

**Precalculus Spring Semester Final Review #1**

1. a. Convert  $\theta = \frac{5\pi}{8}$  to degrees.      b. Convert  $\theta = 175^\circ$  to radians (leave answer in terms of  $\pi$ ).

A  $0.034^\circ$

B  $1.96^\circ$

C  $56.5^\circ$

D  $112.5^\circ$

A  $\frac{35\pi}{36}$

B  $\frac{3\pi}{4}$

C  $\frac{8\pi}{9}$

D  $\frac{5\pi}{3}$

2. If the cosecant of an acute angle is  $\frac{25}{24}$ , find the value of the cosine.

A  $\frac{24}{25}$

B  $\frac{7}{25}$

C  $\frac{25}{7}$

D  $\frac{-24}{25}$

3. a. Find the reference angle for:  $\theta = 340^\circ$

A  $-20^\circ$

B  $20^\circ$

C  $-160^\circ$

D  $160^\circ$

- b. Find the reference angle for:  $\theta = \frac{-17\pi}{3}$

A  $\frac{17\pi}{3}$

B  $\frac{-\pi}{3}$

C  $\frac{\pi}{3}$

D  $\frac{2\pi}{3}$

4. Given the following, find  $\theta$ ,  $0^\circ \leq \theta \leq 360^\circ$ :

a.  $\tan \theta = 3.8958$

A  $1.32^\circ$

B  $0.068^\circ$

C  $356.1^\circ$

D  $75.6^\circ$

b.  $\sin \theta = .2654$

A  $15.39^\circ$

B  $0.269^\circ$

C  $0.0046^\circ$

D  $89.7346^\circ$

c.  $\sec \theta = 2.0717$

A  $61.14^\circ$

B  $0.999^\circ$

C  $28.86^\circ$

D  $1.067^\circ$

5. Given  $\cot \theta = \frac{-12}{5}$  and  $\cos \theta < 0$ ,  
find  $\csc \theta$

- A  $\frac{5}{13}$
- B  $\frac{13}{5}$
- C  $\frac{-13}{5}$
- D  $\frac{-5}{13}$

6. Given  $\sec \theta = \frac{-3}{2}$  and  $\cot \theta > 0$ ,  
find  $\sin \theta$

- A  $\frac{-3\sqrt{5}}{5}$
- B  $\frac{\sqrt{5}}{3}$
- C  $\frac{-\sqrt{5}}{3}$
- D  $\frac{3\sqrt{5}}{5}$

7. Two 100 ft guy wires are attached to the top of a telephone pole, one on each side. The angle of elevation of each wire with the ground is  $57^\circ$ . How tall is the telephone pole (to the nearest foot)?

- A 154 ft
- B 43 ft
- C 54 ft
- D 84 ft

8. From a 75-foot observation tower on the coast, a Coast Guard officer sights a boat in difficulty. The boat is 625 feet from the base of the tower. Find the angle of depression from the top of the tower to the boat.

- A  $6.89^\circ$
- B  $83.11^\circ$
- C  $6.84^\circ$
- D  $83.16^\circ$

9. Find the exact value of the expression:

a:  $\csc\left[\cos^{-1}\left(\frac{-3}{4}\right)\right]$

A  $\frac{4\sqrt{7}}{7}$

B  $-\frac{4\sqrt{7}}{7}$

C  $\frac{\sqrt{7}}{4}$

D  $-\frac{\sqrt{7}}{4}$

b:  $\tan\left[\sin^{-1}\left(\frac{-8}{17}\right)\right]$

A  $\frac{8}{15}$

B  $-\frac{8}{15}$

C  $\frac{15}{8}$

D  $-\frac{15}{8}$

10. a: Find the period in degrees:

$$f(x) = \cos\left(\frac{1}{4}x\right)$$

A  $1440^\circ$

B  $720^\circ$

C  $360^\circ$

D  $90^\circ$

b: Find the period in radians:

$$f(x) = \tan(6x)$$

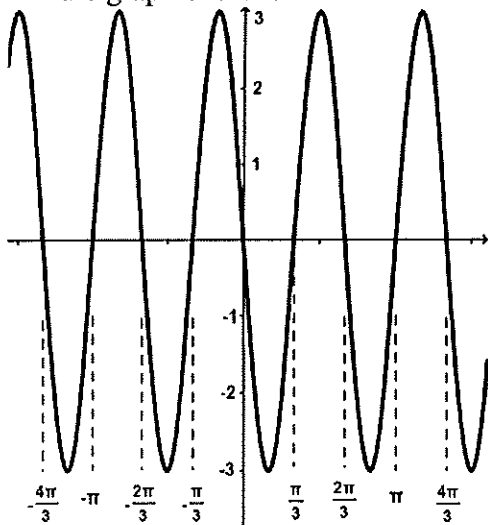
A  $3\pi$

B  $6\pi$

C  $\frac{\pi}{3}$

D  $\frac{\pi}{6}$

11. Which of the following equations describe the graph shown?



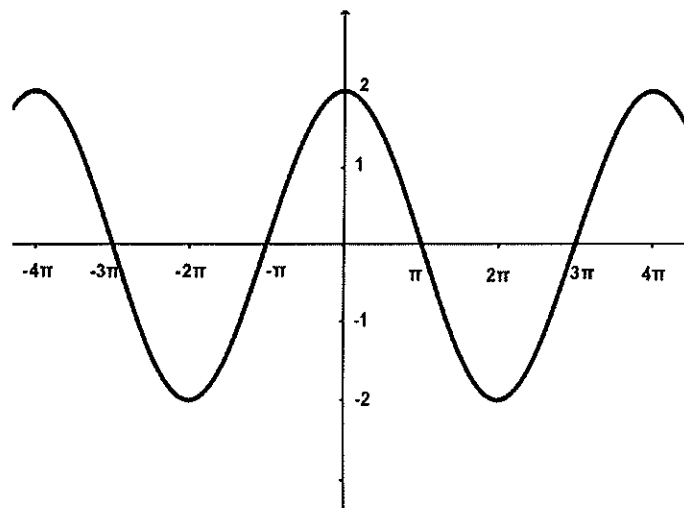
A  $y = -3\sin\left(\frac{x}{3}\right)$

B  $y = \frac{1}{3}\sin(3x)$

C  $y = \sin\left(\frac{x}{3}\right)$

D  $y = -3\sin(3x)$

12. Which of the following equations describe the graph shown?



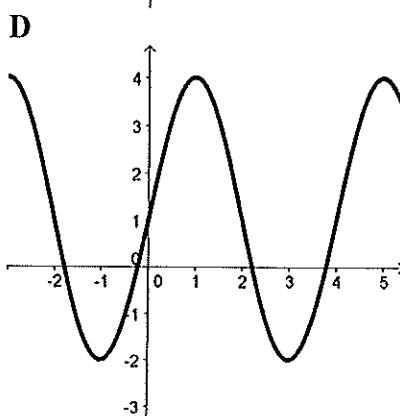
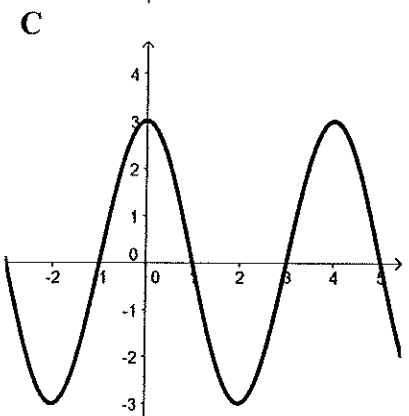
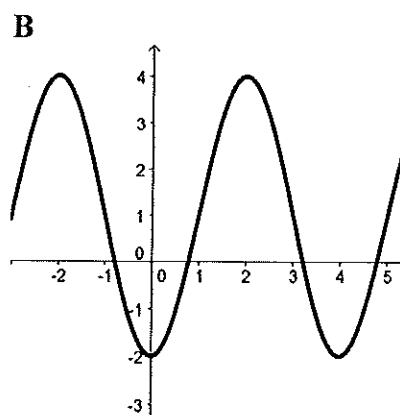
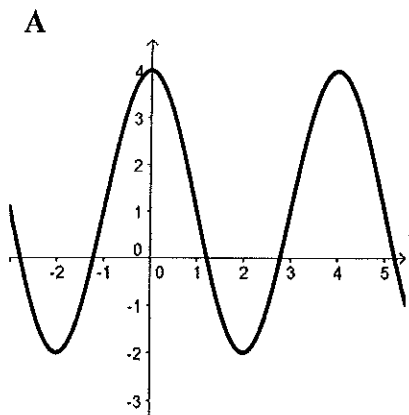
A  $y = \frac{1}{2}\cos\left(\frac{x}{2}\right)$

