
2) Use the amplitude to add maximum and minimum y lines (+/- amplitude from the baseline).
3) Use the inequality procedure to find the start, end of a cycle and add vertical lines at these x values.
4) In the box formed by the boundary lines, draw one cycle of the curve:
   +sin ('snake') -sin ('inverted snake') +cos ('cup') -cos ('inverted cup')



5) Draw in additional periods as needed.

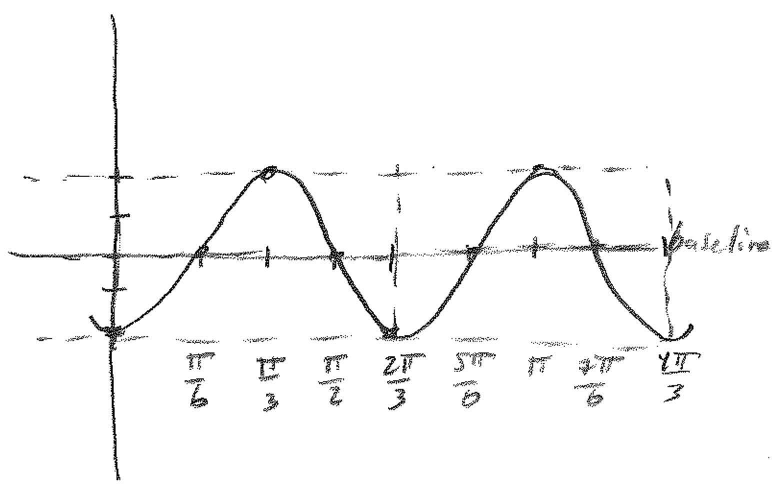
Sketch g(x) by hand $g(x) = \underbrace{2}_{\text{vert. shift}} + \underbrace{2}_{\text{amp}}\sin\left(x+\frac{\pi}{2}\right)$

1) vertical shift = 2
2) amplitude = 2
3) $0 < x+\frac{\pi}{2} < 2\pi$ period $= 2\pi$
   $-\frac{\pi}{2} < x < 2\pi - \frac{\pi}{2}$ phase shift $= -\frac{\pi}{2}$
   start at: $-\frac{\pi}{2}$
   end at: $2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$
4) +sin, 'snake' in box
5) add another period



Sketch by hand: $y = -2\cos 3x$

1) vertical shift: none
2) amplitude = 2
3) $0 < 3x < 2\pi$ period $= \frac{2\pi}{3}$
   $0 < x < \frac{2\pi}{3}$ phase shift = 0
   start at: 0
   end at $\frac{2\pi}{3}$
4) − cos 'inverted cup' in box
5) add another period



don't need graph: $0 < 3x < 2\pi$

$0 < x < \frac{2\pi}{3}$

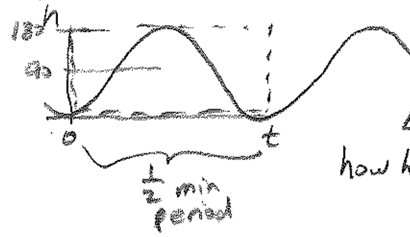
Find the period and amplitude of $y = -2\cos 3x$ $\boxed{\text{amplitude} = 2}$ (not −2, amplitude is always positive) $\boxed{\text{period} = \frac{2\pi}{3}}$

4.5 day 2: Graphs of Sine and Cosine Functions

Sine and cosine functions for modeling:

The radius of this Ferris wheel is 90 ft, and it completes two rotations minute (counter clockwise in this picture). If at t=0 a rider is at ground level, find an equation for the height of the rider in ft as a function of time in minutes.

2 periods = 1 min
1 period = $\frac{1}{2}$ min
$0 < t < \frac{1}{2}$
$0 < 4t < 2$
$0 < 4\pi t < 2\pi$



$\boxed{h(t) = -90\cos(4\pi t) + 90}$

how high is the rider at t = 24 seconds

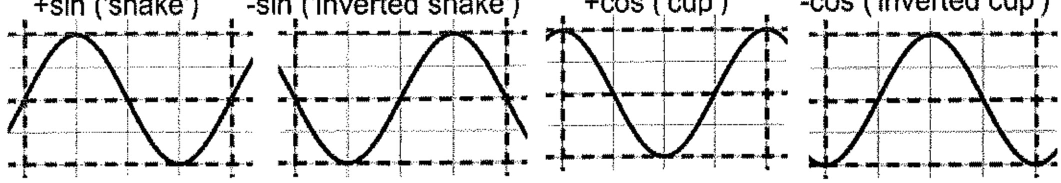
$24 \text{ sec} \cdot \frac{\text{min}}{60 \text{ sec}} = 0.4 \text{ min}$

$h(0.4) = -90\cos(4\pi(0.4)) + 90$

$= \boxed{62.2 \text{ ft}}$

<u>Procedure for sketching by hand</u>:

1) Is there a vertical shift? Draw the baseline.
2) Use the amplitude to add maximum and minimum y lines (+/- amplitude from the baseline).
3) Use the inequality procedure to find the start, end of a cycle and add vertical lines at these x values.
4) In the box formed by the boundary lines, draw one cycle of the curve:

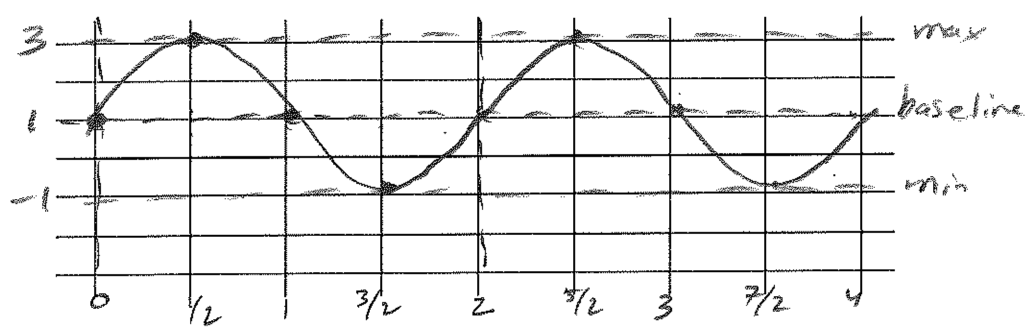


5) Draw in additional periods as needed.

