AP Calculus BC – Practice AP Exam #1

Thank you for taking the practice AP Exam! Previous year students have universally said that these practice exams were very helpful for their AP Exam review and we hope this is very helpful for you as well :)

This exam is structured exactly like an AP Calculus BC Exam with the same number, type and wording-style of questions, and we will give the exam with the same time constraints you'll have on the actual exam.

The exam consists of 2 Sections, each with 2 Parts:

- Section I: Multiple Choice
  - Part A: Calculator Not Permitted: 30 questions, 60 minutes
  - Part B: Graphing Calculator Required: 15 questions, 45 minutes
- Section II: Free Response
  - Part A: Graphing Calculator Required: 2 questions, 30 minutes
  - Part B: Calculator Not Permitted: 4 questions, 60 minutes

Although we will give this practice AP Exam in paper format, the actual AP Exam be in Hybrid Digital format, meaning that for Multiple Choice the questions and the way to mark answers will be online – accessed by your school laptop using the Bluebook testing application. For Free Response, the question prompts will be displayed on your laptop, but you will write your answers in a paper test booklet.

Today, everything is on paper, so for Multiple Choice, simply circle one of the answers so that you can grade your own test later.

The first section is no calculator MCQ, so please put your calculator beneath your desk, read the following directions which match that is on the AP Exam, **but do not start the test until we give further instructions.**

CALCULUS BC, Section I, Part A has 30 multiple-choice questions and lasts 1 hour.

No calculator is allowed for this part of the exam.

Solve each problem. You may use the available paper for scratch work. After examining the choices, select the best of the choices given.

Unless otherwise specified, the domain of the function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g. $\sin^{-1}x = \arcsin x$).

You can go back and forth between questions in this part until time expires. The clock will turn red when 5 minutes remain – **the proctor will not give you any time updates or warnings**.

(For our practice exam today, we will give you one warning when 5 minutes remain because you don't have a visible timer to see turn red).

*** If you need to leave the practice exam early today, check in with the proctor before you leave to get the solutions – you can finish the test at home and score your own exam later ***

#1. If $\int_2^6 g(x)\,dx = 7$ and $\int_2^6 h(x)\,dx = -3$, which of the following cannot be determined from the information given?

(A) $\int_2^6 \left(5h(x) - 2g(x)\right)\,dx$

(B) $\int_6^2 h(x)\,dx$

(C) $\int_2^6 \left(8h(x)g(x)\right)\,dx$

(D) $\int_2^6 \left(9g(x)\right)\,dx$

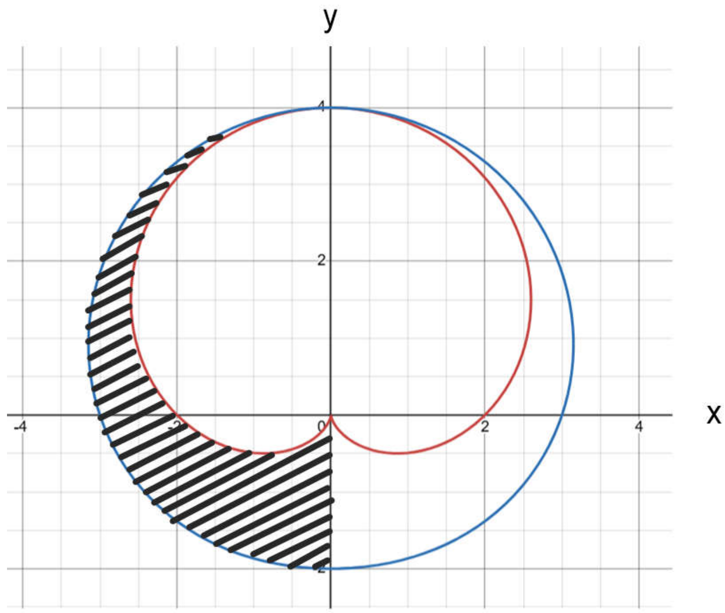
#2. Which of the following statements about the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+2}$ is true?

(A) The series can be shown to diverge by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) The series can be shown to diverge by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(C) The series can be shown to diverge by the p-Series test.

(D) The series can be shown to converge by the alternating series test.



#3. Let R be the region in the second and third quadrants between the graphs of the polar curves $f(\theta)=2+2\sin(\theta)$ and $g(\theta)=3+\sin(\theta)$, as shaded in the figure above.

Which of the following integral expressions gives the area of R ?

(A) $\frac{1}{2}\int_{\pi/2}^{3\pi/2}\left((g(\theta))^2-(f(\theta))^2\right)d\theta$

(B) $\frac{1}{2}\int_{\pi/2}^{3\pi/2}(g(\theta)-f(\theta))^2\,d\theta$

(C) $\int_{\pi/2}^{3\pi/2}(g(\theta)-f(\theta))\,d\theta$

(D) $\int_{-2}^{4}(g(\theta)-f(\theta))\,d\theta$

#4. Which of the following series converges?

(A) $\sum_{n=1}^{\infty}3\left(\frac{1}{n^{0.4}}\right)$

(B) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{n-1}{n^2}\right)$

(C) $\sum_{n=1}^{\infty}3\left(\frac{\pi}{e}\right)^n$

(D) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{3+n^2}{n^2}\right)$

#5. If $\dfrac{dy}{dx} = y + 3,$ and if $y(4) = -2,$ then $y =$

(A) $e^{x} - 3$ (B) $e^{x-4} - 3$ (C) $e^{4-x} + 3$ (D) $e^{-x} + 3$

#6. $\displaystyle\int_{2}^{\infty} \frac{3}{(x-1)^{5/2}}\,dx$ is

(A) $\dfrac{9}{2}$ (B) $\dfrac{-9}{2}$ (C) 2 (D) *divergent*

#7. What is the slope of the line tangent to the curve $2\sqrt{y}-3x^2=7$ at the point $\left(2, \frac{3}{4}\right)$?