B  $y = 2\cos(2x)$

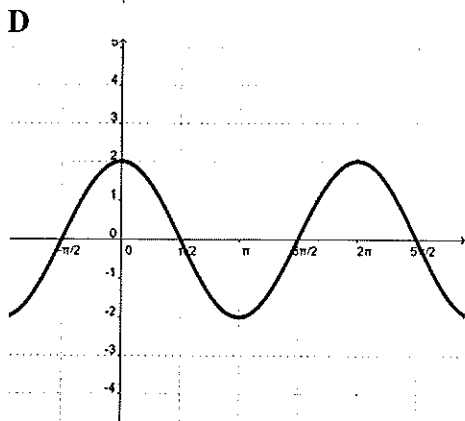
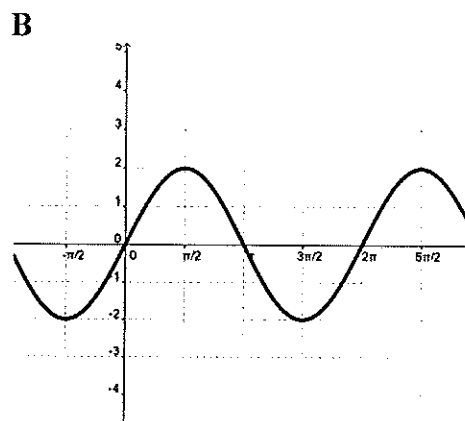
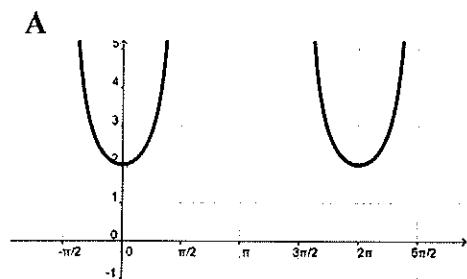
C  $y = 2\cos\left(\frac{x}{2}\right)$

D  $y = \frac{1}{2}\cos(2x)$

13. Which is the graph of  $f(x) = -3\cos\left(\frac{\pi}{2}x + \pi\right) + 1$



14. Which is the graph of  $f(x) = 2\csc(x)$



15. Simplify:  $\tan^2 x - \sin^2 x \tan^2 x$

- A  $\cos^2 x$
- B  $\sec^2 x$
- C 1
- D  $\sin^2 x$

16. Simplify:  $\frac{\sec^2 x - 1}{\sec x - 1}$

- A  $\cos x$
- B  $\sec x + 1$
- C  $\csc x - 1$
- D 1

17. Perform the addition and simplify:  $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}$

- A  $\frac{2 + \cos x}{1 - \cos x}$
- B -1
- C  $2 \csc^2 x$
- D  $\frac{1}{2} \sec^2 x$

18. Perform the subtraction and simplify:  $\cos x \sec x - \frac{\cos x}{\sec x}$

- A  $\sin^2 x$
- B  $1 - \cos x$
- C  $\cos x (\sec^2 x - 1)$
- D -1

19. Find all solutions in the interval  $[0, 2\pi)$ :  $4 \cos^2 x - 1 = 0$

- A  $\frac{\pi}{3}, \frac{5\pi}{3}$
- B  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
- C  $\frac{\pi}{6}, \frac{11\pi}{6}$
- D  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