$y = 2\sin(\pi x) + 1$

$0 < \pi x < 2\pi$

$0 < x < 2$

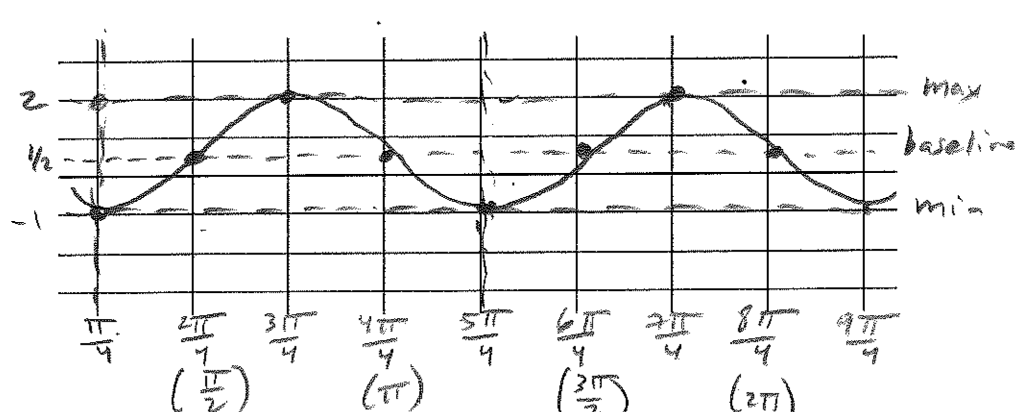


$$y = \frac{1}{2} - \frac{3}{2}\cos\left(2x - \frac{\pi}{2}\right)$$

$0 < 2x - \frac{\pi}{2} < 2\pi$

$\frac{\pi}{2} < 2x < 2\pi + \frac{\pi}{2}$

$\frac{\pi}{4} < x < \underbrace{\underbrace{\pi}_{\text{per}} + \underbrace{\frac{\pi}{4}}_{\text{P.S.}}}_{\frac{5\pi}{4}}$



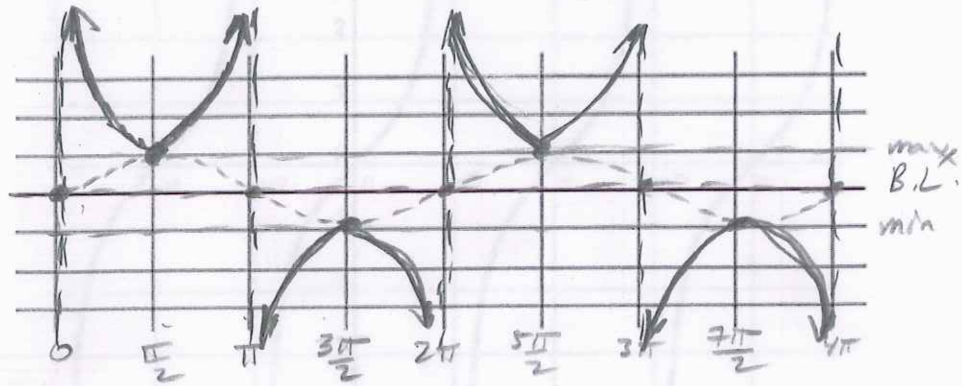
4.6: Graphs of other Trigonometric Functions

Graphing cosecant:

1) First, graph the function this is reciprocal of (sine)
2) At min and max points, cosecant and sine are equal.
3) Wherever the sine crosses the baseline there is a vertical asymptote for the cosecant.
4) Sketch the cosecant from the min and max points approaching the asymptotes.

$y = \csc x$

1st graph: $y = \sin x$



Things to note:

Period: 2π

Domain / Range: D: $\mathbb{R}, x \neq n\pi$

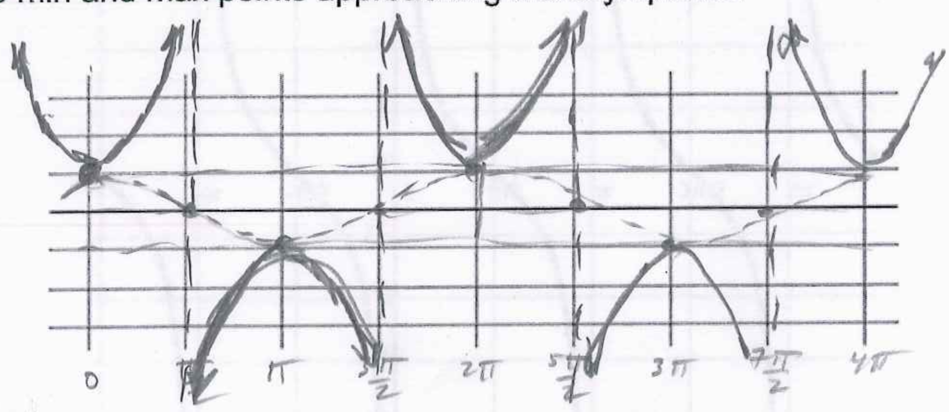
R: $(-\infty, -1] \cup [1, \infty)$

Graphing secant: (same as cosecant, but reciprocal of cosine)

1) First, graph the function this is reciprocal of (cosine)
2) At min and max points, secant and cosine are equal.
3) Wherever the cosine crosses the baseline there is a vertical asymptote for the secant.
4) Sketch the secant from the min and max points approaching the asymptotes.