(A) $\frac{-\sqrt{3}}{18}$ (B) $\frac{\sqrt{3}}{18}$ (C) $-6\sqrt{3}$ (D) $6\sqrt{3}$

#8. Let $y=f(x)$ be the solution to the differential equation $\frac{dy}{dx}=3x-y$ with initial condition $f(4)=5$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x=4$?

(A) -9 (B) -13 (C) 9.3 (D) -12.4

#9. The table below gives selected values for the differentiable function h. In which of the following intervals must there be a number c such that $h'(c) = -3$?

x	0	3	6	9	12
$h(x)$	1	10	7	–2	–5

(A) $(6,9)$

(B) $(0,3)$

(C) $(9,12)$

(D) $(0,12)$

#10. $\int_1^2 \frac{3+\cos(x)}{\sin(x)+3x}\,dx =$

(A) $\ln|2|$ (B) $\ln|2| - \ln|6|$ (C) $\ln\left|\frac{\sin(2)+6}{\sin(1)+3}\right|$ (D) $-\ln|\csc(2)-\cot(2)|$

#11. A function f has a Maclaurin series given by $3+4x+5x^2+\frac{1}{3}x^3+\ldots$, and the Maclaurin series converges to $f(x)$ for all real numbers x. If g is the function defined by $g(x)=e^{f(x)}$, what is the coefficient of x^2 in the Maclaurin series for g ?

(A) $26e^3$ (B) e^3

(C) $13e^3$ (D) $\frac{1}{2}e^3$

#12. Which of the following expressions is equal to

$$\lim_{n\to\infty}\frac{1}{n}\left(\sin\left(4+\frac{1}{n}\right)+\sin\left(4+\frac{2}{n}\right)+\sin\left(4+\frac{3}{n}\right)+\ldots+\sin\left(4+\frac{n}{n}\right)\right)\ ?$$

(A) $\int_0^1 \sin(x)\,dx$

(B) $\int_4^5 \sin(x)\,dx$

(C) $\int_0^4 \sin(x)\,dx$

(D) *none of the above*

#13. If $\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = \ln\left(t^3\right)$, then $\frac{d^2y}{dx^2}$ is

(A) $\frac{3}{4t}$

(B) $\frac{3}{16t}$

(C) $\frac{3}{t}$

(D) *undefined*

#14. Let R be the region in the first quadrant bounded by the graphs of $y = x^2 + 1$, $y = 5$, and the y-axis. Which of the following integrals gives the volume of the solid generated by revolving R about the x-axis?

(A) $\pi\int_0^2 \left(25 - \left(5 - \left(x^2 + 1\right)\right)^2\right) dx$

(B) $\pi\int_0^2 \left(5 - \left(x^2 + 1\right)\right)^2\, dx$

(C) $\pi\int_1^5 \left(4 - \left(\sqrt{y-1}\right)^2\right) dy$

(D) $\pi\int_0^2 \left(25 - \left(x^2 + 1\right)^2\right) dx$

#15. Which of the following statements are true about the two series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ and $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ diverges and $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges conditionally.

(B) Both series converge conditionally.

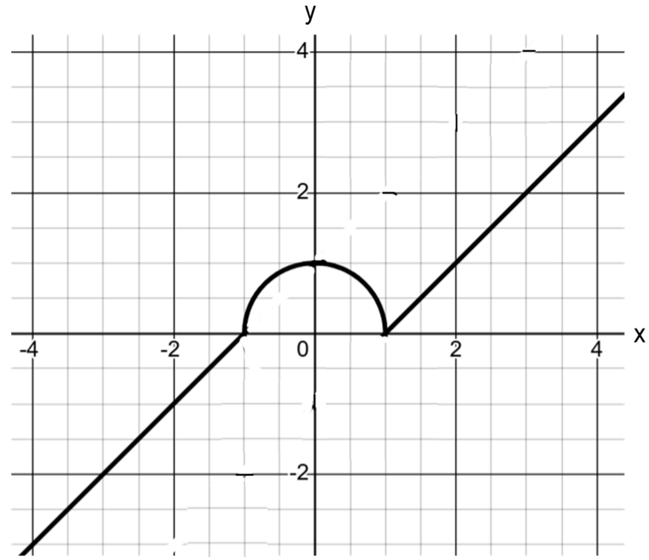
(C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ converges conditionally and $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges absolutely.

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ converges absolutely and $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges conditionally.

#16. The function h has a continuous derivative. If $h(0)=5$, $h(3)=6$, and $\int_0^3 h(x)\,dx=10$,

what is $\int_0^3 x \cdot h'(x)\,dx$?

(A) 5　　(B) 1　　(C) 8　　(D) 16



Graph of g

#17. The graph of the function g is shown above and consists of two line segments and a semi-circle.

What is the value of $\int_{-2}^{4} g'(x)\,dx$?

(A) $4+\frac{\pi}{2}$ (B) $\frac{2}{3}$ (C) 4 (D) 1

#18. The definite integral $\int_{1}^{7} \frac{1}{x}\,dx$ is approximated by a left Riemann sum, a right Riemann sum, and a trapezoidal sum, each with 3 subintervals of equal width. If L is the value of the left Riemann sum, R is the value of the right Riemann sum, and T is the value of the trapezoidal sum, which of the following inequalities is true?

