AP Calculus BC – Practice AP Exam #1 – SOLUTIONS and SCORING

1) Use the solutions provided to score your own AP Practice Exam:

- For MCQs: Score 1 point for every correct answer and 0 points for every incorrect answers (total will be out of 45 points).

- For FRQs: Use the scoring rubric to score each section of your solutions. Each FRQ has a maximum score of 9 points, and the scoring rubric gives very specific requirements to earn each point. If your solution does not fully meet the description for a point, you don't earn that point.

2) To determine your approximate AP Exam score, we've found two websites that we believe give reasonable estimates of your AP Score based on the practice exam…

https://test-ninjas.com/ap-calculus-bc-score-calculator

https://www.albert.io/blog/ap-calculus-bc-score-calculator/

- We recommend entering your scores into each of these sites, and for the 2nd link trying each of the three different years.
- Remember, this is just an approximation based upon previous years scoring. Each year, College Board adjusts the scoring based upon the difficulty of that year's exam using a 'norming' process which maintains consistency from year to year. We can't guarantee exact scores, but this should give you a good idea of roughly how you are currently performing.
- Also note that it is still possible to do even better than this result, especially on Free-Response. If you missed most of the points on just a couple of free-response questions, even if you review just those questions and score significantly better on the actual exam on just these questions, that may have a large impact on your overall score.

Section I, Part A: No Calculator MCQs

#1. C	#11. C	#21. B
#2. D	#12. B	#22. B
#3. A	#13. B	#23. A
#4. B	#14. A	#24. D
#5. B	#15. D	#25. C
#6. C	#16. C	#26. B
#7. D	#17. C	#27. A
#8. B	#18. B	#28. D
#9. A	#19. A	#29. B
#10. C	#20. D	#30. C

Section I, Part B: Calculator Required MCQs

#1. D	#11. A
#2. C	#12. B
#3. B	#13. D
#4. A	#14. B
#5. D	#15. C
#6. C	
#7. B	
#8. A	
#9. D	
#10. C	

Section II, Part A: Calculator Required FRQs

	MODEL SOLUTION	SCORING
#1 (a)	$P'(16) \approx \frac{P(24)-P(8)}{24-8} = \frac{591-117}{16} = \frac{474}{16} = 29.625 \textit{ bacteria per hour}$	**1 pt: Estimate with supporting work** **1 pt: Units**

Scoring notes:

- To earn the first point a response must include a difference and a quotient as the supporting work.
- $\frac{474}{24-8}, \frac{591-117}{24-8}, \textit{or } \frac{591-117}{16}$ is sufficient to earn the first point.
- A response that presents only units without a numerical approximation for $P'(16)$ does not earn the second point.

	MODEL SOLUTION	SCORING
#1 (b)	$\int_8^{30} P(t)\,dt \approx (24-8) \bullet P(24) + (30-24) \bullet P(30)$ $= 16 \bullet 591 + 6 \bullet 1018 = 15564$	**1 pt: Form of right Riemann sum** **1 pt: Estimate**
	$\frac{1}{30-8}\int_8^{30} P(t)\,dt$ is the average number of bacteria in the petri dish over the interval from $t = 8$ to $t = 30$.	**1 pt: Interpretation**

Scoring notes:

- Read $=$ as $\approx$ for the first point.
- To earn the first point at least three of the four factors in the Riemann sum must be correct. If any of the four factors is incorrect, the response does not earn the second point.
- A response of $(24-8) \bullet P(24) + (30-24) \bullet P(30)$ earns the first point. Values must be pulled from the table to earn the second point.
- A response of $16 \bullet 591 + 6 \bullet 1018$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A completely correct right Riemann sum $(\text{e.g., } 16 \bullet 591 + 6 \bullet 1018)$ earns 1 of the first 2 points. An unsupported answer of 15564 does not earn either of the first 2 points.
- Units will not affect scoring for the second point.
- To earn the third point the interpretation must include both "average number of bacteria" and the time interval. The response need not include a reference to units. However, if incorrect units are given in the interpretation, the response does not earn the third point.

