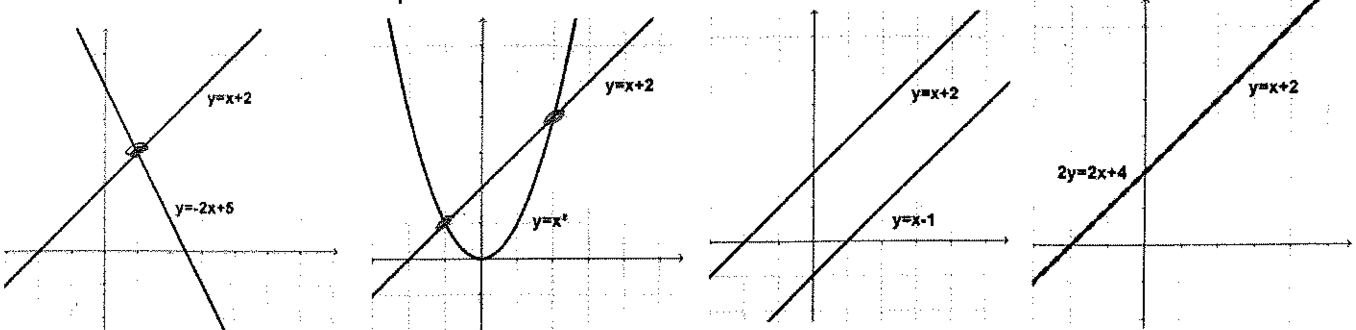
Precalculus – Lesson Notes: Chapter 7 Systems of Equations and Inequalities

7.1 day 1: Solutions of a system; Solving by graphing or substitution

System of Equations - 2 or more equations that describe a system.

What is a solution of a system of equations?

- The values of the variables [(x,y) pairs] that make all the equations in the system true.
- The intersections of the equation curves.

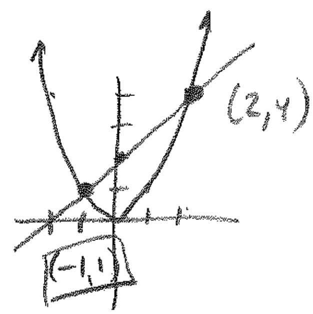


1 solution | multiple solutions | no solution | infinitely many solutions (all the points on the line)

Solving Systems of Equations - Graphically

1) Enter each equation in calculator (Y1=)
2) Find intersections using Calc - Intersection

Example: Solve system graphically: $\begin{cases} y = x^2 \\ y = x+2 \end{cases}$



Solving Systems of Equations - Algebraically using method of substitution

1) Solve one of the equations for one variable in terms of the other.
2) Substitute the expression found in step 1 into the other equation to obtain an equation in one variable.
3) Solve the equation from step 2.
4) Back-substitute the solution into the expression obtained in step 1.

Examples: Solve the systems by substitution

$$\begin{cases} 30x - 40y - 33 = 0 \\ 10x + 20y - 21 = 0 \end{cases}$$

$40y = 30x - 33$

$y = \frac{30}{40}x - \frac{33}{40}$

$10x + 20\left(\frac{30}{40}x - \frac{33}{40}\right) - 21 = 0$

$\left(10x + \frac{600}{40}x - \frac{660}{40} - 21 = 0\right)40$

$400x + 600x - 660 - 840 = 0$

$1000x = 1500$

$x = \frac{1500}{1000} = \boxed{\frac{3}{2}}$

$x = \frac{3}{2}$

$10x + 20y - 21 = 0$

$10\left(\frac{3}{2}\right) + 20y - 21 = 0$

$15 + 20y - 21 = 0$

$20y = 6$

$y = \frac{6}{20} = \boxed{\frac{3}{10}}$

$\boxed{\left(\frac{3}{2}, \frac{3}{10}\right)}$

$$\begin{cases} x^2 + y^2 = 5 \\ x + y = 1 \end{cases}$$

$y = 1 - x$

$x^2 + (1-x)^2 = 5$

$x^2 + (1-x)(1-x) = 5$

$x^2 + 1 - 2x + x^2 = 5$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

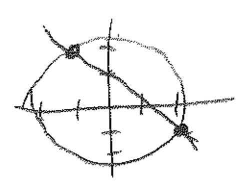
$x = 2$ or $x = -1$

$y = 1 - x$

$y = 1 - 2 \qquad y = 1 - (-1)$

$y = -1 \qquad y = 2$

$\boxed{(2, -1)} \qquad \boxed{(-1, 2)}$



Application examples:

A small business invests $10,000 in equipment to produce a product.
Each unit of the product costs $0.65 to produce and is sold for $1.20.
How many items must be sold before the business breaks even? $C=R$

$C = 0.65x + 10000$

$R = 1.2x$

$C = R$

$0.65x + 10000 = 1.2x$

$10000 = 0.55x$

$\frac{10000}{.55} = x$

$x = 18181.8$

$\boxed{18182 \text{ items}}$

A small business has an initial investment of $5000.
The unit cost of the product is $21.60, and the selling price is $34.10.
(a) Write the cost and revenue functions for x units of product.
(b) Find the break-even point algebraically.

(a) $C = 21.60x + 5000$

$R = 34.1x$

(b) $34.1x = 21.60x + 5000$

$12.5x = 5000$

$x = \frac{5000}{12.5}$

$x = \boxed{400 \text{ units}}$

Choice of 2 jobs: You are offered two different jobs selling college textbooks.
- One company offers an annual salary of $25,000 plus a year-end bonus of 1% of your total sales.
- The other company offers an annual salary of $20,000 plus a year-end bonus of 2% of your total sales.
Determine the annual sales that would make the second offer better.