(A) $R < T < \int_{1}^{7} \frac{1}{x}\,dx < L$

(B) $R < \int_{1}^{7} \frac{1}{x}\,dx < T < L$

(C) $L < T < \int_{1}^{7} \frac{1}{x}\,dx < R$

(D) $L < \int_{1}^{7} \frac{1}{x}\,dx < T < R$

#19. What is the slope of the line tangent to the polar curve $r = 2\theta$ at the point where $\theta = \dfrac{3\pi}{2}$?

(A) $\dfrac{-2}{3\pi}$ (B) $\dfrac{-3\pi}{2}$ (C) 2 (D) 0

#20. Which of the following is the interval of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+3)^n}{n \bullet 4^{n+1}}$?

(A) $[-4, 4)$

(B) $[1, 7)$

(C) $(-\infty, \infty)$

(D) $[-7, 1)$

#21. The second derivative of a function g is given by $g''(x) = x(x-2)^4(x-5)^3$. At which of the following values of x does the graph of g have a point of inflection?

(A) 0, 2, and 5

(B) 0 and 5 only

(C) 2 only

(D) 2 and 5 only

#22. The function v is not differentiable at $t = 3$. Which of the following statements must be true?

(A) $\lim_{t \to 3} v(t)$ does not exist.

(B) $\lim_{t \to 3} \frac{v(t) - v(3)}{t - 3}$ does not exist.

(C) $\int_0^3 v(t)\, dt$ does not exist.

(D) v is not continuous at $t = 3$.

#23. $\lim_{x\to\infty} \dfrac{\ln(x)}{3x^2}$ is

(A) 0 (B) $\dfrac{1}{6}$ (C) 6 (D) *nonexistent*

#24. Which of the following is the Maclaurin series for $x^2 \sin\left(x^3\right)$?

(A) $x^3 - \dfrac{x^5}{3!} + \dfrac{x^7}{5!} - \dfrac{x^9}{7!} + \dots$

(B) $x^5 - \dfrac{x^{11}}{11!} + \dfrac{x^{17}}{17!} - \dfrac{x^{23}}{23!} + \dots$

(C) $x^3 - \dfrac{x^9}{3!} + \dfrac{x^{15}}{5!} - \dfrac{x^{21}}{7!} + \dots$

(D) $x^5 - \dfrac{x^{11}}{3!} + \dfrac{x^{17}}{5!} - \dfrac{x^{23}}{7!} + \dots$

#25. The table below gives values of the twice-differentiable functions f and g and their derivatives at x = 5. If h is the function defined by $h(x) = f(x)\,g'(x)$, what is the value of $h'(5)$?

x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$
5	3	2	–1	–4	5	–2

(A) 15 (B) –20 (C) 4 (D) –12

#26. $\sum_{n=1}^{\infty} \frac{3^n}{\pi^n}$ is

(A) $\frac{\pi}{\pi - 3}$ (B) $\frac{3}{\pi - 3}$ (C) $\frac{\left(\frac{3}{\pi}\right)}{1 + \left(\frac{3}{\pi}\right)}$ (D) *divergent*

#27. The position of a particle is given by the parametric equations $x(t) = e^{1-2t}$ and $y(t) = \ln\left(t^3 + 1\right)$. What is the velocity vector at time $t = 2$?

(A) $\left\langle -2e^{-3},\ \frac{4}{3} \right\rangle$ (B) $\left\langle e^{-3},\ \ln(9) \right\rangle$ (C) $\left\langle e^{-3},\ \ln(9) \right\rangle$ (D) $\left\langle 6e^{-3},\ 0 \right\rangle$

#28. $\frac{d}{dx}\left(3\left(\tan\left(\sqrt{x}\right)\right)^5\right) =$

(A) $15\cot\left(\frac{1}{2\sqrt{x}}\right)$

(B) $15\tan^4\left(\sqrt{x}\right)\sec^2\left(\sqrt{x}\right)$

(C) $\dfrac{15\tan^4\left(\sqrt{x}\right)}{\sqrt{x}}$

(D) $\dfrac{15\tan^4\left(\sqrt{x}\right)\sec^2\left(\sqrt{x}\right)}{2\sqrt{x}}$

#29. Let f be the function defined by $f(x) = \frac{4}{3}x^3 + 4x^2 - 32x + 7$. At which of the following values of x does f attain a local maximum?

(A) -2 (B) -4 (C) 2 (D) 4

#30. $\int_1^3 \left(3x^2 - 4x\right)\,dx$

(A) 16 (B) 8 (C) 10 (D) 14

END of Section 1, Part A

Do not work any further problems until we give instructions to move to the next part, but go ahead and read the instructions below which are provided on the actual AP Exam:

CALCULUS BC, Section I, Part B has 15 multiple-choice questions and lasts 45 minutes.

A graphing calculator is required for some questions on this part of the exam.
You may use a handheld calculator or the calculator available in this application.
Make sure your calculator is in radian mode.

Solve each problem. You may use the available paper for scratch work. After examining the choices, select the best of the choices given.

The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

Unless otherwise specified, the domain of the function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g. $\sin^{-1}x = \arcsin x$).

You can go back and forth between questions in this part until time expires. The clock will turn red when 5 minutes remain – **the proctor will not give you any time updates or warnings**.