	MODEL SOLUTION	SCORING
#1 (c)	$P(65) = P(50) + \int_{50}^{65} P'(t)\,dt$	**1 pt: Integral** **1 pt: Uses initial condition**
	$= 3439 + 10651.39517 = 14090.395$ The number of bacteria at time $t = 65$ hours is 14090.395 bacteria.	**1 pt: Answer**

Scoring notes:

- The first point is earned for a definite integral with integrand $P'(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A linkage error such as $P(65) = \int_{50}^{65} P'(t)\,dt = 3439 + 10651.39517$ or $\int_{50}^{65} P'(t)\,dt = 10651.39517 = 3439$ earns the first 2 points but does not earn the third point.
- Missing differential (dt): $P(65) = \int_{50}^{65} P'(t)\,dt = 3439 + 10651.39517$
    - Unambiguous responses of $P(65) = P(50) + \int_{50}^{65} P'(t)$ or $P(65) = 3439 + \int_{50}^{65} P'(t)$ earn the first 2 points and are eligible for the third point.
    - Ambiguous responses of $P(65) = \int_{50}^{65} P'(t) + P(50)$ or $P(65) = \int_{50}^{65} P'(t) + 3439$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, these responses do not earn the third point.
- The second point is earned for adding $P(65)$ *or* 3439 to a definite integral with lower limit of 8, either symbolically or numerically.
- The third point is earned for an answer of 3439+10651.395 or 10651.395+3439 with no additional simplification, provided there is some supporting work for these values.
- An answer of 14090.395 with no supporting work does not earn any points.

	MODEL SOLUTION	SCORING
#1 (d)	Because $P''(70) < 0$, the rate of change of the number of bacteria, $P'(70)$, is decreasing at time $t = 70$ hours. -or- At time $t = 70$ hours, the number of bacteria is changing at a decreasing rate.	**1 pt: Answer**

Scoring notes:

- This point is earned only for a correct answer with a correct reason that references the sign of the second derivative of *P*.
- A response that uses ambiguous pronouns (such as "it is positive, so increasing") does not earn this point.

Total for question #1: 9 pts

	MODEL SOLUTION	SCORING
#2 (a)	$x'(t) = \frac{d}{dt}(4t^2 - t^3) = 8t - 3t^2$ $x''(2) = \frac{d}{dt}\left(8t - 3t^2\right)\Big\|_{t=2} = -4$ $y''(2) = \frac{d}{dt}\left(e^{2\sin(t)}\right)\Big\|_{t=2} = -5.129580927$ The acceleration vector at time $t = 2$ is $a(t) = \langle -4,\ -5.130\ (or\ -5.129) \rangle$	**1 pt: x"(2) with setup** **1 pt: y"(2) with setup**

Scoring notes:

- The exact answer is $a(t) = \left\langle -4,\ e^{2\sin(2)}\left(2\cos(2)\right)\right\rangle$.
- $\left\langle -4,\ e^{2\sin(2)}\left(2\cos(2)\right)\right\rangle$ together with an incorrect or missing evaluation at $t = 2$ earns 1 of the 2 points.
- A response of $\left\langle -4,\ e^{2\sin(t)}\left(2\cos(t)\right)\right\rangle = \left\langle -4,\ e^{2\sin(2)}\left(2\cos(2)\right)\right\rangle$ or equivalent earns only 1 of the 2 points because it equates an expression to a numeric value.
- An unsupported correct acceleration vector earns 1 of the 2 points.
- The components may be listed separately, as long as they are labeled.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $y''(2) = 2.143$. A response that presents this value with correct setup earns 1 of the 2 points.

	MODEL SOLUTION	SCORING
#2 (b)	$Speed = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\left(8t-3t^2\right)^2 + \left(e^{2\sin(t)}\right)^2}$ $0 \le t \le 5 \;\; and \;\; \sqrt{\left(8t-3t^2\right)^2 + \left(e^{2\sin(t)}\right)^2} = 3$	**1 pt: setup showing correct expression for speed equal to 3**
	$\Rightarrow t = 0.330$ The first time at which the speed of the particle is 3 is $t = 0.330$.	**1 pt: Answer**

Scoring notes:

- A response with an implied equation is eligible for both points. For example, a response of "$Speed = \sqrt{\left(8t-3t^2\right)^2 + \left(e^{2\sin(t)}\right)^2}$ and is first equal to 3 at $t = 0.330$" earns both points.
- $\sqrt{\left(8t-3t^2\right)^2 + \left(e^{2\sin(t)}\right)^2} = 3$ earns the first point. Speed = 3 by itself does not earn the first point. Both of these responses are eligible to earn the second point.
- A response need not consider any later values of t where speed =3.
- A response of "$t = 0.330$" alone does not earn either point.
- A response with a parenthesis error in squaring, such as $\left(e^{2\sin\left(t^2\right)}\right)$ does not earn the first point but does earn the second point for a correct answer. Note: $\sqrt{\frac{dx}{dt}^2 + \frac{dy}{dt}^2}$ is not considered a parenthesis error.
- Degree mode: In degree mode solution by graph intersection will produce the answer 0.419. If setup is shown correctly, this earns 1 of the 2 points.