$y = \sec x$

1st graph $y = \cos x$



Things to note:

Period: 2π

D: $\mathbb{R}, x \neq \frac{\pi}{2} + n\pi$

Domain / Range: R: $(-\infty, -1] \cup (1, \infty)$

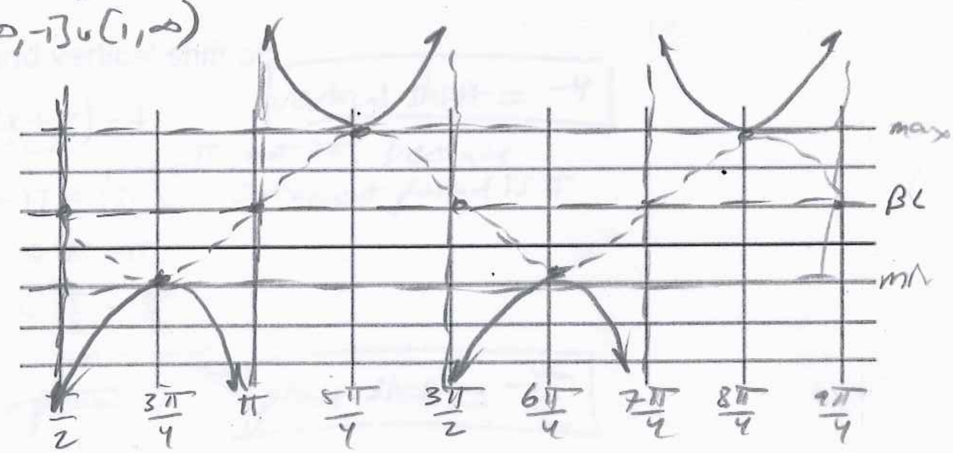
$y = 1 - 2\csc(2x - \pi)$

1st graph $y = 1 - 2\sin(2x - \pi)$

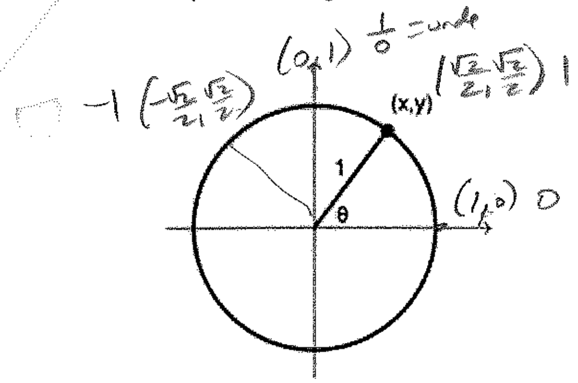
$0 < 2x - \pi < 2\pi$

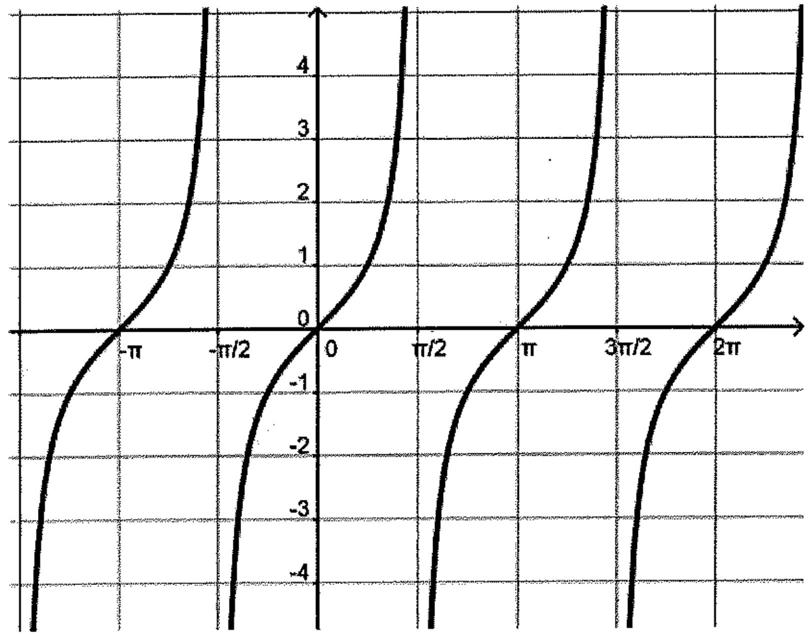
$\pi < 2x < 2\pi + \pi$

$\frac{\pi}{2} < x < \pi + \frac{\pi}{2}$



Graph of tangent function: $y = \tan x$



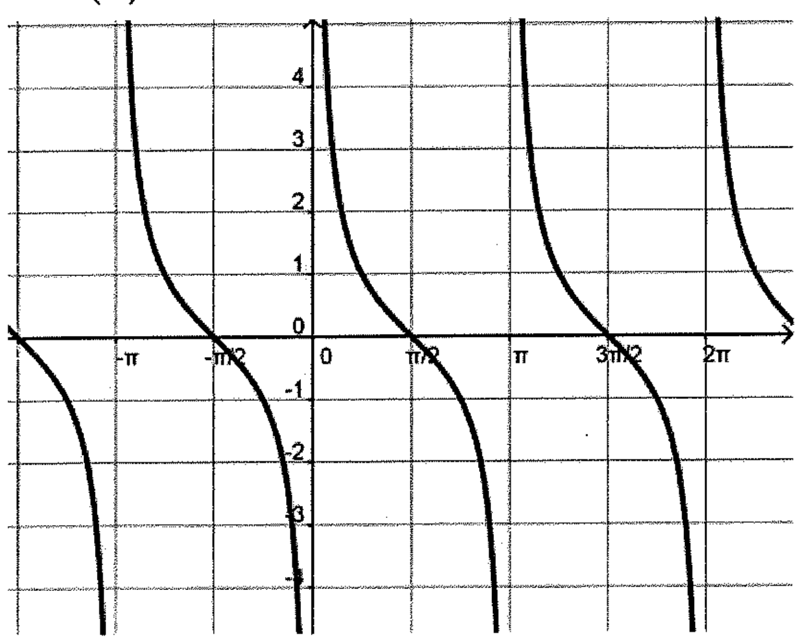


~~Things to note:~~

$\boxed{\text{Period: } \pi}$

Domain / Range: D: $\mathbb{R}$, $x \neq \frac{\pi}{2} + n\pi$
R: $(-\infty, \infty)$