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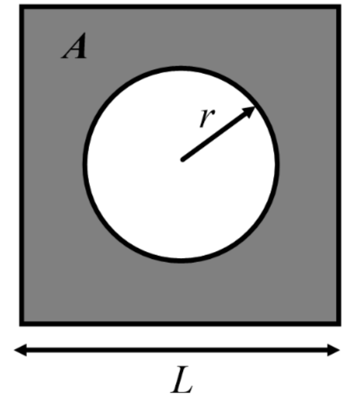
########### STOP! DO NOT PROCEED UNTIL GIVEN INSTRUCTIONS TO DO SO ##########

#1. Circle with radius r is centered in a square with side length L, as shown in the figure. The side length of the square, L, is increasing at a constant rate of 4 inches per second. The radius of the circle, r, is decreasing at a constant rate of 2 inches per second. What is the rate of change, in square inches per second, of the shaded area, A, which is inside the square but not inside the circle, at the instant when L is 3 inches and r is 1 inch?



(A) $24-4\pi$ (B) $36+4\pi$

(C) $36-4\pi$ (D) $24+4\pi$

#2. The population P of wolverines in a nature preserve grows at a rate that is jointly proportional to the size of the wolverine population and the difference between the wolverine population and the carrying capacity of the population. If the carrying capacity of the population is 320 wolverines, which of the following differential equations best models the growth rate of the wolverine population with respect to time t, where k is a constant?

(A) $\frac{dP}{dt}=k(320-P)$

(B) $\frac{dP}{dt}=320-kP$

(C) $\frac{dP}{dt}=kP(320-P)$

(D) $\frac{dP}{dt}=k\frac{1}{P}(320-P)$

#3. Let f be the function defined by $f(x)=\frac{2}{3}x^3-\frac{1}{2}x^2+\frac{5}{3}x+1$.

For how many values of x in the open interval $(0.4,\ 1.8)$ is the instantaneous rate of change of f equal to the average rate of change of f on the closed interval $[0.4, 1.8]$?

(A) 0 (B) 1 (C) 2 (D) 3

#4. The position of a particle moving in the xy-plane is given by the parametric equations $x(t)=\cos(3t+2)$ and $y(t)=\ln(0.5\,t+2)$ for time $t\geq 0$. What is the speed of the particle at time $t=2.5$?

(A) 0.273 (B) 1.544 (C) 0.616 (D) 0.007

#5. The graph of the function f shown consists of four line segments. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$.

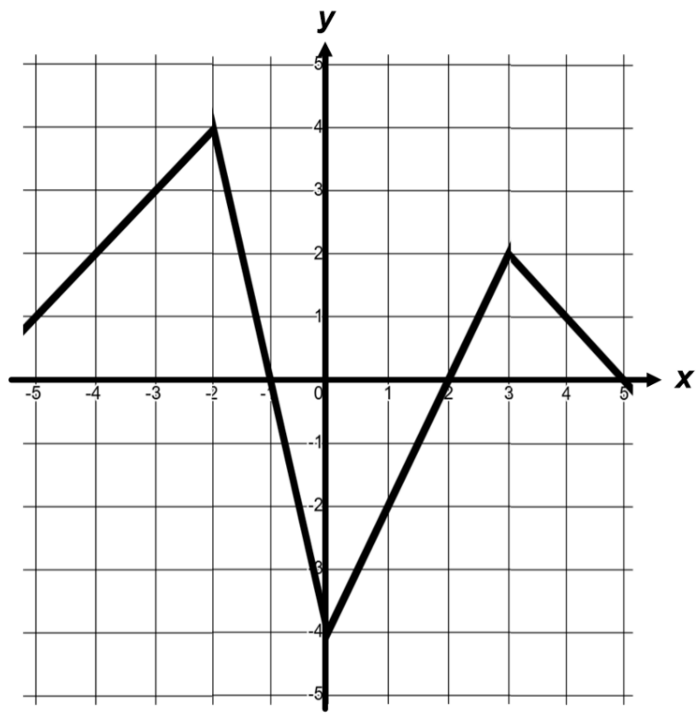
At what value of x does g attain its absolute minimum on the interval $[-5, 5]$?

(A) 2

(B) 0

(C) -1

(D) -5



Graph of f

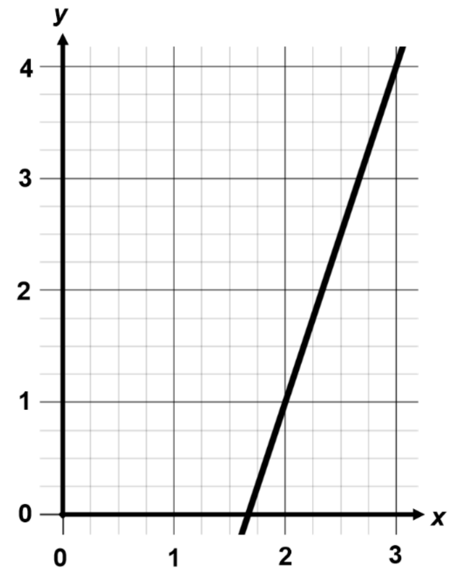
#6. The velocity of a particle is given by $v(t) = 3\ln(t+2)\sin(t-4)$, where t is measured in seconds and $v(t)$ is measured in meters per second. What is the total distance traveled by the particle, in meters, from time $t = 1$ to time $t = 5$?

(A) 2.465 (B) 12.327 (C) 11.436 (D) 0.373

#7. Let $g(x)$ be a differentiable function.

The figure to the right shows the graph of the line tangent to the graph of g at $x = 2$.

Of the following, which must be true?