	MODEL SOLUTION	SCORING
#2 (c)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^{2\sin(t)}}{8t-3t^2}$ $\left.\frac{dy}{dx}\right\vert_{t=2} = \frac{e^{2\sin(2)}}{8(2)-3(2)^2} = 1.541$ The slope of the line tangent to the curve at $t = 2$ is 1.541.	**1 pt: Slope with supporting work**
	$y(2) = y(0) + \int_0^2 \frac{dy}{dt}\, dt = 3 + \int_0^2 e^{2\sin(t)}\, dt = 12.613 \;\; (or\ 12.612)$ The y-coordinate of the position of the particle at $t = 2$ is 12.613 (or 12.612).	**1 pt: Correct integral part setup** **1pt: Correct answer**

Scoring notes:

- To earn the first point, the response must communicate $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$; for example:

  $\frac{dy}{dx} = \frac{e^{2\sin(t)}}{8t - 3t^2}$

  $\frac{dy/dt}{dx/dt} = 1.541$

  $x'(2) = 6.163,\ \ y'(2) = 4,\ \ slope = 1.541$

  $\frac{dx}{dt} = 8t - 3t^2,\ \ slope = 1.541$

- A response may import an incorrect expression for $x'(t)$ or value of $x'(1)$ from part (a), provided it was declared in part (a).
- The second point is earned for a response that presents the definite integral $\int_0^2 e^{2\sin(t)}\,dt\ \ or\ \ \int_0^2 \frac{dy}{dt}\,dt$ with or without the initial condition.
- For the second point, if the differential is missing:
  - $\int_0^2 e^{2\sin(t)}$ earns the second point and is eligible for the third point.
  - $y(2) = \int_0^2 e^{2\sin(t)}$ earns the second point but is not eligible for the third point.
  - $y(2) = 3 + \int_0^2 e^{2\sin(t)}$ earns the second point and is eligible for the third point.
  - $y(2) = \int_0^2 e^{2\sin(t)} + 3$ does not earn the second point but earns the third point for the correct answer.
- The third point is not earned for a response that presents an incorrect statement such as

  $y(2) = \int_0^2 \frac{dy}{dt}\,dt = 3 + 9.613$

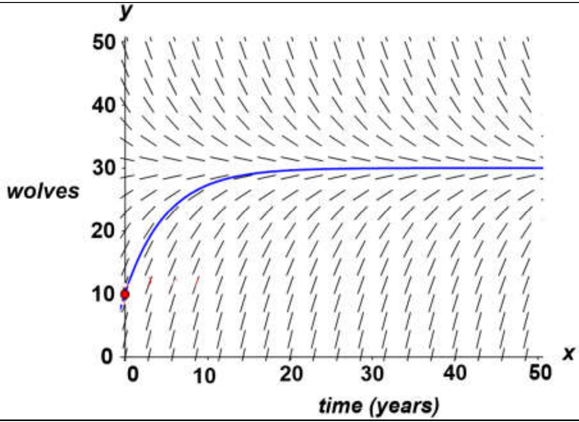
- Degree mode: In degree mode, $\frac{dy}{dx} = 0.268\ \ and\ \ y(2) = 6.482$. This earns only the second point for a correct setup.

	MODEL SOLUTION	SCORING
#2 (d)	$\int_0^5 \sqrt{\left(8t-3t^2\right)^2+\left(e^{2\sin(t)}\right)^2}\ dt$	**1 pt: Correct integral setup**
	$=50.707$ The total distance traveled by the particle over $0 \le t \le 5$ is 50.707.	**1 pt: Answer**

Scoring notes:

- The first point is earned for presenting the correct integrand in a definite integral.
- Parentheses errors were assessed in part (b) and, therefore, will not affect the scoring in part (d).
- If the integrand is an incorrect speed function imported from part (b), the response earns the first point and does not earn the second point.
- As unsupported answer of 50.707 does not earn either point.
- Degree mode: In degree mode, total distance is 44.799 or 44.780. This is eligible for the integral setup point if correct, but does not earn the second point.