20. Find all solutions in the interval  $[0, 2\pi)$ :  $2 \sin x \cos x = \sqrt{2} \sin x$

A  $\frac{\pi}{4}, \frac{7\pi}{4}$

B  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

C  $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}$

D  $0, \pi, \frac{\pi}{4}, \frac{7\pi}{4}$

21. Find all solutions in the interval  $[0, 2\pi)$ :  $\sec^2 x - \sec x = 2$

A  $\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

B  $\frac{\pi}{2}, \frac{\pi}{6}, \frac{11\pi}{6}$

C  $\pi, \frac{\pi}{3}, \frac{2\pi}{3}$

D  $0, \pi, \frac{2\pi}{3}, \frac{5\pi}{3}$

22. Find all solutions in the interval  $[0, 2\pi)$ :  $2 \sin^2 x - 5 \sin x + 2 = 0$

A  $\frac{\pi}{3}, \frac{2\pi}{3}$

B  $\frac{\pi}{3}, \frac{5\pi}{3}$

C  $\frac{\pi}{6}, \frac{11\pi}{6}$

D  $\frac{\pi}{6}, \frac{5\pi}{6}$

23. Evaluate:  $\cos 255^\circ$  (Use the fact that  $255^\circ = 315^\circ - 60^\circ$ )

A  $\frac{\sqrt{2} + \sqrt{6}}{4}$

B  $\frac{\sqrt{6} - \sqrt{2}}{4}$

C  $\frac{\sqrt{2} - \sqrt{6}}{4}$

D  $\frac{\sqrt{1} + \sqrt{3}}{2}$

24. Evaluate:  $\sin 105^\circ$  (Use the fact that  $105^\circ = 150^\circ - 45^\circ$ )

A  $\frac{\sqrt{2} + \sqrt{6}}{4}$

B  $\frac{\sqrt{6} - \sqrt{2}}{4}$

C  $\frac{\sqrt{2} - \sqrt{6}}{4}$

D  $\frac{\sqrt{1} + \sqrt{3}}{2}$

25. Given  $\cos u = \frac{-12}{13}$ ,  $\pi < u < \frac{3\pi}{2}$  and  $\csc v = \frac{\sqrt{10}}{3}$ ,  $\frac{\pi}{2} < v < \pi$ , find  $\cos(u + v)$

A  $\frac{-36 + 13\sqrt{10}}{39}$

B  $\frac{9\sqrt{10}}{10}$

C  $\frac{27\sqrt{10}}{130}$

D  $\frac{41\sqrt{10}}{130}$

26. Given  $\cot u = \frac{2}{5}$ ,  $0 < u < \frac{\pi}{2}$  and  $\cos v = \frac{-3}{5}$ ,  $\pi < v < \frac{3\pi}{2}$ , find  $\tan(u + v)$

A 5

B  $-\frac{15}{24}$

C  $\frac{5}{2}$

D  $\frac{2\pi}{3}$

27. Find the area of the triangle (to the nearest tenth) with sides of length 5, 9, and 10.

A  $504.0 u^2$

B  $95760.0 u^2$

C  $22.5 u^2$

D  $309.5 u^2$

28. In  $\triangle ABC$ ,  $BC = 90$ ,  $AC = 52$ , and  $m\angle C = 102^\circ$ . What is the area of  $\triangle ABC$  to the nearest tenth?

- A  $2327.9 u^2$
- B  $22915.6 u^2$
- C  $4577.8 u^2$
- D  $2288.9 u^2$

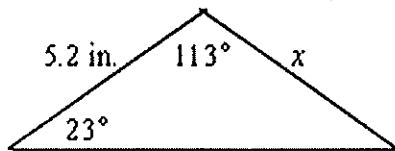
29. Given a triangle with sides  $a=6$ ,  $b=8$ , and  $c=12$ , find  $m\angle C$  to the nearest degree.