(A) $g'(2) < g(2)$

(B) $g'(2) > g(2)$

(C) $g'(2) = g(2)$

(D) $g'(2) = -g(2)$

#8. Let $g(x)$ be the function defined by $g(x) = \begin{cases} k^2 - x, & x < 2 \\ \dfrac{3}{k+x}, & x \geq 2 \end{cases}$, where k is a positive constant.

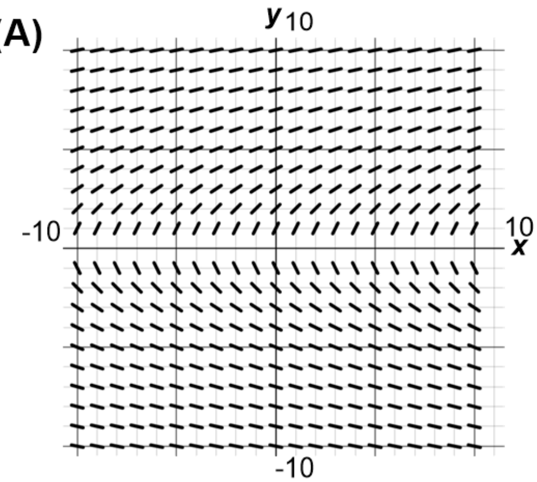
For what value of k, if any, is $g(x)$ continuous?

(A) 1.678 (B) 3.606 (C) 1.414

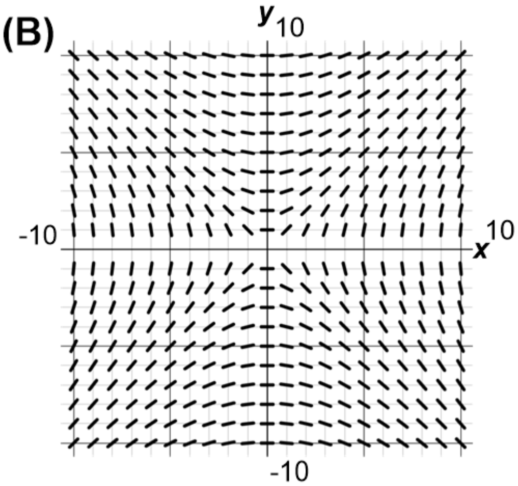
(D) There is no such value of k.

#9. Let f be a continuous function of x. Which of the following could be a slope field for the differential equation $\dfrac{dy}{dx} = \dfrac{1}{5}y$?

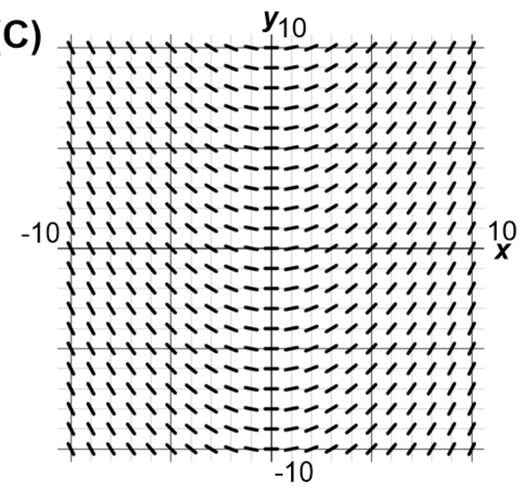
(A)



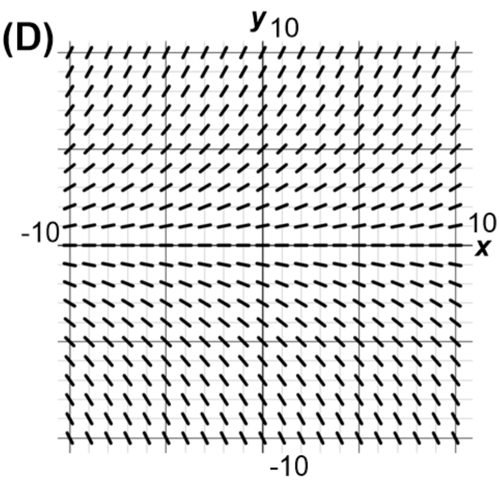
(B)



(C)



(D)



#10. Let $g(x)$ be a function with derivative given by $g'(x) = e^{-x} + \cos(x)$. What is the length of the graph of $y = g(x)$ from $x = 0$ to $x = 1.3$?

(A) 2.412 (B) 1.963 (C) 2.161 (D) 1.691

#11. The function h has derivatives of all orders for all real numbers with $h(3) = 5,\ h'(3) = -2,\ h''(3) = 4,$ and $h'''(3) = 7.$ Using the third-degree Taylor polynomial for h about $x = 3$, what is the approximation of $h(3.4)$?

(A) 4.595 (B) 4.669 (C) 67.175 (D) 4.200

#12. The temperature of water in a tea kettle is modeled by the differentiable function $T(t) = 100e^{-0.064t}$ where T is measured in degrees Celsius and t is measured in minutes from the time the tea kettle is taken off the stove with the water boiling. Which of the following expressions gives the average temperature of the water in the tea kettle from $t = 5$ to $t = 15$?

(A) $536\ °C$ (B) $53.634\ °C$ (C) $-3.433\ °C$ (D) $34.326\ °C$

#13. The continuous function g is positive and has domain $x > 0$. If the asymptotes of the graph of g are $x = 3$ and $y = -1$, which of the following statements must be true?