Graph of cotangent function: $y = \cot(x)$



Things to note:
Period: π

Domain / Range: D: $\mathbb{R}$ $x \neq n\pi$
R: $(-\infty, \infty)$

Find the period, phase shift, and vertical shift of:

$$y = 3\cot(2x + \pi) - 4$$

$\boxed{\text{Vertical shift} = -4}$

$0 < 2x + \pi < \boxed{\pi}$ ← π, not 2π, because cotangent period is π

$-\pi < 2x < \pi - \pi$

$-\frac{\pi}{2} < x < \frac{\pi}{2} - \frac{\pi}{2}$

$\boxed{\text{period} = \frac{\pi}{2}}$

$\boxed{\text{phase shift} = -\frac{\pi}{2}}$

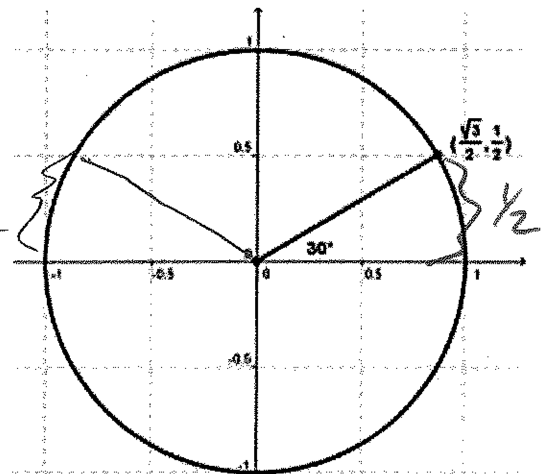
4.7 day 1: Inverse Trigonometric Functions

In solving right triangle problems, we sometimes needed to find an angle: $\sin\theta = 0.5$
and we used our inverse trig calculator functions.

$\sin^{-1}(\sin\theta) = \sin^{-1}(0.5)$

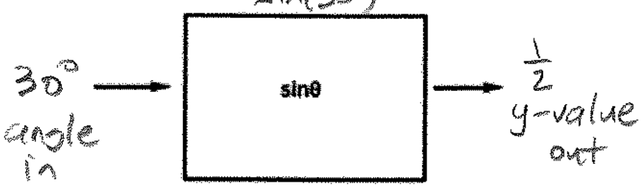
$\theta = \sin^{-1}(0.5)$

Let's look at this from our definition of the sine function with the unit circle:



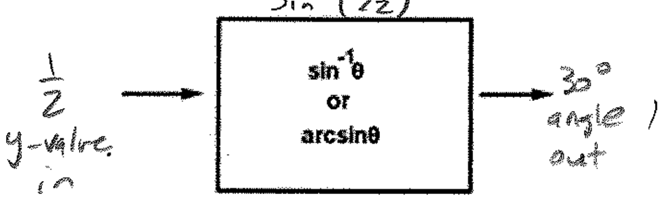
The sine function takes an angle (30°) as input and returns as output the number $\frac{1}{2}$ which is the y-coordinate on the unit circle. We could think of the sine function like a 'machine':

sin(30°)



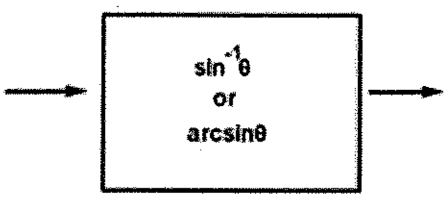
What would an inverse sine function do? The reverse:

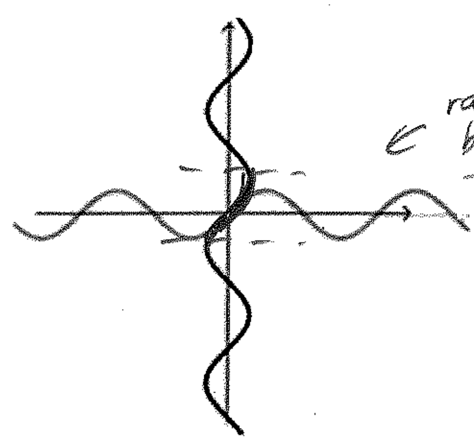
$\sin^{-1}(\frac{1}{2})$



, 150°, 390°, 510°, −330°, ...

The input would be the 'height' (y-coordinate) and the inverse function would return the angle. But there are multiple angles that have this y-value (sine value). Which one do we use?





range must be restricted to $-\frac{\pi}{2}, \frac{\pi}{2}$

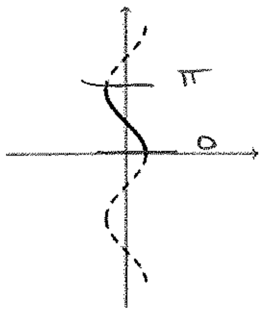
Since this is a function, there can be only one output. For arcsin:

Domain: $[-1, 1]$

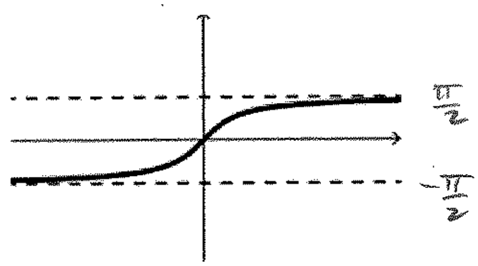
Range: $[-90°, 90°]$ or $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

The same is true for arccos and arctan:

For arccos:



For arctan:



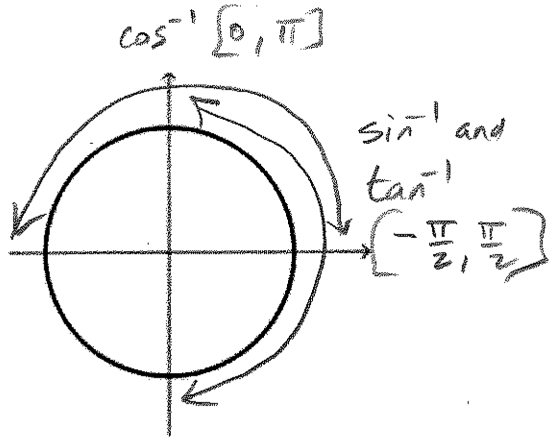
Domain: $[-1, 1]$

Range: $[0°, 180°]$ *or* $[0, \pi]$

Domain: $[-\infty, \infty]$

Range: $[-90°, 90°]$ *or* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

<u>Ranges of the inverse trig functions</u>:



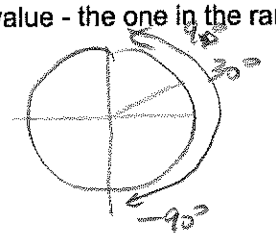
Using inverse trig functions:

We can use inverse trig function to solve equations for an angle, but we have to be aware that using a calculator will only provide one value - the one in the range of the inverse sine function:

$$\sin\theta = 0.5$$

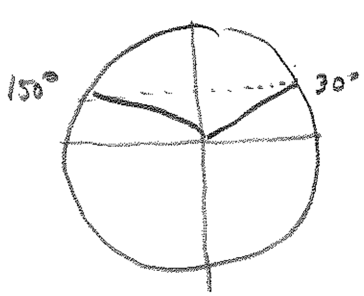
$\sin^{-1}(\sin\theta) = \sin^{-1}(0.5)$

$\theta = 30°$

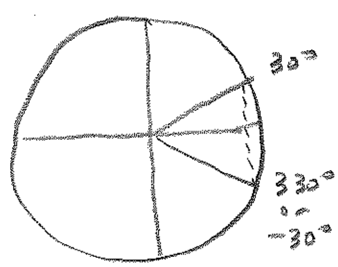


where is other place on unit circle with same value?