Total for question #2: 9 pts

	MODEL SOLUTION	SCORING
#3 (a)		**1 pt: Solution curve**

Scoring notes:

- The solution curve must pass through the point (0, 10), extend reasonably close to the left and right edges of the rectangle, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.
- The solution curve must lie entirely below the horizontal line segments at $W = 30$.

	MODEL SOLUTION	SCORING
#3 (b)	$\left.\frac{dW}{dt}\right\|_{t=0} = \frac{1}{5}(30-10) = \frac{20}{5} = 4$	**1 pt: Derivative value with supporting work**
	The tangent line equation is $W = 4t + 10$ $W(5) = 4(5) + 10 = 30$ The number of wolves at time $t = 5$ years is approximately 30 wolves.	**1 pt: Approximation**

Scoring notes:

- The value of the slope may appear in a tangent line equation or approximation.
- A response of $4(5)+10$ is the minimal response to earn both points.
- A response of $\frac{1}{5}(30-10)$ earns the first point. If there are any subsequent errors in simplification, the response does not earn the second point.
- In order to earn the second point the response must present an approximation found by using a tangent line that:
  - passes through the point (0,10) and
  - has slope 4 or a nonzero slope that is declared to be the value of $\frac{dW}{dt}$.
- An unsupported approximation does not earn the second point.
- The approximation need not be simplified, but the response does not earn the second point if the approximation is simplified incorrectly.

	MODEL SOLUTION	SCORING
#3 (c)	$\frac{d^2W}{dt^2} = -\frac{1}{5}\frac{dW}{dt} = -\frac{1}{5}\left(\frac{1}{5}(30-W)\right) = -\frac{1}{25}(30-W)$	**1 pt: 2nd derivative**
	Because $W(t) < 30$, $\frac{d^2W}{dt^2} < 0$, so the graph of W is concave down. Therefore, the tangent line approximation of $W(5)$ is an overestimate.	**1 pt: Overestimate with reason**

Scoring notes:

- The first point is earned for either $\frac{d^2W}{dt^2} = -\frac{1}{5}\left(\frac{1}{5}(30-W)\right)$ or $\frac{d^2W}{dt^2} = -\frac{1}{25}(30-W)$ (or equivalent). A response that presents any subsequent simplification error does not earn the second point.
- A response that presents an expression for $\frac{d^2W}{dt^2}$ in terms of $\frac{dW}{dt}$ but fails to continue to an expression in terms of W does not earn the first point but is eligible for the second point.
- If the response presents an expression for $\frac{d^2W}{dt^2}$ that is incorrect, the response is eligible for the second point only if the expression is a nonconstant linear function that is negative for $10 < W < 30$. Special case: A response that presents $\frac{d^2W}{dt^2} = \frac{1}{25}(30-W)$ does not earn the first point but is eligible to earn the second point for a consistent answer and reason.
- To earn the second point a response must include $\frac{d^2W}{dt^2} < 0$, or $\frac{dW}{dt}$ is decreasing, or "the graph of W is concave down", as well as the conclusion that the approximation is an overestimate.
- A response the presents an argument based on $\frac{d^2W}{dt^2}$ or concavity at a single point does not earn the second point.

	MODEL SOLUTION	SCORING
#3 (d)	$\frac{1}{30-W}dW = \frac{1}{5}dt$ $\int \frac{1}{30-W}dW = \int \frac{1}{5}dt$	**1 pt: Separates variables**
	$-\ln\lvert 30-W\rvert = \frac{1}{5}t + C$	**1 pt: Finds antiderivatives**
	$-\ln\lvert 30-10\rvert = \frac{1}{5}(0) + C \Rightarrow C = -\ln(20)$ $W(t) < 30 \Rightarrow 30-W > 0 \Rightarrow \lvert 30-W\rvert = 30-W$ $-\ln(30-W) = \frac{1}{5}(t) - \ln(20)$ $\ln(30-W) = -\frac{1}{5}t + \ln(20)$	**1 pt: Constant of integration and uses initial condition**
	$30-W = 20e^{-\frac{1}{5}t}$ $W = 30-20e^{-\frac{1}{5}t}$	**1 pt: Solves for *W***