- A  $88^\circ$
- B  $117^\circ$
- C  $17^\circ$
- D  $2^\circ$

30. Given a triangle with sides  $A = 115^\circ$ ,  $b = 15$ , and  $c = 10$ , find  $a$  to the nearest tenth.

- A 21.3
- B 15.9
- C 53.1
- D 451.8

31. Solve for  $x$  to the nearest tenth, in the given triangle:



- A  $-248.6 in.$
- B  $0.1 in.$
- C  $2.9 in.$
- D  $9.2 in.$

32. Given a triangle with  $B = 61^\circ$ ,  $c = 18$ , and  $b = 17$  find  $m\angle C$  to the nearest tenth.

- A  $68^\circ$
- B  $68^\circ$  and  $112^\circ$
- C  $56^\circ$
- D  $56^\circ$  and  $124^\circ$

33. Rewrite the following in trigonometric form.

a.  $-3$

b.  $-2 - 2i\sqrt{3}$

A  $3\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

A  $4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

B  $3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

B  $4\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

C  $3\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

C  $4\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

D  $3(\cos\pi + i\sin\pi)$

D  $4(\cos\pi + i\sin\pi)$



34. Use DeMoivre's theorem to evaluate:  $[2(\cos 300^\circ + i \sin 300^\circ)]^6$

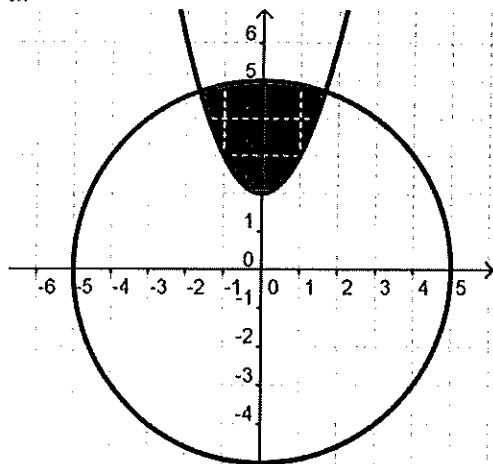
- A  $64(\cos 0^\circ + i \sin 0^\circ)$
- B  $2(\cos 0^\circ + i \sin 0^\circ)$
- C  $12(\cos 50^\circ + i \sin 50^\circ)$
- D  $64(\cos 200^\circ + i \sin 200^\circ)$

35. Use DeMoivre's theorem to find a cube root of:  $-27i$

- A  $3(\cos 60^\circ + i \sin 60^\circ)$
- B  $27(\cos 60^\circ + i \sin 60^\circ)$
- C  $3(\cos 90^\circ + i \sin 90^\circ)$
- D  $27(\cos 90^\circ + i \sin 90^\circ)$

49. Which system of inequalities matches the graph?

a.



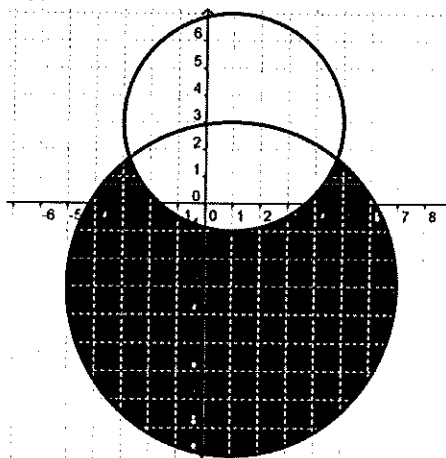
A  $\begin{cases} x^2 + y^2 \leq 25 \\ y \leq x^2 + 2 \end{cases}$

B  $\begin{cases} x^2 + y^2 \geq 25 \\ y \geq x^2 + 2 \end{cases}$

C  $\begin{cases} x^2 + y^2 \geq 25 \\ y \leq x^2 + 2 \end{cases}$

D  $\begin{cases} x^2 + y^2 \leq 25 \\ y \geq x^2 + 2 \end{cases}$

b.