(sin = y)
across y-axis

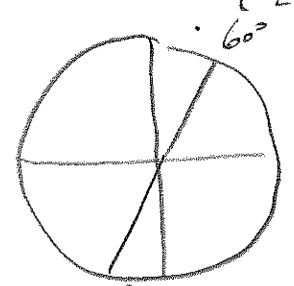


(cos = x)
across x-axis



(tan)
across circle

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



$\tan 60° = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

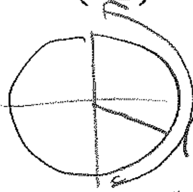
60+180

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $\tan 240° = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$

Examples:
If possible, find the exact value:

"angle where $y = -\frac{1}{2}$"

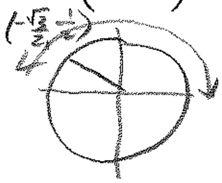
$\arcsin\left(-\frac{1}{2}\right)$



$\boxed{-30^\circ \text{ or } -\frac{\pi}{6}}$

"angle where $x = -\frac{\sqrt{3}}{2}$"

$\arccos\left(-\frac{\sqrt{3}}{2}\right)$



$\boxed{150^\circ \text{ or } \frac{5\pi}{6}}$

"angle where $y = 2$"

$\arcsin(2)$

not possible

"angle where $\frac{y}{x} = \frac{\sqrt{3}}{3}$"

$\arctan\frac{\sqrt{3}}{3}$

not on unit circle, use calculator

$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = 30^\circ$

? wait, this is on unit circle

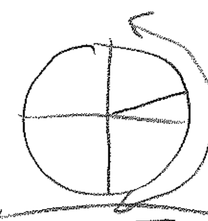
2 "tricks" that sometimes give unit circle values for tangents: $\arctan\frac{\sqrt{3}}{3}$

1) Divide numerator and denominator by 2:

$\frac{y}{x} = \frac{\sqrt{3}/2}{3/2}$ ← on unit circle / ← not

2) Rationalize, then divide numerator and denominator by 2:

$\frac{y}{x} = \frac{\sqrt{3}}{3}\frac{\sqrt{3}}{\sqrt{3}} = \frac{3}{3\sqrt{3}} = \frac{\cancel{3}\cdot 1}{\cancel{3}\sqrt{3}} = \frac{1/2}{\sqrt{3}/2}$ y ← on circle / x ← on circle



$\boxed{30^\circ \text{ or } \frac{\pi}{6}}$

Write each trigonometric expression in inverse function form, or vice versa:

$\sin(45^\circ) = \frac{\sqrt{2}}{2} \leftrightarrow$ $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$ or $\arcsin\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \leftrightarrow$ $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} \leftrightarrow$ $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\tan\left(\frac{\pi}{4}\right) = 1 \leftrightarrow$ $\tan^{-1}(1) = \frac{\pi}{4}$

4.7 day 2: Inverse Trigonometric Functions

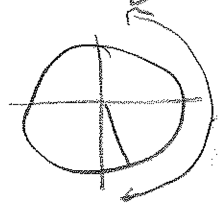
Evaluate the expression <u>without</u> using a calculator: rewrite in non-inverse form, then try...

...unit circle look-up:

$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{\pi}{3}}$$

"y"

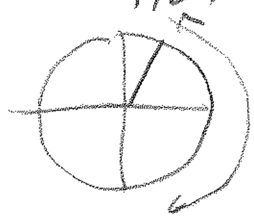
$\sin\theta = -\frac{\sqrt{3}}{2}$



...divide top & bottom by 2 'trick':

$$\arctan\left(\sqrt{3}\right) = \boxed{\frac{\pi}{3}}$$

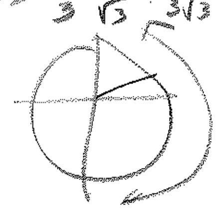
$\tan\theta = \frac{\sqrt{3}/2}{1/2}$ $\begin{matrix} y \\ x \end{matrix}$



...moving radical to other side 'trick':

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \boxed{\frac{\pi}{6}}$$

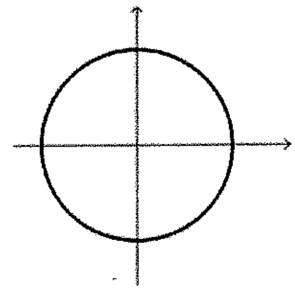
$\tan\theta = \frac{\sqrt{3}}{3}\frac{\sqrt{3}}{\sqrt{3}} = \frac{3}{3\sqrt{3}} = \frac{1/2}{\sqrt{3}/2}$ $\begin{matrix} y \\ x \end{matrix}$



Composites of Inverse Trig Functions:

arcsin and sin are inverses, so, in general, they 'cancel each other out':

$$\cancel{\sin}\left(\cancel{\arcsin}\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$$



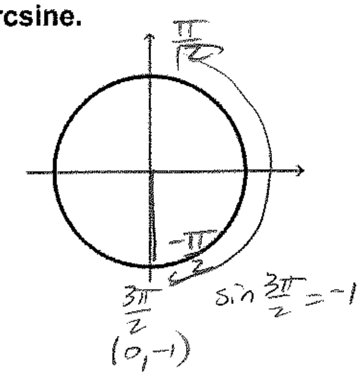
also...

$$\cancel{\arcsin}\left(\cancel{\sin}\frac{\pi}{4}\right) = \frac{\pi}{4}$$

..but only if the argument is in the range of arcsine.

$$\arcsin\left(\sin\frac{3\pi}{2}\right) = \boxed{-\frac{\pi}{2}}$$

$\arcsin(-1)$



$\sin\frac{3\pi}{2} = -1$

Same is true for arccos and arctan...

they 'cancel cos and tan' but be careful that inverse functions can only give values in their ranges.