Scoring notes:

- A response with no separation of variables earns 0 out of 4 points.
- A response that presents an antiderivative of $-\ln(30-W)$ without absolute value symbols is eligible for all 4 points.
- A response with no constant of integration can earn at most the first 2 points.
- A response is eligible for the third point only if it has earned the first 2 points. Special case: A response that presents $+\ln\lvert 30-W\rvert = \frac{1}{5}(t) + C$ (or equivalent) does not earn the second point, is eligible for the third point, but not eligible for the fourth point.
- An eligible response earns the third point by correctly including the constant of integration in an equation and substituting 0 for t and 10 for W.
- A response is eligible for the fourth point only if it has earned the first 3 points.
- A response earns the fourth point only for an answer of $W = 30-20e^{-\frac{1}{5}t}$ or equivalent.

Total for question #3: 9 pts

	MODEL SOLUTION	SCORING
#4 global point	$G'(x) = f(x)$ in any part of the response.	**1 pt:** $G'(x) = f(x)$

Scoring notes:

- This "global point" can be earned in any one part. Expressions that show this connection and therefore earn this point include: $G' = f$, $G'(x) = f(x)$, $G''(x) = f'(x)$ in part (a), $G'(1) = f(1)$ in part (b), or $G'(4) = f(4)$ in part (c).

	MODEL SOLUTION	SCORING
#4 (a)	$G'(x) = f(x)$ The graph of G is concave down for $-2 < x < 0$ and $4 < x < 5$, because $G' = f$ is decreasing on these intervals.	**1 pt: Answer with reason**

Scoring notes:

- Intervals may also include one or both endpoint.

	MODEL SOLUTION	SCORING
#4 (b)	$P'(x) = G(x) \bullet f'(x) + f(x) \bullet G'(x)$ $P'(1) = G(1) \bullet f'(1) + f(1) \bullet G'(1)$	**1 pt: Product rule**
	Substituting $G(1) = \int_0^1 f(t)\,dt = \left[-(1)(2) - \frac{1}{2}(1)(2)\right]$ $G'(1) = f(1) = -2$ into the above expression for $P'(1)$ gives the following:	**1 pt:** $G(1)$ *or* $G'(1)$
	$\left[-(1)(2) - \frac{1}{2}(1)(2)\right](2) + (2)(2)$ $= -2$	**1 pt: Answer**

Scoring notes:

- The first point is earned for the correct application of the product rule in terms of x or in the evaluation of $P'(3)$. Once earned, this point cannot be lost.
- The second point is earned by correctly evaluating $G(1) = -(1)(2) - \frac{1}{2}(1)(2) - 3$, $G'(1) = -2$, or $f(1) = -2$.
- To be eligible to earn the third point a response must have earned the first two points.
- Simplification of the numerical value is not required to earn the third point.

	MODEL SOLUTION	SCORING
#4 (c)	$\lim_{x\to 4}\left(4x-x^2\right)=0$ because G is continuous for $0\le x\le 5$, $\lim_{x\to 4}G(x)=\lim_{x\to 4}\int_0^4 f(t)dt=0.$ Therefore, the limit $\lim_{x\to 4}\frac{G(x)}{4x-x^2}$ is an indeterminant form of type $\frac{0}{0}$.	**1 pt: Uses L'Hopital's Rule**
	Using L'Hopital's Rule, $\lim_{x\to 4}\frac{G(x)}{4x-x^2}=\lim_{x\to 4}\frac{G'(x)}{4-2x}=\lim_{x\to 4}\frac{f(x)}{4-2x}=\lim_{x\to 4}\frac{f(4)}{4-2(4)}=\frac{4}{-4}=-1$	**1 pt: Answer with justification**

Scoring notes:

- To earn the first point the response must show $\lim_{x\to 4}\left(4x-x^2\right)=0$ and $\lim_{x\to 4}G(x)=0$ and must show a ratio of the two derivatives, $G'(x)$ and $4-2x$. The ratio may be shown as evaluations of the derivatives at $x = 4$, such as $\frac{G'(4)}{-4}$.
- To earn the second point the response must evaluate correctly with appropriate limit notation. In particular the response must include $\lim_{x\to 4}\frac{G'(x)}{4-2x}$ or $\lim_{x\to 4}\frac{f(x)}{4-2x}$.
- With any linkage errors (such as $\frac{G'(x)}{4-2x}=\frac{f(4)}{-4}$), the response does not earn the second point.