(A) $\lim_{x \to 3^+} g(x) = -1$

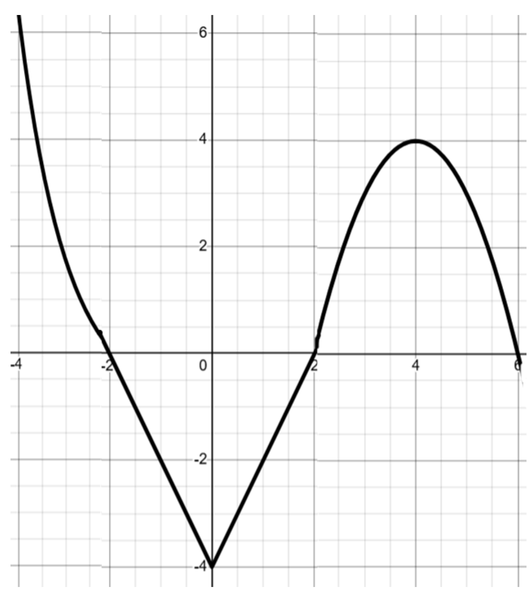
(B) $\lim_{x \to \infty} g(x) = \infty$ and $\lim_{x \to -\infty} g(x) = -\infty$

(C) $\lim_{x \to 3^+} g(x) = -1$ and $\lim_{x \to \infty} g(x) = \pm\infty$

(D) $\lim_{x \to 3^+} g(x) = \pm\infty$ and $\lim_{x \to \infty} g(x) = -1$

#14. If $\dfrac{dx}{dt} = 6\sqrt{\cos(0.125\,t + 7)}$, by how much does x change as t changes from $t = 1$ to $t = 3$?

(A) -0.824 (B) 9.019 (C) 11.952 (D) 0.824



Graph of f'

#15. The function f is continuous on the closed interval $[-4,\ 6]$. The graph of f', the derivative of f, is shown above. On which of the following intervals is f decreasing?

(A) $[-4,\ 0]$ and $[4,\ 6]$ (B) $[0,\ 4]$

(C) $[-2,\ 2]$ (D) $[-4,\ -2]$ and $[2,\ 6]$

END of Section 1, Part B

Do not work any further problems until we give instructions to move to the next part.

Next, will be a short break before we begin the Free Response Section.

This is a good time to use the bathroom and you can talk quietly in the hallways.

Please maintain a silent test environment inside the classroom until all students are finished.

########### <u>STOP</u>! DO NOT PROCEED UNTIL GIVEN INSTRUCTIONS TO DO SO ##########

Do not work any further problems until we give instructions to move to the next part, but go ahead and read the instructions below which are provided on the actual AP Exam:

CALCULUS BC, Section II, Part A has 2 free-response questions and lasts 30 minutes.

A graphing calculator is required for some questions on this part of the exam.
You may use a handheld calculator or the calculator available in this application.
Make sure your calculator is in radian mode.

You may use the available paper for scratch work, but you must write your answers in the free-response booklet. In the free-response booklet, write your solution to each part of each question in the space provided for the part. For questions that have sub-parts, be sure to label those clearly in your solutions. Use a pencil or a pen with black or dark blue ink.

You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as $\text{fnInt}\left(\text{X}^2,\text{X},1,5\right)$.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

Solve each problem. You may use the available paper for scratch work. After examining the choices, select the

Unless otherwise specified, the domain of the function *f* is assumed to be the set of all real numbers *x* for which *f(x)* is a real number.

You can go back and forth between questions in this part until time expires. The clock will turn red when 5 minutes remain – **the proctor will not give you any time updates or warnings**.

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########## <u>STOP</u>! DO NOT PROCEED UNTIL GIVEN INSTRUCTIONS TO DO SO #########

t (hours)	0	8	24	30	50
$P(t)$ (bacteria)	50	117	591	1018	3439

#1. A petri dish contains a population of bacteria that is increasing over time which can be modeled by an increasing differentiable function P, where $P(t)$ is measured in bacteria and t is measured in hours. For $0 \le t \le 50,$ selected valued of $P(t)$ are given in the table shown.

(a) Approximate $P'(16)$ using the average rate of change of P over the interval $8 \le t \le 24.$ Show the work that leads to your answer and include units of measure.

(b) Use a right Riemann sum with the subintervals indicated by the data in the table to approximate the value of $\int_8^{30} P(t)\,dt.$ Interpret the meaning of $\frac{1}{30-8}\int_8^{30} P(t)\,dt$ in the context of the problem.

t (hours)	0	8	24	30	50
$P(t)$ (bacteria)	50	117	591	1018	3439

#1 (continued). A petri dish contains a population of bacteria that is increasing over time which can be modeled by an increasing differentiable function *P*, where $P(t)$ is measured in bacteria and *t* is measured in hours. For $0 \le t \le 50$, selected valued of $P(t)$ are given in the table shown.

(c) For $50 \le t \le 80$, the rate of change of the population of bacteria is modeled by $P'(t) = \dfrac{(495{,}000)e^{-0.1077t}}{\left(1+99e^{-0.1077t}\right)^2}$, where $P'(t)$ is measured in bacteria per hour and *t* is measured in hours.

Find the number of bacteria in the petri dish at time $t = 65$ hours. Show the setup for your calculations.

(d) For the model defined in part (c), it can be shown that $P''(t) = \dfrac{(-53311.5)e^{-0.1077t}\left[1-99e^{-0.1077t}\right]}{\left(1+99e^{-0.1077t}\right)^3}$.

At time $t = 70$ hours, determine whether the number of bacteria is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

#2. For $0 \le t \le 5,$ a particle is moving along a curve so that is position at time t is $\left(x(t),\ y(t)\right),$ where $x(t) = 4t^2 - t^3$ and $y(t)$ is not explicitly given, but it is known that $\frac{dy}{dt} = e^{2\sin(t)}$. At time $t = 0$, the particle is at position $(2, 3)$.