$E_1 = 25000 + .01S$

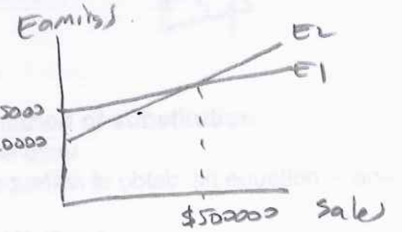
$E_2 = 20000 + .02S$

$E_1 = E_2$

$25000 + .01S = 20000 + .02S$

$5000 = .01S$

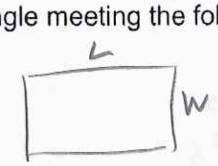
$S = \frac{5000}{.01} = \$500{,}000$



if sell > $500,000 worth of textbooks then 20,000/2% job better, otherwise 25000/1% better

(textbook ≈ $100)
500,000 = 5000 textbooks

Geometry: Find the dimensions of the rectangle meeting the following conditions:
- The perimeter is 42 inches.
- The width is three-fourths the length.



$P = 2L + 2W = 42$

$W = \frac{3}{4}L$

substitution:

$4\left(2L + 2\left(\frac{3}{4}L\right) = 42\right)$

$8L + 6L = 168$

$14L = 168$

$L = \frac{168}{14}$

$\boxed{L = 12 \text{ in}}$

$W = \frac{3}{4}L$

$W = \frac{3}{4}(12)$

$\boxed{W = 9 \text{ in}}$

7.1/7.2: Solving by elimination; more application examples

Solving System of Equations - Algebraically using Method of Elimination

1) Obtain coefficients for x (or y) that differ only in sign by multiplying all terms of one or both equations by suitably chosen constants.
2) Add the equations to eliminate one variable. Solve the resulting equation.
3) Back-substitute the value obtained into either original equation to solve for the other variable.
4) optional: Check your solution in both of the original equations.

Examples: Solve each system using elimination

$$\begin{cases} 2x-3y=-15 \\ 5x+2y=10 \end{cases}$$

$2(2x-3y=-15)$, $3(5x+2y=10)$

$4x-6y=-30$

$15x+6y=30$

$19x=0$

$x=0$

$2x-3y=-15$

$2(0)-3y=-15$

$y=\frac{-15}{-3}=5$

$\boxed{(0,5)}$

$$\begin{cases} 3x+4y=11 \\ x+2y=5 \end{cases}$$

$-3(x+2y=5)$

$3x+4y=11$

$-3x-6y=-15$

$-2y=-4$

$y=2$

$x+2y=5$

$x+2(2)=5$

$x+4=5$

$x=1$

$\boxed{(1,2)}$

$$\begin{cases} x-2y=3 \\ -2x+4y=1 \end{cases}$$

$2(x-2y=3)$

$2x-4y=6$

$-2x+4y=1$

$0=7$

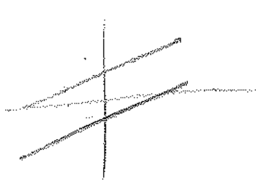
contradiction (can never be true)

so system has $\boxed{\text{no solution}}$

'inconsistent system'

$x-2y=3 \rightarrow y=\frac{1}{2}x-\frac{3}{2}$

$-2x+4y=1 \rightarrow y=\frac{1}{2}x+\frac{1}{4}$



$$\begin{cases} 2x-y=1 \\ 4x-2y=2 \end{cases}$$

$-2(2x-y=1)$

$-4x+2y=-2$

$4x-2y=2$

$0=0$

not a contradiction but means equations were identical (same line)

$\boxed{\text{infinitely many solutions}}$

but not every point, only points on line

one way to write this is 'parametric form'

$2x-y=1$

$2x=y+1$

$x=\frac{1}{2}y+\frac{1}{2}$

Solution is: $\boxed{\left(\frac{1}{2}y+\frac{1}{2}, y\right)}$

Some example solution points

$\left(\frac{1}{2},0\right)$ $(y=0)$

$(1,1)$ $(y=1)$

$(0,-1)$ $(y=-1)$

A total of $32,000 in invested in two municipal bonds that pay 5.75% and 6.25% simple interest. The investor wants an annual interest income of $1900 from the investments. What is the most that can be invested in the 5.75% bond?

X = amt in 5.75%
y = amt in 6.25%

$$\begin{cases} x + y = 32000 \\ .0575x + .0625y = 1900 \end{cases}$$

$$\begin{cases} .0575(x+y = 32000) \\ .0575x + .0625y = 1900 \end{cases}$$

$$\begin{cases} -.0575x - .0575y = -1840 \\ .0575x + .0625y = 1900 \end{cases}$$

$$.005y = 60 \rightarrow y = \frac{60}{.005} = 12000$$

$y = 12000$
$x + (12000) = 32000$
$x = 20000$

$\boxed{(\$20000, \$12000)}$ amt in 5.75% and in 6.25%

Five hundred gallons of 89 octane gasoline is obtained by mixing 87 octane gas with 92 octane gas.
(a) Write equations for total amount of fuel, and octane of fuel mix.
(b) How much of each type of gasoline is required to obtain the 500 gallons of 89 octane gas?