	MODEL SOLUTION	SCORING
#4 (d)	$G(5)=\int_0^5 f(t)\,dt=2$ and $G(0)=\int_0^0 f(t)\,dt=0$ Average rate of change = $\frac{G(5)-G(0)}{5-0}=\frac{2-0}{5}=\frac{2}{5}$	**1 pt: Average rate of change**
	Yes, $G'(x)=f(x)$ so G is differentiable on (0,5) and continuous on [0,5]. Therefore, the Mean Value Theorem applies and guarantees a value c, $0<c<5$, such that $G'(c)=\frac{2}{5}$.	**1 pt: Answer with justification**

Scoring notes:

- To earn the first point the response must present at least a difference and a quotient and a correct evaluation. For example, $\frac{2-0}{5}$ *or* $\frac{G(5)-G(0)}{5}=\frac{2}{5}$.
- Simplification of the numerical value is not required to earn the first point, but any simplification must be correct.
- The second point can be earned without the first point if the response has the correct setup but an incorrect or no evaluation of the average rate of change. The Mean Value Theorem need not be explicitly stated provided the conditions and conclusion are stated.

Total for question #4: 9 pts

	MODEL SOLUTION	SCORING
#5 (a)	Area = $\int_0^3 [f(x) - g(x)]\,dx = \int_0^3 f(x)\,dx - \int_0^3 g(x)\,dx$	**1 pt: Integrand**
	$= 12.5 - \int_0^3 \frac{4}{4-x}\,dx = 12.5 + 4\left[\ln\lvert 4-x\rvert\right]_0^3$	**1 pt: Antiderivative of** $g(x)$
	$= 12.5 + 4[\ln(1) - \ln(4)] = 12.5 - 4\ln(4)$	**1 pt: Answer**

Scoring notes:

- The first point is earned with an implied integrand for f and explicit integrand for g, such as $12.5 - \int_0^3 g(x)\,dx.$
- The second point is earned for finding $a\int \frac{1}{4-x}\,dx = a \bullet \ln\lvert 4-x\rvert$ *or* $a \bullet \ln\lvert 4-x\rvert$.
- A response is eligible for the third point only if it has earned the first 2 points. The third point is earned only for the correct answer. The answer does not need to be simplified; however, if simplification is attempted, it must be correct.
- A response is not eligible for the third point with incorrect limits of integration for u-substitution, for example, $\int_0^3 \frac{4}{4-x}\,dx = \int_0^3 \frac{4}{u}\,du = 4\left[\ln\lvert 4-x\rvert\right]_0^3$.
- A response with incorrect communication, such as "Area = $\int_0^3 [g(x) - x(x)]\,dx = 12.5 + 4\ln(4)$" does not earn the third point. However, a response of "$\int_0^3 [g(x) - x(x)]\,dx = -4\ln(4) - 12.5$ so the area is $12.5 - (-4\ln(4))$" earns all 3 points.

	MODEL SOLUTION	SCORING
#5 (b)	$\int_0^{\infty}(g(x))^2\,dx = \lim_{b\to\infty}\int_0^b\left(\frac{4}{4-x}\right)^2 dx$	**1 pt: Limit notation**
	$= \lim_{b\to\infty}\left(\frac{16}{4-x}\Big\|_0^b\right)$	**1 pt: Antiderivative**
	$= \lim_{b\to\infty}\left(\frac{16}{4-b}-\frac{16}{4-0}\right) = 0-\frac{16}{4} = -4$	**1 pt: Answer**

Scoring notes:

- To earn the first point a response must correctly use limit notation throughout the problem and not include arithmetic with infinity, for example, $\left[\frac{16}{4-x}\right]_0^{\infty}$ *or* $\frac{16}{4-\infty}-4$ are not allowed.
- The second point can be earned by finding an antiderivative of the form $\frac{a}{4-x}$ for $a>0$, from an indefinite or improper integral, with or without correct limit notation. If $a \neq 16$, the response does not earn the third point.
- The third point is earned only for an answer of -4 (or equivalent value).
- A response is not eligible for the third point with incorrect limits of integration for *u*-substitution, for example $\lim_{b\to\infty}\int_0^b \frac{16}{u^2}\,du = \lim_{b\to\infty}\left[\frac{16}{4-x}\right]_0^b$.