A  $\begin{cases} (x-1)^2 + (y-3)^2 \geq 16 \\ (x-1)^2 + (y+3)^2 \leq 36 \end{cases}$

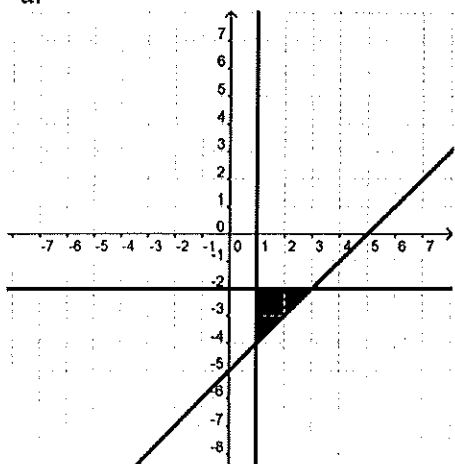
B  $\begin{cases} (x-1)^2 + (y-3)^2 \leq 16 \\ (x-1)^2 + (y+3)^2 \geq 36 \end{cases}$

C  $\begin{cases} (x-1)^2 + (y-3)^2 \geq 16 \\ (x-1)^2 + (y+3)^2 \geq 36 \end{cases}$

D  $\begin{cases} (x-1)^2 + (y-3)^2 \leq 16 \\ (x-1)^2 + (y+3)^2 \leq 36 \end{cases}$

37. Which system of inequalities matches the graph?

a.



$$5x - 3y \geq 15$$

**A**  $x \geq 1$   
 $y \leq -2$

$$5x - 3y \leq 15$$

**B**  $x \geq 1$   
 $y \leq -2$

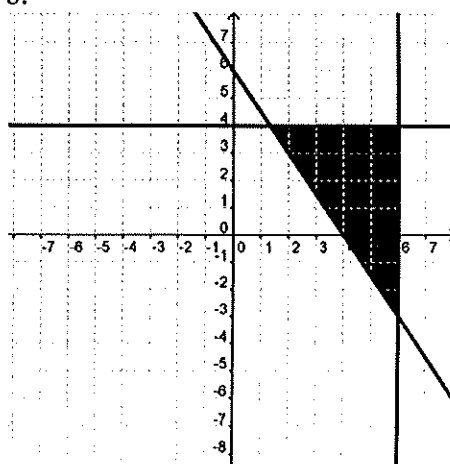
$$5x + 3y \leq 15$$

**C**  $x \geq 1$   
 $y \leq -2$

$$5x + 3y \geq 15$$

**D**  $x \geq 1$   
 $y \leq -2$

b.



$$3x - 2y \leq 12$$

**A**  $x \leq 6$   
 $y \leq 4$

$$3x - 2y \geq 12$$

**B**  $x \leq 6$   
 $y \leq 4$

$$3x + 2y \geq 12$$

**C**  $x \leq 6$   
 $y \leq 4$

$$3x + 2y \leq 12$$

**D**  $x \leq 6$   
 $y \leq 4$

38. A small business has a initial investment of \$5000. The unit cost of the product is \$21.60, and the selling price is \$34.10. Find the sales necessary to break even.

- A** 90
- B** 400
- C** 4945
- D** 62500

39. Solve the following system:  $x + y = 4$   
 $x^2 + y^2 = 4x$

- A (2,2)
- B (0,-4)
- C (2,2), (4,0)
- D (2,2), (0,-4)

40. Solve the following system:  $x^2 + y^2 = 169$   
 $x^2 - 8y = 104$

- A (0,-13)
- B (12,5)
- C (0,-13), (12,5)
- D (0,-13), (12,5), (-12,5)

41. Solve the system of linear equations:

$$\begin{aligned} 2x + 3y + 3z &= 3 \\ 6x + 6y + 12z &= 13 \\ 12x + 9y - z &= 2 \end{aligned}$$

- A  $\left(\frac{1}{2}, -\frac{1}{3}, 1\right)$
- B (1,-3,1)
- C  $\left(\frac{1}{2}, -\frac{1}{3}, 0\right)$
- D *no solution*