X = gals of 87 octane gas
Y = gals of 92 octane gas

$$x + y = 500$$
$$87x + 92y = 89(500)$$

$$-87(x + y = 500)$$
$$87x + 92y = 44500$$

$$-87x - 87y = -43500$$
$$87x + 92y = 44500$$
$$5y = 1000$$
$$y = \frac{1000}{5}$$
$$y = 200$$

$x + y = 500$
$x + (200) = 500$
$x = 300$

$\boxed{(300 \text{ gals}, 200 \text{ gals})}$ 87 octane, 92 octane

A man in a boat can row 8 miles downstream in one hour.
He can row 6 miles upstream in three hours.
How fast can the man row in still water and what is the rate of the current?

variables for what we are finding

rowing speed = r
water speed = w

$\xrightarrow{w}\xrightarrow{r}$ $r+w$ total speed downstream

$\xleftarrow{r}\xrightarrow{w}$ $r-w$ total speed upstream

$d = rt$
(distance = rate × time)

$d = r\,t$
$8 = (r+w)(1) \rightarrow (r + w = 8)3$
$6 = (r-w)(3) \rightarrow 3r - 3w = 6$

$$3r + 3w = 24$$
$$3r - 3w = 6$$
$$6r = 30$$
$$r = 5$$

$r + w = 8$
$(5) + w = 8$
$w = 3$

$\boxed{\text{rowing, } r = 5 \text{ mph; water, } w = 3 \text{ mph}}$

An airplane flying into a headwind travels the 1800-mile flying distance between two cities in 3 hours and 36 minutes. On the return flight, the distance is traveled in 3 hours. Find the airspeed of the plane and the speed of the wind, assuming both remain constant.

p w

$\xrightarrow{p}\xrightarrow{w}$ $p+w$ takes 3hrs

$\xrightarrow{p}\xleftarrow{w}$ $p-w$ takes 3hrs, 36min = 3.6hrs

$d = rt$
$$\begin{cases} 1800 = (p+w)3 \\ 1800 = (p-w)3.6 \end{cases}$$

$(3.6p - 3.6w = 1800)3$
$(3p + 3w = 1800)3.6$

$$10.8p - 10.8w = 5400$$
$$10.8p + 10.8w = 6480$$
$$21.6p = 11880$$
$$p = 550$$

$3p + 3w = 1800$
$3(550) + 3w = 1800$
$3w = 150$, $w = 50$

$\boxed{\text{plane} = 550 \text{ mph}, \text{ wind} = 50 \text{ mph}}$

7.2 day2: More application examples

HW #61 - Find the point of equilibrium of the demand and supply equations:
Demand: $p = 50 - 0.5x$
Supply: $p = 0.125x$

$D = S$

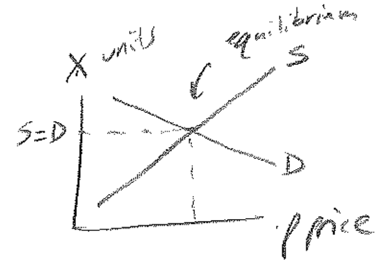
$50 - 0.5x = 0.125x$

$+0.5x \quad +0.5x$

$50 = 0.625x$

$x = \frac{50}{0.625} = 80$

$\boxed{80 \text{ units}}$ $\quad p = 0.125(80) = \boxed{\$10}$



HW #67 - Ten liters of a 30% acid solution is obtained by mixing a 20% and a 50% solution. Write a system of equations, and solve the system to determine how much of each solution is required to obtain the specified concentration in the final mixture.

x = liters of 20%
y = liters of 50%

$x + y = 10$

$.20x + .50y = .30(10)$

$-.2(x + y = 10)$

$.2x + .5y = 3$

$-.2x - .2y = -2$

$.2x + .5y = 3$

$.3y = 1$

$y = \frac{1}{.3} = 3.\overline{33}$

$x + (3.33) = 10$

$x = 6.6\overline{6}$

x = 6.66 liters 20% acid
y = 3.33 liters 50% acid

HW #69 - A total of $12,000 is invested in two corporate bonds that pay 7.5% and 9% simple interest. The investor wants an annual interest income of $990 from the investments. What is the most that can be invested in the 7.5% bond? $\boxed{\$6000}$

x = 7.5% amt
y = 9% amt

$(x + y = 12000)(-.075)$

$.075x + .09y = 990$

$-.075x - .075y = -900$

$.075x + .09y = 990$

$.015y = 90$

$y = \frac{90}{.015}$

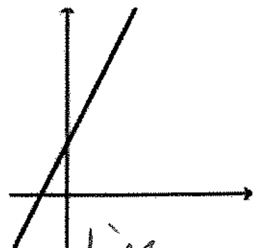
$y = \$6000 \quad x = 12000 - y = \$6000$

7.3 day 1: Multivariable Systems; Row operations; Gaussian Elimination

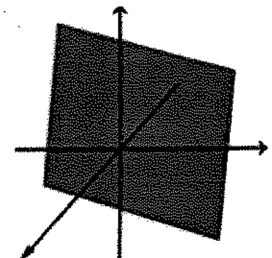
Multivariable Linear Systems:

1-dimension	2-dimensions	3-dimensions
x=2	4x - 2y = -2	4x - 2y + 3z = 5

point



line

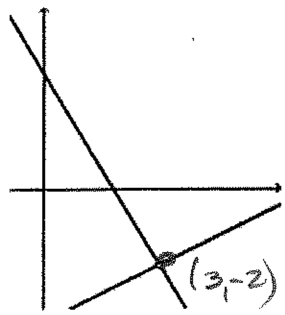


plane

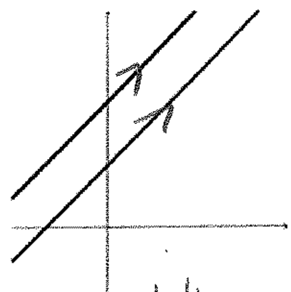
2-D systems of equations:

$$\begin{cases} 5x+3y=9 \\ 2x-4y=14 \end{cases}$$

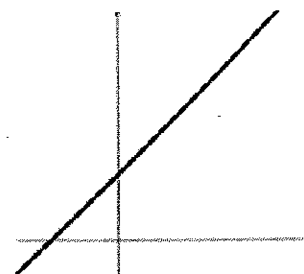
solution: (3, -2)



point



no solution

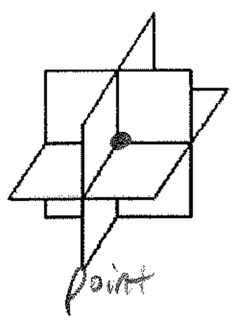


infinite solutions (same line)