Examples:

always ok

$$\sin(\arcsin(-0.2)) = -0.2$$

always ok

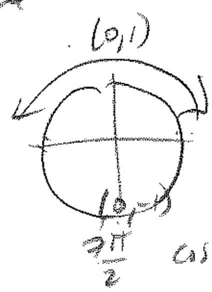
$$\tan(\arctan 25) = 25$$

inverse function on outside

CAUTION!

$$\arccos\left(\cos\left(\frac{7\pi}{2}\right)\right) = \boxed{\frac{\pi}{2}}$$

$\arccos(0)$



$\cos\left(\frac{7\pi}{2}\right) = 0$

Evaluating inverse functions using triangle sketches:

Find the exact value of the expression:

1) Rewrite the inside expression in non-inverse form.
2) Determine range where this angle can occur from arcsin, arccos, arctan rules
3) Use sign to determine quadrant and draw in a radius, then draw x and y to make a triangle.
4) Fill in two sides of the triangle using the sketching rules:

$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r} \qquad \tan\theta = \frac{y}{x}$$

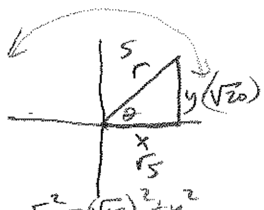
5) Use Pythagorean Theorem to find missing side.
6) Use sketch to evaluate the outside expression of the angle.

$$\sin\left(\arccos\left(\frac{\sqrt{5}}{5}\right)\right)$$

$\cos\theta = \frac{\sqrt{5}}{5}$ $\quad \frac{x}{r}$

$\sin(\theta) = \frac{y}{r}$

$= \boxed{\frac{\sqrt{20}}{5}}$



$5^2 = (\sqrt{5})^2 + y^2$

$25 = 5 + y^2$

$20 = y^2$

$y = \sqrt{20}$

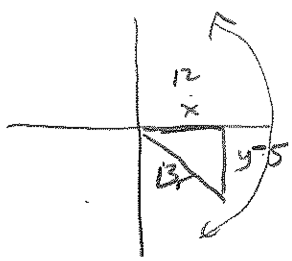
$$\csc\left(\arctan\left(-\frac{5}{12}\right)\right)$$

$\tan\theta = \frac{-5}{12}$ $\quad \frac{y}{x}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sin\theta = \frac{y}{r} = \frac{-5}{13}$

$\csc\theta = \boxed{\frac{-13}{5}}$



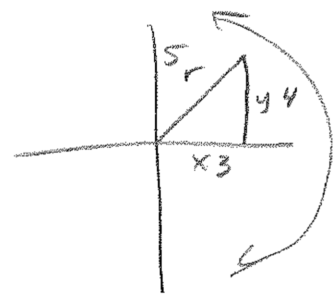
$$\sec\left(\arcsin\left(\frac{4}{5}\right)\right)$$

$\sin\theta = \frac{4}{5}$ $\quad \frac{y}{r}$

$\sec\theta = \frac{1}{\cos\theta}$

$\cos\theta = \frac{x}{r} = \frac{3}{5}$

$\sec\theta = \boxed{\frac{5}{3}}$

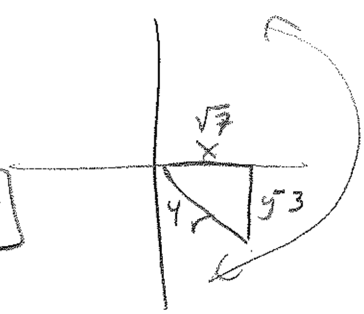


$$\tan\left(\arcsin\left(-\frac{3}{4}\right)\right)$$

$\sin\theta = \frac{-3}{4}$ $\quad \frac{y}{r}$

$\tan\theta = \frac{y}{x} = \frac{-3}{\sqrt{7}}$

$\frac{-3\sqrt{7}}{\sqrt{7}\sqrt{7}} = \boxed{\frac{-3\sqrt{7}}{7}}$



$x^2 + 3^2 = 4^2$

$x^2 + 9 = 16$

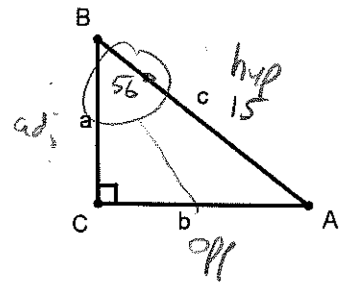
$x^2 = 7$

$x = \sqrt{7}$

4.8 day 1: Applications of Trigonometry

Many real-world problems can be modeled using right triangles, and missing information can be found by 'solving the triangle'.

Sometimes, the problem just gives you a generic triangle to 'solve'.
Convention for side, angle naming:



Example: B=56°, c=15, solve the triangle.

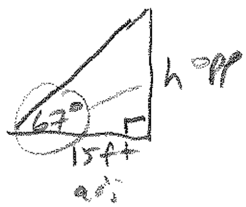
$\cos\theta = \frac{adj}{hyp}$ $\quad \sin\theta = \frac{opp}{hyp}$ $\quad A = 180° - 56° - 90° = \boxed{34°}$

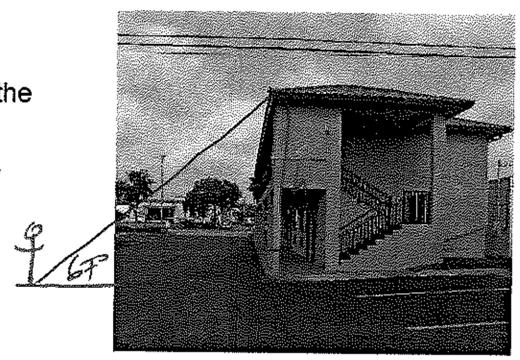
$\cos 56° = \frac{a}{15}$ $\quad \sin 56° = \frac{b}{15}$

$a = 15\cos 56°$ $\quad b = 15 \sin 56°$

$\boxed{a = 8.4}$ $\quad \boxed{b = 12.4}$

Example:
A man at ground level measures the angle of elevation to the top of a building to be 67°. If, at this point, he is 15 feet from the building, what is the height of the building?