	MODEL SOLUTION	SCORING
#5 (c)	Using integration by parts, $u = x \qquad dv = f'(x)$ $du = dx \qquad v = f(x)$	**1 pt: u and dv**
	$\int h(x)\,dx = \int x \bullet f'(x)\,dx = x \bullet f(x) - \int f(x)\,dx$	**1 pt: Correct expression for indefinite integral using integration by parts**
	$\int_0^3 h(x)\,dx = \int_0^3 x \bullet f'(x)\,dx = \left[x \bullet f(x)\right]_0^3 - \int_0^3 f(x)\,dx$ $= \left(3f(3) - 0f(0)\right) - 12.5 = 3(4) - 12.5 = -0.5$	**1 pt: Answer**

Scoring notes:

- The first and second points are earned with an implied *u* and *dv* in the presence of

  $x \bullet f(x) - \int f(x)\,dx$ or $\left[x \bullet f(x)\right]_0^3 - 12.5.$

- Limits of integration may be present, omitted, or partially present in the work for the first and second points.
- The tabular method may be used to show integration by parts. In this case, the first point is earned by having columns (labeled or unlabeled) that begin with x and $f'(x)$. The second point is earned for

  $x \bullet f(x) - \int f(x)\,dx$.

- The third point is earned only for the correct answer and can only be earned if the first 2 points were earned.

Total for question #5: 9 pts

	MODEL SOLUTION	SCORING
#6 (a)	At $x = 4$, the series is $$\sum_{n=1}^{\infty}\frac{(n+1)(4)^{n+1}}{n^3(4)^n}=\sum_{n=1}^{\infty}\frac{(n+1)4(4)^n}{n^3(4)^n}=\sum_{n=1}^{\infty}\frac{(n+1)4}{n^3}\frac{(4)^n}{(4)^n}$$ $$=\sum_{n=1}^{\infty}\frac{4(n+1)}{n^3}$$	**1 pt: Considers the series obtained by plugging 4 in for x**
	$\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{4(n+1)}{n^3}$ *evaluate by Limit Comparison with* $\sum_{n=1}^{\infty}b_n=\sum_{n=1}^{\infty}\frac{4(n)}{n^3}=\sum_{n=1}^{\infty}\frac{4}{n^2}=4\sum_{n=1}^{\infty}\frac{1}{n^2}$ *is a* $p-$*Series with* $p=2$ *and therefore converges by the* $p-$*Series test.* *Limit Comparison*: $$\lim_{n\to\infty}\left\|\frac{a_n}{b_n}\right\|=\lim_{n\to\infty}\left\|\frac{4\left(\frac{n+1}{n^3}\right)}{\left(\frac{n}{n^3}\right)}\right\|=\lim_{n\to\infty}\left\|\frac{4(n+1)}{n}\right\|=4>0\ (\textit{finite, positive})$$ $\therefore$ *because* $\sum_{n=1}^{\infty}\frac{4(n)}{n^3}$ *converges,* $\sum_{n=1}^{\infty}\frac{4(n+1)}{n^3}=\sum_{n=1}^{\infty}\frac{(n+1)x^{n+1}}{n^3 4^n}$ *also converges at* $x=4$.	**1 pt: Answer with reason and supporting work**

Scoring notes:

- To earn the first point using the limit comparison test, a response must consider the term $\frac{(n+1)4}{n^3}\frac{(4)^n}{(4)^n}$. This could be shown by considering the term $\frac{(n+1)4}{n^3}$, either individually or as part of a sum.
- The direct comparison test cannot be used to show convergence here (the 'direction' is incorrect).
- To earn the second point using the limit comparison test, a response must correctly write the limit of the ratio of the terms in the given series to the terms of the convergent series and demonstrate that the limit of this ratio is a finite, positive number.
- The response does not need to use the term "limit comparison test", but the response cannot declare use of an incorrect test.