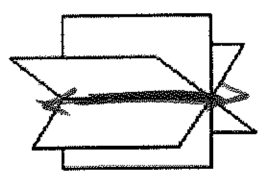
3-D systems of equations:

$$\begin{cases} x-2y+3z=9 \\ -x+3y \qquad =-4 \\ 2x-5y+5z=17 \end{cases}$$

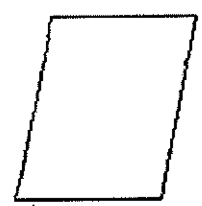
solution: (1, -1, 2)



point

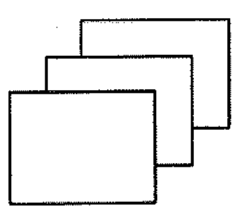


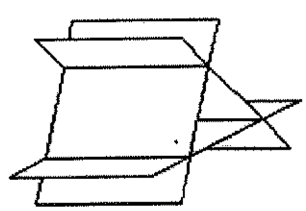
line

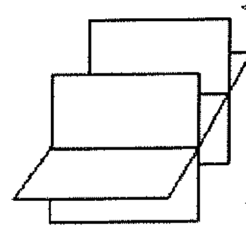


plane (coinciding planes)

consistent







inconsistent (no solutions)

Row-Echelon Form / Back-Substitution

Systems of equations are equivalent if they have the same solution set.

$$\begin{cases} x-2y+3z=9 \\ -x+3y \quad =-4 \\ 2x-5y+5z=17 \end{cases} \xrightarrow[\text{equivalent}]{} \begin{cases} x-2y+3z=9 \\ \quad y+3z=5 \\ \quad\quad z=2 \end{cases}$$

'back substitution'

$y+3(2)=5$

$y=-1$

$x-2(-1)+3(2)=9$

$x+8=9$

$x=1$

$(1,-1,2)$ solution

Gaussian Elimination - a way to get equivalent systems that are easier to solve

Elementary Row Operations - each of these produces an equivalent system

1) Interchange any 2 equations.

2) Multiply one equation by a nonzero constant.

3) Add a multiple of one equation to another equation.

This last one is just 'elimination method':

$$-2(x+2y+z=3)$$
$$2x-3y-2z=5$$
$$-7y-4z=-1$$

$$\begin{cases} x+2y+z=3 & \leftarrow R1 \text{ 'row 1'} \\ 2x-3y-2z=5 & \leftarrow R2 \text{ 'row 2'} \end{cases}$$

$$\begin{array}{r} \\ -2R_1+R_2 \end{array}\begin{cases} x+2y+z=3 \\ -7y-4z=-1 \end{cases}$$

Gaussian Elimination procedure:

- Get 'x' in first row (by dividing by coefficient or swapping rows)
- Use First row to clear x's in other rows.
- Get either 'y' or 'z' in 2nd row (whichever is easier)
- Use this to clear that variable in 3rd row.

(cancel)

$$\begin{cases} x-2y+3z=9 \\ -x+3y \quad =-4 \\ 2x-5y+5z=17 \end{cases}$$

$$\begin{array}{r} \\ R_1+R_2 \\ -2R_1+R_3 \end{array}\begin{cases} x-2y+3z=9 \\ y+3z=5 \\ -y-z=-1 \end{cases}$$

$$\begin{array}{r} \\ \\ R_2+R_3 \end{array}\begin{cases} x-2y+3z=9 \\ y+3z=5 \\ 2z=4 \end{cases}$$

3rd eqn: $2z=4$

$z=\frac{4}{2}$

$\underline{z=2}$

2nd eqn: $y+3z=5$

$y+3(2)=5$

$y+6=5$

$\underline{y=-1}$

1st eqn: $x-2y+3z=9$

$x-2(-1)+3(2)=9$

$x+2+6=9$

$x+8=9$

$\underline{x=1}$

$$\boxed{\text{Solution: } (1,-1,2)}$$

$$\begin{cases} 4x + y - 3z = 11 \\ 2x - 3y + 2z = 9 \\ x + y + z = -3 \end{cases}$$

$$\begin{matrix} R_3 \\ \\ R_1 \end{matrix}\begin{cases} x + y + z = -3 \\ 2x - 3y + 2z = 9 \\ 4x + y - 3z = 11 \end{cases}$$

$$\begin{matrix} \\ -2R_1+R_2 \\ -4R_1+R_3 \end{matrix}\begin{cases} x + y + z = -3 \\ -5y = 15 \\ -3y - 7z = 23 \end{cases}$$

$$\begin{matrix} \\ R_3 \\ R_2 \end{matrix}\begin{cases} x + y + z = -3 \\ -3y - 7z = 23 \\ -5y = 15 \end{cases}$$

$-5y = 15$, $\underline{y = -3}$

$-3y - 7z = 23$
$-3(-3) - 7z = 23$
$9 - 7z = 23$
$-7z = 14$
$\underline{z = -2}$

$x + y + z = -3$
$x + (-3) + (-2) = -3$
$x = 2$ $\boxed{(2, -3, -2)}$

$$\begin{cases} 2x + 2z = 6 \\ 5x + 3y = 11 \\ 3y - 4z = 1 \end{cases}$$

$$\begin{matrix} \frac{1}{2}R_1 \\ \\ \\ \end{matrix}\begin{cases} x + z = 3 \\ 5x + 3y = 11 \\ 3y - 4z = 1 \end{cases}$$

$$\begin{matrix} \\ -5R_1+R_2 \\ \\ \end{matrix}\begin{cases} x + z = 3 \\ 3y - 5z = -4 \\ 3y - 4z = 1 \end{cases}$$

$$\begin{matrix} \\ \\ -R_2+R_3 \end{matrix}\begin{cases} x + z = 3 \\ 3y - 5z = -4 \\ z = 5 \end{cases}$$