$\tan\theta = \frac{opp}{adj}$

$\tan 67° = \frac{h}{15}$

$h = 15 \tan 67°$

$\boxed{h = 35.3 \text{ ft}}$

Things to remember:

angle of elevation is above a horizontal

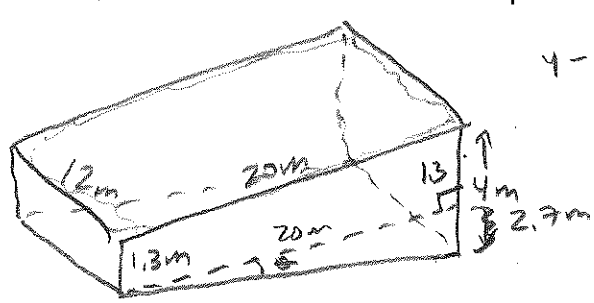
angle of depression is below a horizontal

$\sin\theta = \frac{opp}{hyp}$ $\quad \cos\theta = \frac{adj}{hyp}$ $\quad \tan\theta = \frac{opp}{adj}$

Sometimes, you want to find the angle, instead of a side length. Use inverse trig functions:

Example: A swimming pool is 20 meters long and 12 meters wide. The bottom of the pool has a constant slant so that the water depth is 1.3 meters at the shallow end and 4 meters at the deep end. Find the angle of depression of the bottom of the pool.

$4 - 1.3 = 2.7\text{m}$



$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{2.7}{20}$

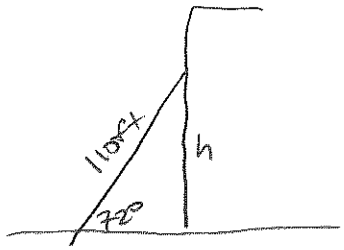
$\tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{2.7}{20}\right)$

$\theta = \tan^{-1}\left(\frac{2.7}{20}\right)$

$\boxed{\theta = 7.7°}$

Practice problems:

#1. A safety regulation states that the maximum angle of elevation for a rescue ladder is 72°. If a fire department's longest ladder is 110 feet, what is the maximum safe rescue height?



$\sin\theta = \frac{opp}{hyp}$

$\sin 72° = \frac{h}{110}$

$h = 110\sin 72°$

$\boxed{h = 104.6 ft}$

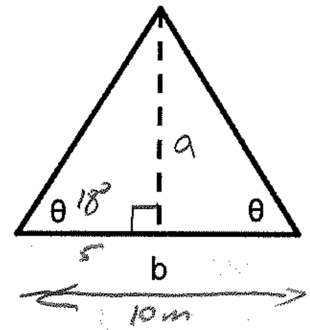
#2. Find the altitude of the isosceles triangle if b=10 meters and $\theta = 18°$

$\tan\theta = \frac{opp}{adj}$

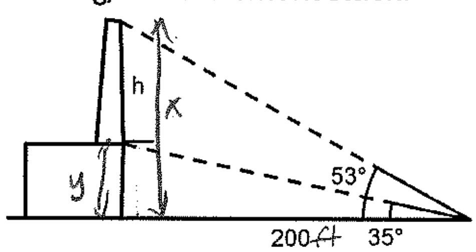
$\tan 18° = \frac{a}{5}$

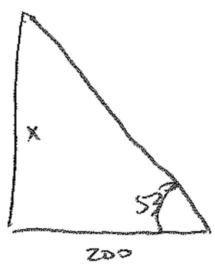
$a = 5\tan 18°$

$\boxed{a = 1.6 m}$



#3. At a point 200 feet from the base of a building, the angle of elevation to the bottom of a smokestack is 35°, and the angle of elevation to the top is 53°, as shown. Find the height of the smokestack.

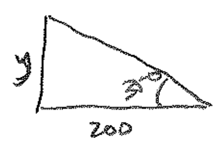




$\tan 53° = \frac{x}{200}$

$x = 200\tan 53°$

$x = 265.4 ft$



$\tan 35° = \frac{y}{200}$

$y = 200\tan 35°$

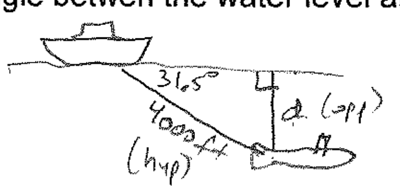
$y = 140.0 ft$

$h = x - y$

$h = 265.4 - 140.0$

$\boxed{h = 125.4 ft}$

#4. The sonar of a navy cruiser detects a nuclear submarine that is 4000 feet from the cruiser. The angle betwen the water level and the submarine is 31.5°. How deep is the submarine?



$\sin\theta = \frac{opp}{hyp}$

$\sin 31.5° = \frac{d}{4000}$

$d = 4000\sin 31.5°$

$\boxed{d = 2090 ft}$

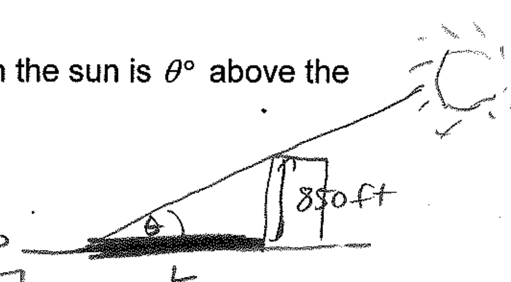
#5. A shadow of length L is created by an 850-foot building when the sun is $\theta°$ above the horizon.