	MODEL SOLUTION	SCORING
#6 (b)	$f(-2)=\sum_{n=1}^{\infty}\frac{(n+1)(-2)^{n+1}}{n^3 4^n}=(-2)\sum_{n=1}^{\infty}\frac{(n+1)}{n^3}\left(-\frac{1}{2}\right)^n$ is an alternating series with terms that decrease in magnitude to 0. By the alternating series error bound: $\left\|f(-2)-S_2\right\| \le \left\|\text{1st neglected term}\right\|$ $\left\|f(-2)-S_2\right\| \le \left\|(-2)\frac{(3+1)}{(3)^3}\left(-\frac{1}{2}\right)^3\right\|$ $\left\|f(-2)-S_2\right\| \le \left\|(-2)\frac{4}{27}\left(-\frac{1}{8}\right)\right\|$ $\left\|f(-2)-S_2\right\| \le \frac{1}{27}$. Therefore, $\left\|f(-2)-S_2\right\| < \frac{1}{25}$	**1 pt: Uses 3rd term** **1pt: Verification**

Scoring notes:

- The first point is earned for correctly using $x = -2$ in the third term. (Listing the third term as part of a polynomial is not sufficient.) Using $x = -2$ in any term of degree four or higher does not earn this point.
- The expression $(-2)\frac{(3+1)}{(3)^3}\left(-\frac{1}{2}\right)^3$ that is subsequently simplified incorrectly earns the first point but not the second.
- To earn the second point the response must state that the series for $f(-2)$ is alternating or that the alternating series error bound is being used.
- A response that declares the error is equal to $\frac{1}{25}$ (or any equivalent form of this value) does not earn the second point.

	MODEL SOLUTION	SCORING
#6 (c)	The general term of the Maclaurin series for f' *is* $\frac{(n+1)}{n^3 4^n}(n+1)x^n$ *or* $\frac{(n+1)^2}{n^3 4^n}x^n$	**1 pt: General term**
	Because the radius of convergence of the Maclaurin series for *f* is 4, the radius of convergence of the Maclaurin series for f' is also 4.	**1 pt: Radius**

Scoring notes:

- A response of $\frac{(n+1)}{n^3 4^n}(n+1)x^n$ or $\frac{(n+1)^2}{n^3 4^n}x^n$ earns the first point. Any expression mathematically equivalent to this also earns the first point.
- The response need not simplify, but any presented simplification must be correct in order to earn the first point.
- The second point is earned only for a supported answer of 4. The second point can be earned without the first.
- The question can also be answered by using a test such as the ratio test on the resulting series.

	MODEL SOLUTION	SCORING
#6 (d)	$\left\lvert \frac{\left(\frac{(n+2)x^{2n+2}}{(n+1)^3 5^{n+1}}\right)}{\left(\frac{(n+1)x^{2n}}{(n)^3 5^n}\right)} \right\rvert = \left\lvert \frac{(n+2)x^{2n+2}}{(n+1)^3 5^{n+1}} \bullet \frac{(n)^3 5^n}{(n+1)x^{2n}} \right\rvert$	**1 pt: Sets up ratio**
	$\lim_{n\to\infty} \left\lvert \frac{(n+2)x^{2n+2}}{(n+1)^3 5^{n+1}} \bullet \frac{(n)^3 5^n}{(n+1)x^{2n}} \right\rvert = \lim_{n\to\infty} \frac{(n+2)n^3}{(n+1)^3(n+1)} \left\lvert \frac{x^2}{5} \right\rvert = \left\lvert \frac{x^2}{5} \right\rvert$	**1 pt: Limit**
	$\left\lvert \frac{x^2}{5} \right\rvert < 1 \Rightarrow x^2 < 5 \Rightarrow \lvert x \rvert < \sqrt{5}$ The radius of convergence of *h* is $\sqrt{5}$.	**1 pt: Radius of convergence**

Scoring notes:

- The first point is earned by presenting a correct ratio with or without absolute values. Once earned, this point cannot be lost. Any errors in simplification or evaluation of the limit will not earn the second point.
- The first point is earned for any ratios mathematically equivalent to the solution ratios.
- The first point is also earned for ratios mathematically equivalent to the reciprocal ratios (numerator and denominator flipped). Responses including a reciprocal ratio can earn the second point for using limit notation to correctly find a limit of the absolute value of their ratio to be $\left\lvert \frac{5}{x^2} \right\rvert$. Such responses earn the third point only for a final answer of $\sqrt{5}$ with a valid explanation for reporting the reciprocal of $\frac{1}{\sqrt{5}}$.
- To earn the second point a response must use the ratio and correctly evaluate the limit of the ratio, using correct limit notation.
- The third point is earned only for an answer of $\sqrt{5}$ with supporting work.

Total for question #6: 9 pts