$\underline{z = 5}$

$3y - 5z = -4$
$3y - 5(5) = -4$
$3y - 25 = -4$
$3y = 21$
$\underline{y = 7}$

$x + z = 3$
$x + (5) = 3$
$x = -2$
$\boxed{(-2, 7, 5)}$

$$\begin{cases} x - 3y + z = 1 \\ 2x - y - 2z = 2 \\ x + 2y - 3z = -1 \end{cases}$$

$$\begin{matrix} \\ -2R_1+R_2 \\ -R_1+R_3 \end{matrix}\begin{cases} x - 3y + z = 1 \\ 5y - 4z = 0 \\ 5y - 4z = -2 \end{cases}$$

$$\begin{matrix} \\ \\ -R_2+R_3 \end{matrix}\begin{cases} x - 3y + z = 1 \\ 5y - 4z = 0 \\ \underline{0 = -2} \end{cases}$$

inconsistent
(0 can never $= -2$)
so original system has $\boxed{\text{no solution}}$

Application example:

Find the position equation: $s = at^2 + bt + c$

for an object at the given heights moving vertically at the specified times.

At $t = 1$ second, $s = 128$ feet
At $t = 2$ seconds, $s = 80$ feet
At $t = 3$ seconds, $s = 0$ feet

plug each data point in:

$a(1)^2 + b(1) + c = 128 \Rightarrow$
$a(2)^2 + b(2) + c = 80 \Rightarrow$
$a(3)^2 + b(3) + c = 0 \Rightarrow$

$$\begin{cases} a + b + c = 128 \\ 4a + 2b + c = 80 \\ 9a + 3b + c = 0 \end{cases}$$

the 'constants' (parameters) become the 'variables' in a system which can be solved

$$\begin{cases} a + b + c = 128 \\ 4a + 2b + c = 80 \\ 9a + 3b + c = 0 \end{cases}$$

$$\begin{matrix} \\ -4R_1+R_2 \\ -9R_1+R_3 \end{matrix}\begin{cases} a + b + c = 128 \\ -2b - 3c = -432 \\ -6b - 8c = -1152 \end{cases}$$

$$\begin{matrix} \\ \\ -3R_2+R_3 \end{matrix}\begin{cases} a + b + c = 128 \\ -2b - 3c = -432 \\ \underline{c = 144} \end{cases}$$

$-2b - 3(144) = -432$
$-2b - 432 = -432$
$-2b = 0$
$\underline{b = 0}$

$a + b + c = 128$
$a + (0) + (144) = 128$
$\underline{a = -16}$

equation: $s = -16t^2 + 0t + 144$
$\boxed{s = -16t^2 + 144}$

7.3 day 2: Infinitely many solutions case; Non-square systems

Example: A system with infinitely many solutions

$$\begin{cases} x + y - 3z = -1 \\ y - z = 0 \\ -x + 2y = 1 \end{cases}$$

$$\begin{cases} x + y - 3z = -1 \\ y - z = 0 \\ 3y - 3z = 0 \end{cases} \quad R_1 + R_3$$

$$\begin{cases} x + y - 3z = -1 \\ y - z = 0 \\ 0 = 0 \end{cases} \quad -3R_2 + R_3$$

consistent
but no new information

Really only 2 equations
in this system.

Solve for everything
in terms of z:

$$\begin{cases} x + y - 3z = -1 \\ y - z = 0 \end{cases}$$

$$\begin{aligned} y - z &= 0 \\ +z \quad &+z \\ \underline{y} &\underline{= z} \end{aligned}$$

$$\begin{aligned} x + y - 3z &= -1 \\ x + (z) - 3z &= -1 \\ x - 2z &= -1 \\ \underline{x} &\underline{= 2z - 1} \end{aligned}$$

general solution:

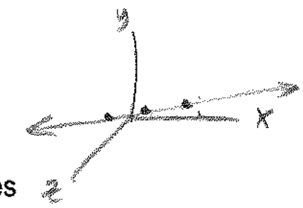
$(2z-1, z, z)$

sometimes another letter is used:

$\boxed{(2a-1, a, a)}$

Some specific points on
the solution line:

$a=0$	$(-1, 0, 0)$
$a=1$	$(1, 1, 1)$
$a=2$	$(3, 2, 2)$
$a=100$	$(199, 100, 100)$



Nonsquare System = a system in which # of equations is different than # of variables

Example: nonsquare system

$$\begin{cases} x - 2y + z = 2 \\ 2x - y - z = 1 \end{cases}$$

$$\begin{cases} x - 2y + z = 2 \\ 3y - 3z = -3 \end{cases} \quad -2R_1 + R_2$$

$$\begin{cases} x - 2y + z = 2 \\ y - z = -1 \end{cases} \quad \tfrac{1}{3}R_1$$

2 equations, 3 variables
so infinitely many solutions.

$$\begin{aligned} y - z &= -1 \\ \underline{y} &\underline{= z - 1} \end{aligned}$$

$$\begin{aligned} x - 2y + z &= 2 \\ x - 2(z-1) + z &= 2 \\ x - 2z + 2 + z &= 2 \\ x - z &= 0 \\ \underline{x} &\underline{= z} \end{aligned}$$

general solution:

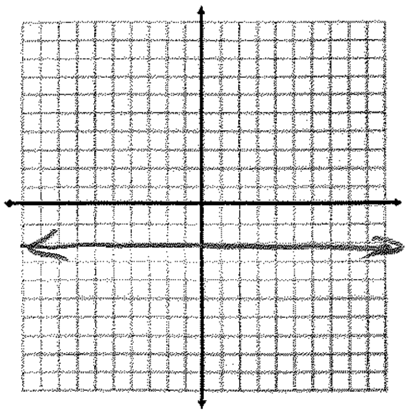
$(z, z-1, z)$ or $\boxed{(a, a-1, a)}$

7.4 day 1: Systems of inequalities

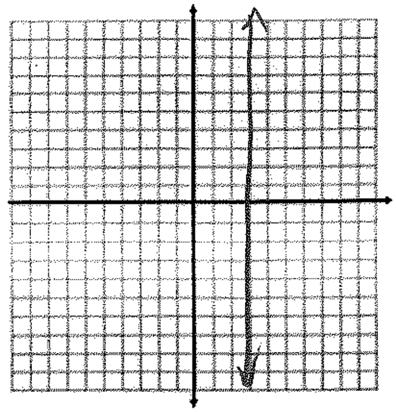
intercept method
$3x+\cancel{2y}=6$
$x=2$
$\cancel{3x}+2y=6$
$y=3$

Quick review of graphing: Graph each equation.

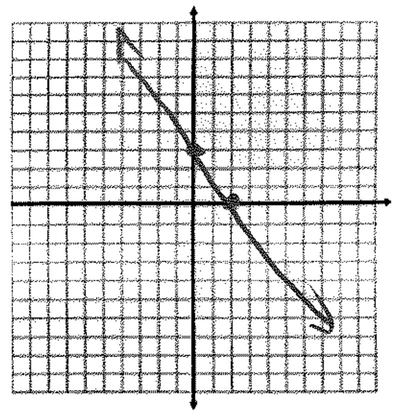
$y=-2$



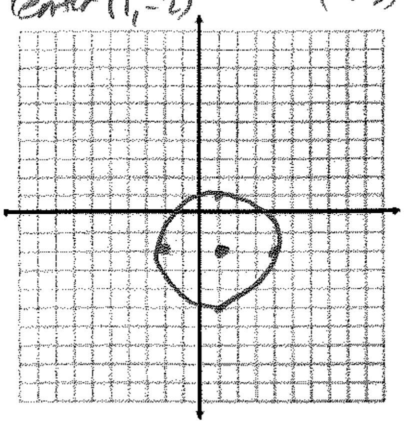
$x=3$



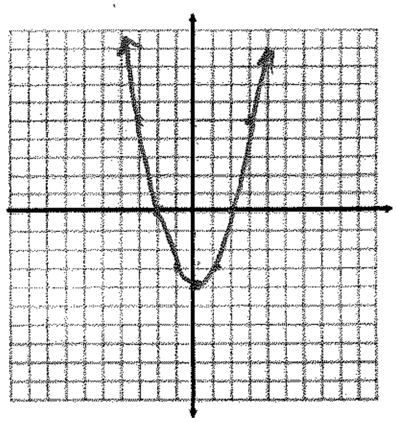
$3x+2y=6$



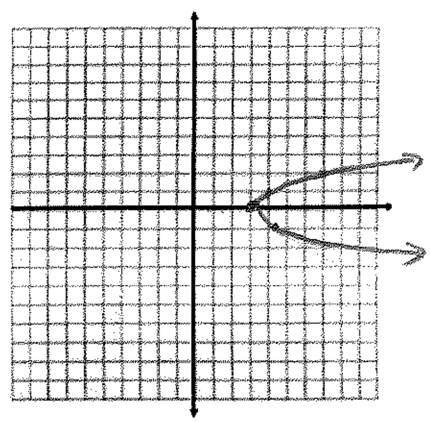
$(x-1)^2+(y+2)^2=9$ r^2
center $(1,-2)$ $(r=3)$



$x^2=y+4$ $y=x^2-4$



$y^2=x-3$ $x=y^2+3$



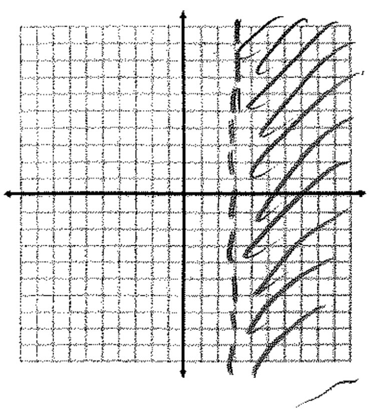
Graphing inequalities - Turn into an equation, and graph, then shade appropriate side.

$\geq or \leq$ solid line Shading: $\geq or >$ above or to the right

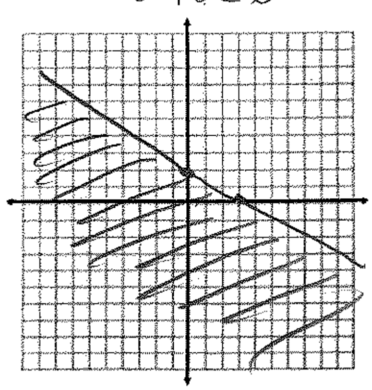
$> or <$ dotted line $\leq or <$ below or to the left

-or-
use a test point

$x>3$

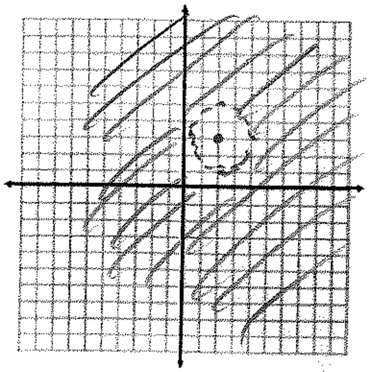


$2x+3y\leq 6$
$0+0<6$ ✓



center $(2,3)$ $r=2$

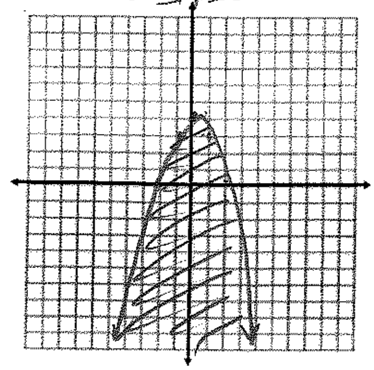
$$(x-2)^2+(y-3)^2>4$$



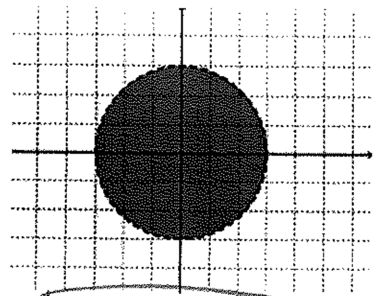
$(y=-x^2+4)$

$$y \le 4-x^2$$

$0 \le y - 0$



Write an inequality for the shaded region:

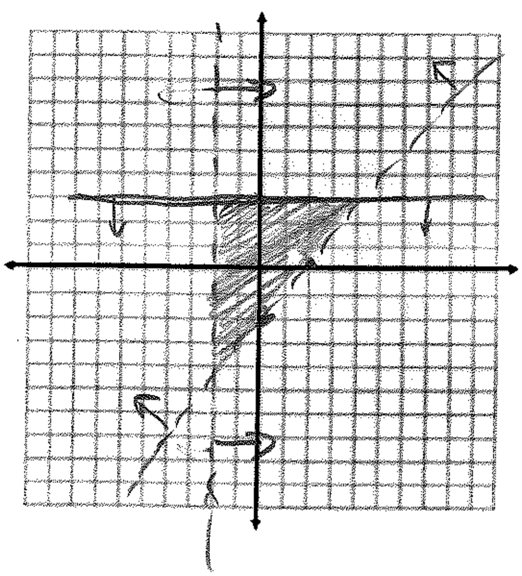


$\boxed{x^2+y^2<9}$

Graphing systems of inequalities:

- Graph each inequality in the system, including shading (use different colors, or different cross-hatching marks.)
- The solution is a region - the area that 'overlaps' (is shaded for all inequalities in the system.)
- The solution may be: bounded or unbounded, or there may be no solution (no overlap.)

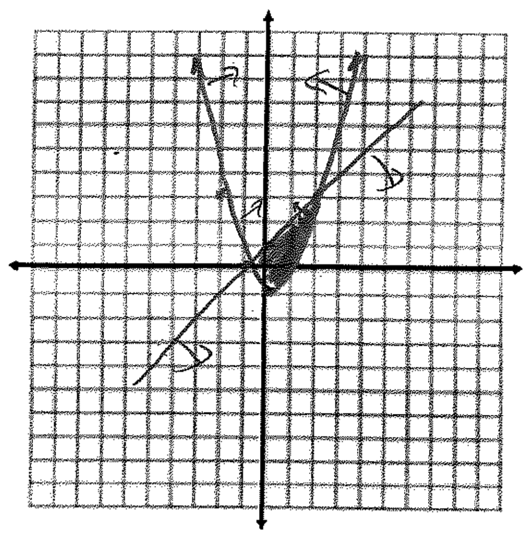
$$\begin{cases} x-y<2 \\ x>-2 \\ y\le 3 \end{cases}$$



$x^2-y=1$

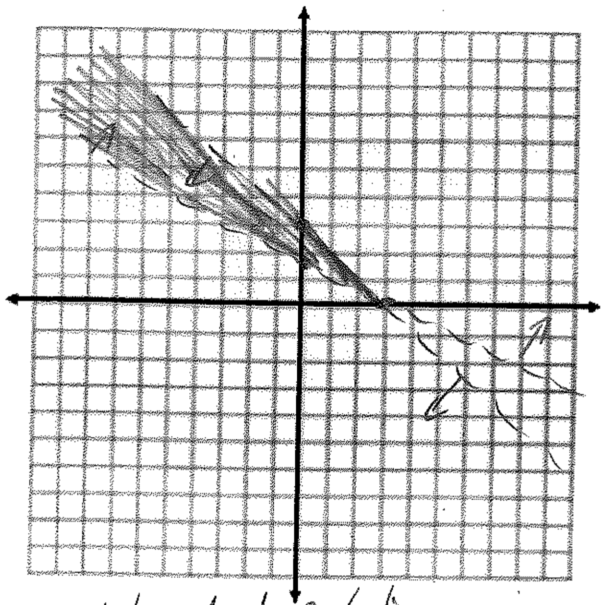
$y=x^2-1$

$$\begin{cases} x^2-y\le 1 \\ -x+y\le 1 \end{cases}$$

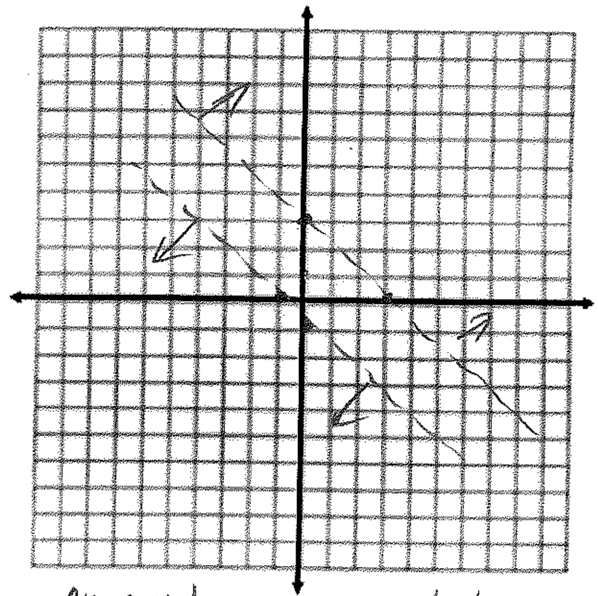


$$\begin{cases} x+y<3 \\ x+2y>3 \end{cases}$$

$$\begin{cases} x+y>3 \\ x+y<-1 \end{cases}$$



unbounded solution



no overlap = no solution

7.4 day 2: Application Examples of Inequality Systems

Concert Ticket Sales - One type of concert ticket costs $15 and another costs $25. The promoter of a concert must sell at least 15,000 tickets, including at least 8,000 of the $15 tickets and at least 4,000 of the $25 tickets, and the gross receipts must total at least $275,000 in order for the concert to be held.

