Honors Algebra 3-4 Name Key

9.1-9.3 Review #2 Period ______

$a_n = a_1 + (n-1)d$ $\qquad a_n = a_1(r)^{n-1}$

$S = \frac{n}{2}(a_1 + a_n)$ $\qquad S = a_1\left(\frac{1-r^n}{1-r}\right)$ $\qquad S = \frac{a_1}{1-r}$

1. Write the first five terms of the sequence where $a_1 = 3$ and $a_{n+1} = 2a_n(3n-1)$

$a_2 = 2(3)[3(1)-1] = 12$

$a_3 = 2(12)[3(2)-1] = 120$

$a_4 = 2(120)[3(3)-1] = 1920$

$a_5 = 2(1920)[3(4)-1] = 42240$

$\boxed{3, 12, 120, 1920, 42240}$

2. Find a formula for the nth term of this sequence: $1, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{24}{5}, 20 \ldots$

(hint: make the first term a fraction and note a pattern in the denominators)

$\frac{1}{1} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{6}{4} \quad \frac{24}{5} \quad \frac{120}{6}$

n:	1	2	3	4	5	6
$n!$	1	2	6	24	120	
$(n-1)!$	1	1	2	6	24	120

$\boxed{a_n = \frac{(n-1)!}{n}}$

3. Use sigma notation to write the given sum: $3+1-1-3-5$ arithmetic

$\overset{-2}{\rightarrow}\overset{-2}{\rightarrow}\overset{-2}{\rightarrow}$

(don't compute the sum – write using a $\sum$ and assume n begins with 1)

$a_n = 3 - 2(n-1)$ so $\boxed{\sum_{n=1}^{5} 3-2(n-1)}$ or $\boxed{\sum_{n=1}^{5} 5-2n}$

4. Find the sums:

(a) $\sum_{n=1}^{4} \frac{3n^2}{n+1}$

$\frac{3(1)^2}{(1)+1} + \frac{3(2)^2}{(2)+1} + \frac{3(3)^2}{(3)+1} + \frac{3(4)^2}{(4)+1}$ use calculator

$\frac{3}{2} + \frac{12}{3} + \frac{27}{4} + \frac{48}{5} = \boxed{\frac{437}{20}}$

(b) $\sum_{n=0}^{\infty} 10\left(\frac{1}{2}\right)^n$ infinite geometric w/ $|r|<1$ converges to $S = \frac{a_1}{1-r} = \frac{10}{1-\frac{1}{2}} = \boxed{20}$

5. Given $a_3 = -2,\ a_6 = -11$ for an ***arithmetic sequence*** find:

(a) d

$a_6 = a_1 + d(6-1) = -11$

$-a_3 = -(a_1 + d(3-1)) = -(-2)$

$5d$

$-2d$

$3d = -9$

$\boxed{d = -3}$

(b) a_1

$a_3 = a_1 + d(3-1)$

$-2 = a_1 - 3(2)$

$-2 = a_1 - 6$

$+6 \quad +6$

$\boxed{4 = a_1}$

(c) a_n

$\boxed{a_n = 4 - 3(n-1)}$

(d) a_{100}

$a_{100} = 4 - 3(100-1)$

$= 4 - 3(99)$

$= \boxed{-293}$

6. Given $a_2 = 8,\ a_5 = 64$ for an ***geometric sequence*** find:

(a) r (b) a_1 (c) a_n (d) a_8

(a) $\frac{a_5}{a_2} = \frac{a_1 r^{5-1}}{a_1 r^{2-1}} = \frac{64}{8}$

$= \frac{r^4}{r^1} = r^3 = 8$

so $\boxed{r=2}$

(b) $a_2 = a_1 r^{2-1}$

$8 = a_1 (2)^1$

$\boxed{a_1 = 4}$

(c) $a_n = a_1 (r)^{n-1}$

$\boxed{a_n = 4(2)^{n-1}}$

(d) $a_8 = 4(2)^{8-1}$

$a_8 = 4(128) = \boxed{512}$

7. Find the sum of the first 40 terms of the sequence which begins: $1, 5, 9, 13, 17...$

$+4\ +4\ +4$ arithmetic $d=4$

$a_1 = 1$

$a_n = 1 + 4(n-1)$

so $a_{40} = 1 + 4(40-1)$

$= 157$

$S = \frac{n}{2}(a_1 + a_n)$

$S = \frac{40}{2}(1 + 157)$

$\boxed{S = 3160}$

8. Simplify each ratio fully:

(a) $\frac{15!}{8!4!}$ (b) $\frac{(n+2)!}{(n-1)!}$

(a) $\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot \cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 15 \cdot 7 \cdot 13 \cdot 11 \cdot 10 \cdot 9$