42. Solve the system of linear equations:

$$\begin{aligned} -x + 2y &= 1.5 \\ 2x - 4y &= 3 \end{aligned}$$

- A (2,-1)
- B (1,-2)
- C (1-2a,a)
- D *no solution*

43. Solve the system of linear equations:  $2x + y - 7z = 4$   
 $2x - y - z = 0$

- A (2a+1, 3a+2, a)
- B (2, 3, 1)
- C (1-2a, 1-3a, a)
- D *no solution*

44. Given:  $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} 6 & -3 & 2 \\ -4 & 1 & -2 \end{bmatrix}$

Find: a. A+B

- A  $\begin{bmatrix} -4 & 4 & -1 \\ 3 & -2 & 7 \end{bmatrix}$
- B  $\begin{bmatrix} 8 & -2 & 3 \\ -5 & 0 & 3 \end{bmatrix}$
- C  $\begin{bmatrix} 12 & -3 & 2 \\ 4 & -1 & -10 \end{bmatrix}$
- D *not possible*

b. 5A - 4B

- A  $\begin{bmatrix} 4 & -1 & -4 \\ -2 & 7 & 3 \end{bmatrix}$
- B  $\begin{bmatrix} -14 & 17 & -3 \\ 11 & -9 & 33 \end{bmatrix}$
- C  $\begin{bmatrix} -80 & 80 & -20 \\ 60 & -40 & 140 \end{bmatrix}$
- D *not possible*

45. Find the product of matrices:

a.  $\begin{bmatrix} -5 & 4 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 6 & 8 \end{bmatrix}$

A  $\begin{bmatrix} 9 & 37 \\ 48 & 54 \end{bmatrix}$

B  $\begin{bmatrix} -2 & 3 \\ 8 & 15 \end{bmatrix}$

C  $\begin{bmatrix} -17 & 5 \\ -14 & 80 \end{bmatrix}$

D *not possible*

b.  $\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}$

A  $\begin{bmatrix} -9 & 1 \\ -15 & 10 \end{bmatrix}$

B  $\begin{bmatrix} -4 & 6 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}$

C  $\begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$

D *not possible*

c.  $\begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}$

A  $\begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$

B  $\begin{bmatrix} -4 & 5 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}$

C  $\begin{bmatrix} 1 & -9 \\ 6 & -4 \end{bmatrix}$

D *not possible*

46. Given the following, Find  $A^{-1}$

a.  $A = \begin{bmatrix} -2 & 7 \\ -4 & 11 \end{bmatrix}$

A  $\begin{bmatrix} \frac{11}{6} & -\frac{7}{6} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

B  $\begin{bmatrix} -11 & 7 \\ -4 & 2 \end{bmatrix}$

C  $\begin{bmatrix} 11 & -7 \\ 4 & -2 \end{bmatrix}$

D *no solution*

b.  $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ -4 & 4 & 3 \end{bmatrix}$

A  $\begin{bmatrix} 1 & -1 & 0 \\ 7 & -\frac{17}{2} & 1 \\ -8 & 10 & -1 \end{bmatrix}$

B  $\begin{bmatrix} 1 & -1 & 0 \\ 7 & -\frac{17}{2} & 1 \\ -8 & 10 & -1 \end{bmatrix}$

C  $\begin{bmatrix} 1 & -1 & 0 \\ 7 & -\frac{17}{2} & 1 \\ -8 & 10 & -1 \end{bmatrix}$

D *no solution*

47. Find the determinant of the following matrices:

a.  $\begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{bmatrix}$

A -243

B -75

C 75

D 243

b.  $\begin{bmatrix} 1 & -1 & 8 & 4 \\ 2 & 6 & 0 & 4 \\ 2 & 0 & 2 & 6 \\ 0 & 2 & 8 & 0 \end{bmatrix}$

A -124

B -16

C 16

D 124

## Precalculus 2<sup>nd</sup> Semester Final Exam Formulas

### Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

### Law of Sines / Law of Cosines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

### Area of a Triangle

$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$s = \frac{(a+b+c)}{2}$$

### DeMoivre's Theorem

$$z^n = [r(\cos \theta + i \sin \theta)]^n$$

$$z^n = r^n (\cos n\theta + i \sin n\theta)$$

**Remember: Check your mode when doing trig problems!! Good luck have a great summer!!**