(a) Find the acceleration vector of the particle at time $t = 2$. Show the setup for your calculations.

(b) For $0 \le t \le 5,$ find the first time t at which the speed of the particle is 3. Show the work that leads to your answer.

#2 (continued). For $0 \le t \le 5$, a particle is moving along a curve so that is position at time t is $\left(x(t),\ y(t)\right)$, where $x(t) = 4t^2 - t^3$ and $y(t)$ is not explicitly given, but it is known that $\frac{dy}{dt} = e^{2\sin(t)}$. At time $t = 0$, the particle is at position $(2, 3)$.

(c) Find the slope of the line tangent to the path of the particle at time $t = 2$. Find the y-coordinate of the position of the particle at time $t = 2$. Show the work that leads to your answers.

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le 5$. Show the setup for your calculations.

END of Section II, Part A

Do not work any further problems until we give instructions to move to the next part, but go ahead and read the instructions below which are provided on the actual AP Exam:

CALCULUS BC, Section II, Part B has 4 free-response questions and lasts 1 hour.

A calculator is not allowed for this part of the exam.

You may use the available paper for scratch work, but you must write your answers in the free-response booklet. In the free-response booklet, write your solution to each part of each question in the space provided for the part. For questions that have sub-parts, be sure to label those clearly in your solutions. Use a pencil or a pen with black or dark blue ink.

Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as $\text{fnInt}\left(X^2, X, 1, 5\right)$.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

Solve each problem. You may use the available paper for scratch work. After examining the choices, select the

Unless otherwise specified, the domain of the function *f* is assumed to be the set of all real numbers *x* for which *f(x)* is a real number.

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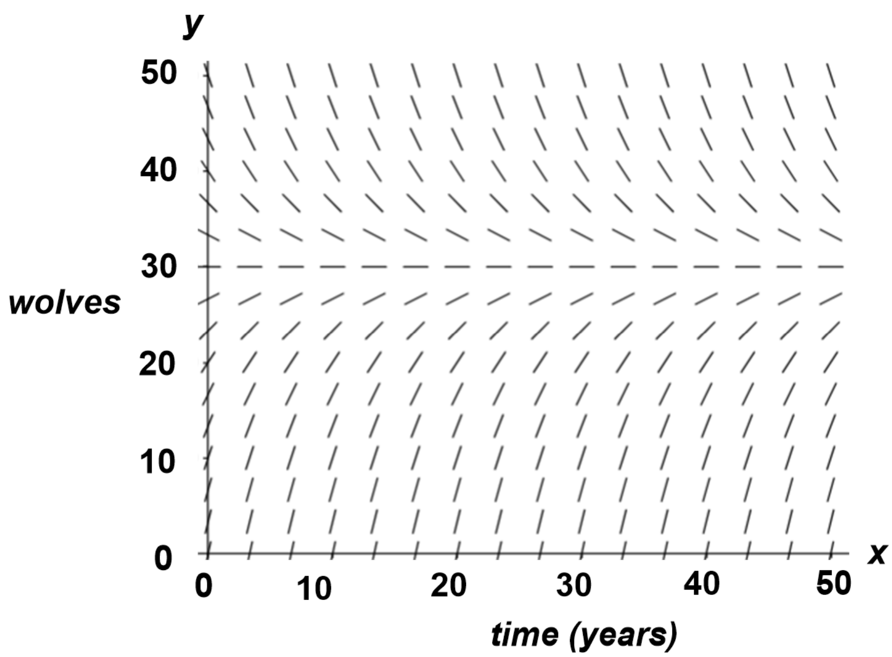
########### STOP! DO NOT PROCEED UNTIL GIVEN INSTRUCTIONS TO DO SO ##########

#3. The number of wolves in a particular forest can be modeled by the differential equation $\frac{dW}{dt}=\frac{1}{5}(30-W)$, where w is the number of wolves in the forest at time t in years from the time the population of wolves was first measured. At time $t = 0$, the wolf population was 10 wolves.

It can be shown that $W(t)<30$ for all values of t.

(a) A slope field for the differential equation $\frac{dW}{dt}=\frac{1}{5}(30-W)$ is shown. Sketch the solution curve through the point $(0,10)$.

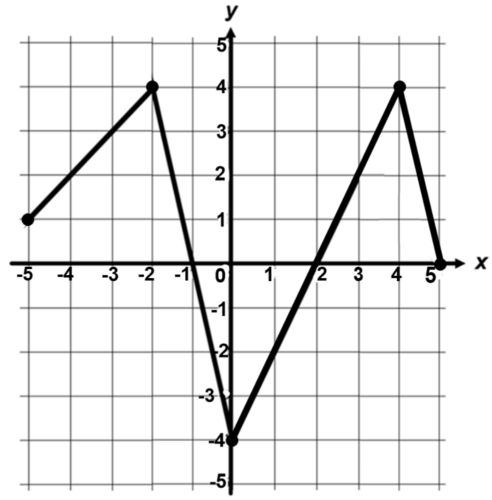


(b) Use the line tangent to the graph of W at $t = 0$ to approximate $W(5)$, the number of wolves at time t = *5 years.*

#3 (continued). The number of wolves in a particular forest can be modeled by the differential equation $\frac{dW}{dt} = \frac{1}{5}(30 - W)$, where w is the number of wolves in the forest at time t in years from the time the population of wolves was first measured. At time $t = 0$, the wolf population was 10 wolves. It can be shown that $W(t) < 30$ for all values of t.