(a) Write L as a function of θ

$\tan\theta = \frac{850}{L}$

$\frac{\tan\theta}{1} \cancel{=} \frac{850}{L}$

$(\tan\theta)L = 850$



(b) Use a calculator to complete the table:

θ	10°	20°	30°	40°	50°
L	4821	2335	1472	1013	713

$\boxed{L = \frac{850}{\tan\theta}}$

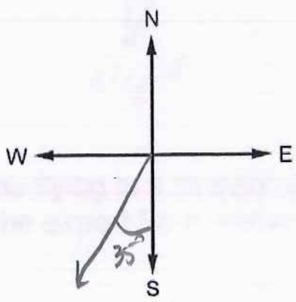
4.8 day 2: More applications; Navigational bearings

Bearings

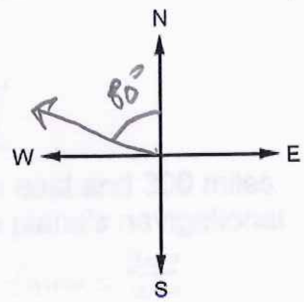
Sometimes, in navigation problems, direction is given as a 'bearing'. Bearing is defined as an acute angle from a North-South reference line, given as 'reference direction' followed by' angle from the reference direction'.

Examples of bearings:

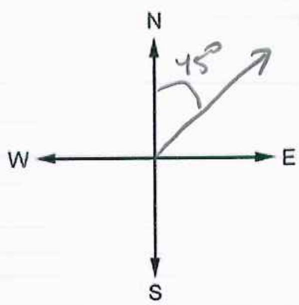
S 35° W



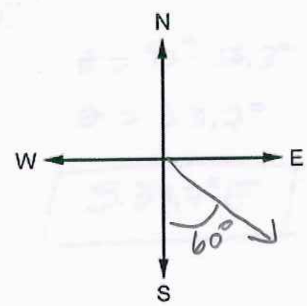
N 80° W



N 45° E



S 60° E

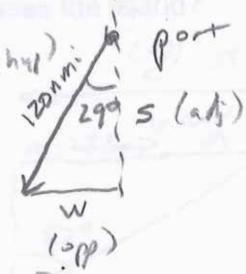


Example:

A ship leaves port at noon and has a bearing of S 29° W. If the ship sails at 20 knots, how many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 p.m?

travels for 6 hrs at $\frac{20 \text{ nmi}}{\text{hr}}$

$d = \frac{20 \text{ nmi}}{\text{hr}} \cdot 6 \text{ hr} = 120 \text{ nmi}$



$\cos\theta = \frac{adj}{hyp}$

$\cos 29° = \frac{s}{120}$

$s = 120 \cos 29°$

$\boxed{s = 105 \text{ nmi}}$

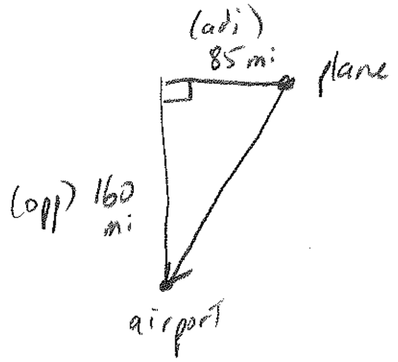
$\sin\theta = \frac{opp}{hyp}$

$\sin 29° = \frac{w}{120}$

$w = 120 \sin 29°$

$\boxed{w = 58 \text{ nmi}}$

A plane is 160 miles north and 85 miles east of an airport. If the pilot wants to fly directly to the airport, what bearing should be taken?



$\tan\theta = \frac{opp}{adj}$

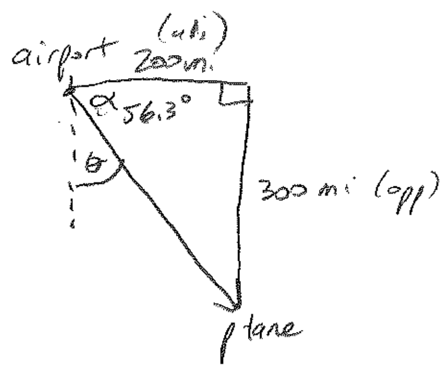
$\tan\theta = \frac{160}{85}$

$\theta = \tan^{-1}\left(\frac{160}{85}\right)$

$\theta = 28^\circ$

$\boxed{S28W^\circ}$

An airplane flying in a straight line is currently 200 miles east and 300 miles south of the airport from which it departed. What is the plane's navigational bearing?



$\tan\alpha = \frac{300}{200}$

$\alpha = \tan^{-1}\left(\frac{300}{200}\right)$

$\alpha = 56.3^\circ$

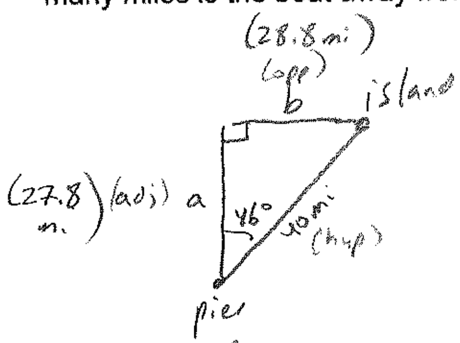
$\theta = 90^\circ - 56.3^\circ$

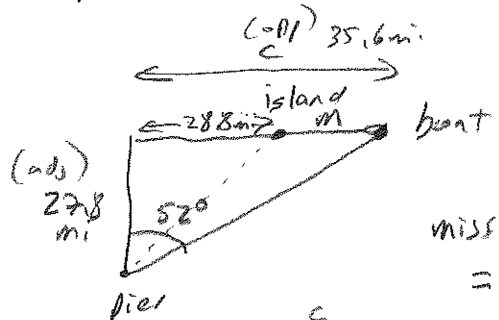
$\theta = 33.7^\circ$

$\boxed{S\,33.7^\circ E}$

Challenge problem:

A small boat set out on a N46°E course from a landing pier in order to reach an island 40 miles from the pier. The wind pushed the boat off course so that its actual bearing was N52°E, causing the boat to pass by the island. How many miles is the boat away from the island when it passes the island?





$\cos 46^\circ = \frac{a}{40}$

$a = 40\cos 46^\circ$

$\boxed{a = 27.8 \text{ mi}}$

$\sin 46^\circ = \frac{b}{40}$

$b = 40\sin 46^\circ$

$\boxed{b = 28.8 \text{ m}}$

$\tan 52^\circ = \frac{c}{27.8}$

$c = 27.8\tan 52^\circ$

$c = 35.6 \text{ mi}$

miss distance

$= 35.6 - 28.8$

$= \boxed{6.8 \text{ miles}}$