x = number of $15 tickets
y = number of $25 tickets

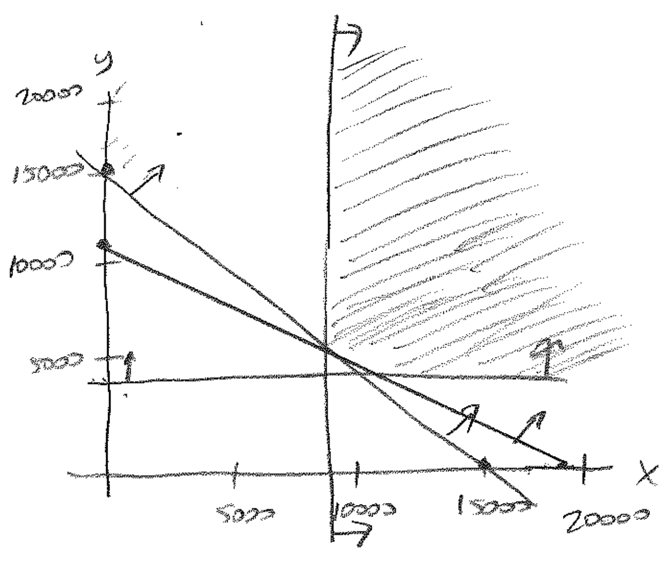
($15 tickets) ($25 tickets)

total number: $x + y \geq 15000$

$15 num: $x \geq 8000$

$25 num: $y \geq 4000$

total sales: $15x + 25y \geq 275000$

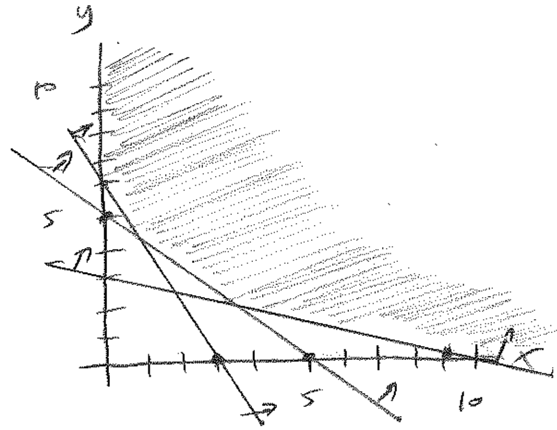


Nutrition - The liquid portion of a diet is to provide at least 300 calories, 36 units of vitamin A, and 90 units of vitamin C daily.

2 dietary drinks are available:

	drink X	drink Y
calories	60	60
vitamin A	12	6
vitamin C	10	30

Set up a system of linear inequalities that must be satisfied in order to meet the minimum daily requirements for calories and vitamins.



(drink X) (drink Y)

(calories) $60x + 60y \geq 300$

(vit A) $12x + 6y \geq 36$

(vit C) $10x + 30y \geq 90$

$x \geq 0$

$y \geq 0$

Economics - Mike's Famous Toy Trucks manufactures two kinds of toy trucks - a standard model and a deluxe model. The manufacturing process has two stages: cutting and finishing (different for each model):

	Standard model	Deluxe model
Cutting:	2 hrs	2 hrs
Finishing:	2 hrs	4 hrs

The company has 2 workers who do cutting and 3 who do finishing, each of whom works at most 40 hours per week.
Each standard model toy truck brings a profit of \$3 and each deluxe model a profit of \$4. Assuming that every truck made will be sold, how many of each should be made to maximize profit?

x = number std trucks
y = number deluxe truck

(std) (deluxe)

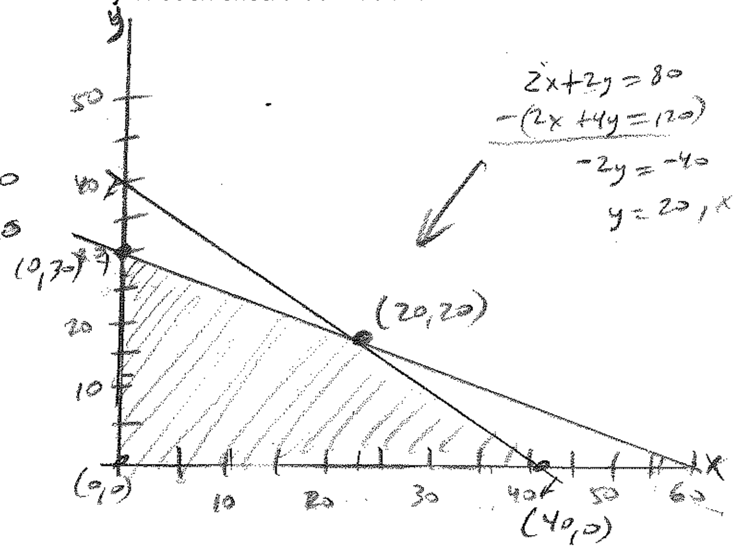
(cutting) $2x + 2y \leq 80$

(finishing) $2x + 4y \leq 120$

$x \geq 0$

$y \geq 0$

$2x+2y=80$
$-(2x+4y=120)$
$-2y=-40$
$y=20, x=20$



Profit $= P = 3x+4y$

corner	profit
(0,0)	\$0
(0,30)	\$120
(20,20)	\$140
(40,0)	\$120