$= \boxed{1351350}$

(b) $= \frac{(n+2)(n+1)(n)\cancel{(n-1)(n-2)\cdots 1}}{\cancel{(n-1)(n-2)\cdots 1}}$

$= \boxed{(n+2)(n+1)(n)}$

or $(n^2+3n+2)(n) = \boxed{n^3+3n^2+2n}$

9. Logs are stacked in a pile. Each row of logs has one less log than the row below it. The top row has 8 logs and the bottom row has 20 logs. How many total logs are in the pile?

$(a_1 = 8)$

8 logs $)+1$

9

10 $)+1$

20 logs

$(a_{13} = 20)$

arithmetic

$a_1 = 8$

number of rows

$(20-8)+1$

$n = 13$ rows

$S = \frac{n}{2}(a_1 + a_n)$

$S = \frac{13}{2}(8 + 20)$

$\boxed{S = 182 \text{ logs}}$

10. A city of 150,000 people is growing at a rate of 5% per year. The city's population can be modeled using a geometric sequence.

(a) Write a formula for the population, P, of the city versus t if P=150,000 when t=1.

(b) Use this formula to find the population when t=10.

(a) $P = P_0 (r)^t$

$\boxed{P = 150{,}000\,(1.05)^t}$

(b) $P = 150{,}000\,(1.05)^{10} = \boxed{244{,}334}$

Honors Algebra 3-4 /Precalculus

9.1-9.3 Review #2

Name______________________

Period______

$a_n = a_1 + (n-1)d$ $\qquad$ $a_n = a_1(r)^{n-1}$

$S = \frac{n}{2}(a_1 + a_n)$ $\qquad$ $S = a_1\left(\frac{1-r^n}{1-r}\right)$ $\qquad$ $S = \frac{a_1}{1-r}$

1. Write the first five terms of the sequence where $a_1 = 3$ and $a_{n+1} = 2a_n(3n-1)$

2. Find a formula for the nth term of this sequence: $1, \frac{1}{2}, \frac{2}{3}, \frac{3}{2}, \frac{24}{5}, 20...$
(hint: make the first term a fraction and note a pattern in the denominators)

3. Use sigma notation to write the given sum: $3+1-1-3-5$
(don't compute the sum – write using a $\sum$ and assume n begins with 1)

4. Find the sums:
(a) $\sum_{n=1}^{4} \frac{3n^2}{n+1}$ $\qquad$ (b) $\sum_{n=0}^{\infty} 10\left(\frac{1}{2}\right)^n$

5. Given $a_3 = -2$, $a_6 = -11$ for an ***arithmetic sequence*** find:
(a) d $\qquad$ (b) a_1 $\qquad$ (c) a_n $\qquad$ (d) a_{100}

6. Given $a_2 = 8,\ a_5 = 64$ for an *geometric sequence* find:

(a) r (b) a_1 (c) a_n (d) a_8

7. Find the sum of the first 40 terms of the sequence which begins: $1, 5, 9, 13, 17...$

8. Simplify each ratio fully:

(a) $\dfrac{15!}{8!4!}$ (b) $\dfrac{(n+2)!}{(n-1)!}$

9. Logs are stacked in a pile. Each row of logs has one less log than the row below it. The top row has 8 logs and the bottom row has 20 logs. How many total logs are in the pile?

10. A city of 150,000 people is growing at a rate of 5% per year. The city's population can be modeled using a geometric sequence.

(a) Write a formula for the population, P, of the city versus t if P=150,000 when t=1.

(b) Use this formula to find the population when t=10.