(c) Write an expression for $\frac{d^2W}{dt^2}$. Use $\frac{d^2W}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $W(5)$. Give a reason for your answer.

(d) Use separation of variables to find an expression for $W(t)$, the particular solution to the differential equation $\frac{dW}{dt} = \frac{1}{5}(30 - W)$ with initial condition $W(0) = 10$.

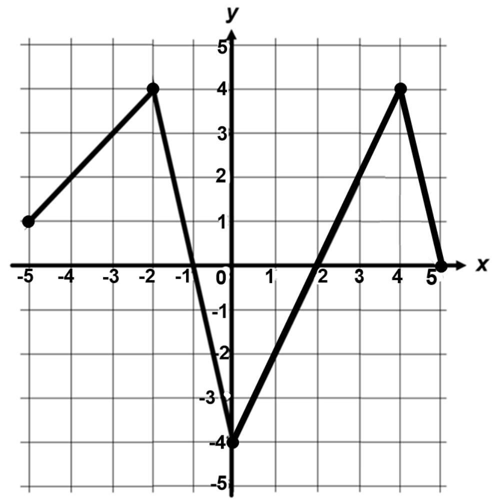


Graph of f

#4. Let f be a continuous function defined on the closed interval $-5 \le x \le 5$. The graph of f, consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t)\,dt$.

(a) On what open interval(s) is the graph of G concave down? Give a reason for your answer.

(b) Let P be the function defined by $P(x) = G(x) \bullet f(x)$. Find $P'(1)$.



Graph of f

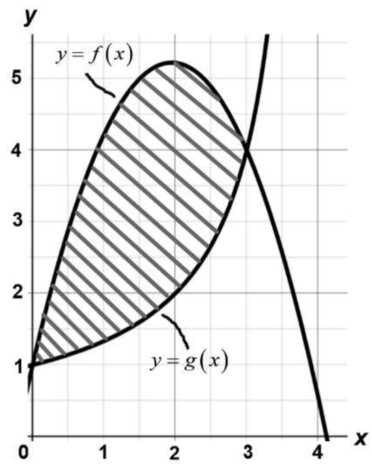
#4 (continued). Let f be a continuous function defined on the closed interval $-5 \le x \le 5$. The graph of f, consisting of four line segments, is shown above. Let G be the function defined by

$$G(x) = \int_0^x f(t)\,dt.$$

(c) Find $\lim\limits_{x \to 4} \dfrac{G(x)}{4x - x^2}$.

(d) Find the average rate of change of G on the interval $[0,5]$. Does the Mean Value Theorem guarantee a value c, $0 \le x \le 5$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

#5. The graphs of the functions f and g are shown in the figure for $0 \le x \le 4$. It is known that $g(x) = \dfrac{4}{4-x}$ for $x \ge 0$. The twice-differentiable function f, which is not explicitly given, satisfies $f(3) = 4$ and $\int_0^3 f(x)\,dx = 12.5$

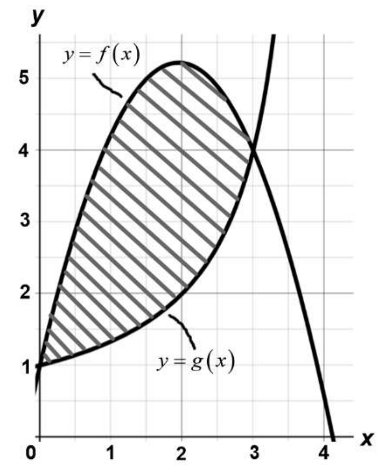


(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Evaluate the improper integral $\int_0^\infty (g(x))^2\,dx$, or show that the integral diverges.

#5 (continued). The graphs of the functions f and g are shown in the figure for $0 \le x \le 4$. It is known that $g(x) = \dfrac{4}{4-x}$ for $x \ge 0$. The twice-differentiable function f, which is not explicitly given, satisfies

$f(3) = 4$ and $\int_0^3 f(x)\,dx = 12.5$



(c) Let h be the function defined by $h(x) = x \cdot f'(x)$. Find the value of $\int_0^3 h(x)\,dx$.

#6. The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(n+1)x^{n+1}}{n^3 4^n}$ and converges to $f(x)$ for all x in the interval of convergence. It can be shown that the Maclaurin series for f has a radius of convergence of 4.

(a) Determine whether the Maclaurin series for f converges or diverges at $x = 4$. Give a reason for your answer.

(b) It can be shown that $f(-2) = \sum_{n=1}^{\infty} \frac{(n+1)(-2)^{n+1}}{n^3 4^n} = (-2)\sum_{n=1}^{\infty} \frac{n+1}{n^3}\left(-\frac{1}{2}\right)^n$ and that the first two terms of this series sum to $S_2 = \frac{29}{16}$. Show that $\left|f(-2) - S_2\right| < \frac{1}{25}$.

#6 (continued). The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(n+1)x^{n+1}}{n^3 4^n}$ and converges to $f(x)$ for all x in the interval of convergence. It can be shown that the Maclaurin series for f has a radius of convergence of 4.

(c) Find the general term of the Maclaurin series for f', the derivative of f. Find the radius of convergence of the Maclaurin series for f'.

(d) Let $h(x) = \sum_{n=1}^{\infty} \frac{(n+1)x^{2n}}{n^3 5^n}$. Use the ratio test to determine the radius of convergence of the Maclaurin series for h.

END of EXAM

Congratulations on finishing this Practice AP Calculus BC Exam!

Please check in with the proctor to get the solutions packet and instructions for scoring.