APCalcBC-HomeworkQuiz-#2

1. Which of the following gives the length of the path described by the parametric equations $x = \sin(t^3)$ and $y = e^{5t}$ fromt=0 tot=π ?

(A) $\int_0^{\pi} \sqrt{\sin^2(t^3) + e^{10t}}\, dt$

(B) $\int_0^{\pi} \sqrt{\cos^2(t^3) + e^{10t}}\, dt$

(C) $\int_0^{\pi} \sqrt{9t^4\cos^2(t^3) + 25e^{10t}}\, dt$

(D) $\int_0^{\pi} \sqrt{3t^2\cos(t^3) + 5e^{5t}}\, dt$

(E) $\int_0^{\pi} \sqrt{\cos^2(3t^2) + e^{10t}}\, dt$

arclength $= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_0^{\pi} \sqrt{(3t^2\cos(t^3))^2 + (5e^{5t})^2}\, dt$

$= \int_0^{\pi} \sqrt{9t^4\cos(t^3) + 25e^{10t}}\, dt$

2.

What is the average rate of change of the function f given by $f(x) = x^4 - 5x$ on the closed interval [0, 3]?

(not average value)

(A) 8.5

(B) 8.7

(C) 22

(D) 33

(E) 66

avg rate of change $= \frac{f(3)-f(0)}{3-0} = \frac{[3^4 - 5(3)] - (0^4 - 5(0))}{3}$ $\quad 81 - 15 = 66$

$= \frac{81-15}{3} = \frac{66}{3} = 22$

3.

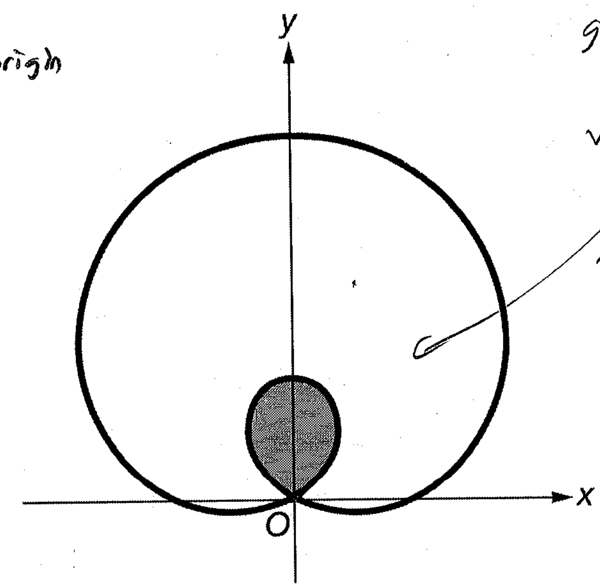
area starts/stop at origin where $r=0$

$r = 2+4\sin\theta = 0$

$4\sin\theta = -2$

$\sin\theta = -\frac{1}{2}$

$7\pi/6$ $\quad 11\pi/6$



graph in calculator... (polar mode)

verify $\frac{7\pi}{6}$ ~~&~~ $\frac{11\pi}{6}$ ✓

then:

$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2\, d\theta$

$= \int_{7\pi/6}^{11\pi/6} \frac{1}{2}(2+4\sin\theta)^2\, d\theta$

$= 2.174$

The figure above shows the graph of the polar curve $r = 2 + 4\sin\theta$. What is the area of the shaded region?

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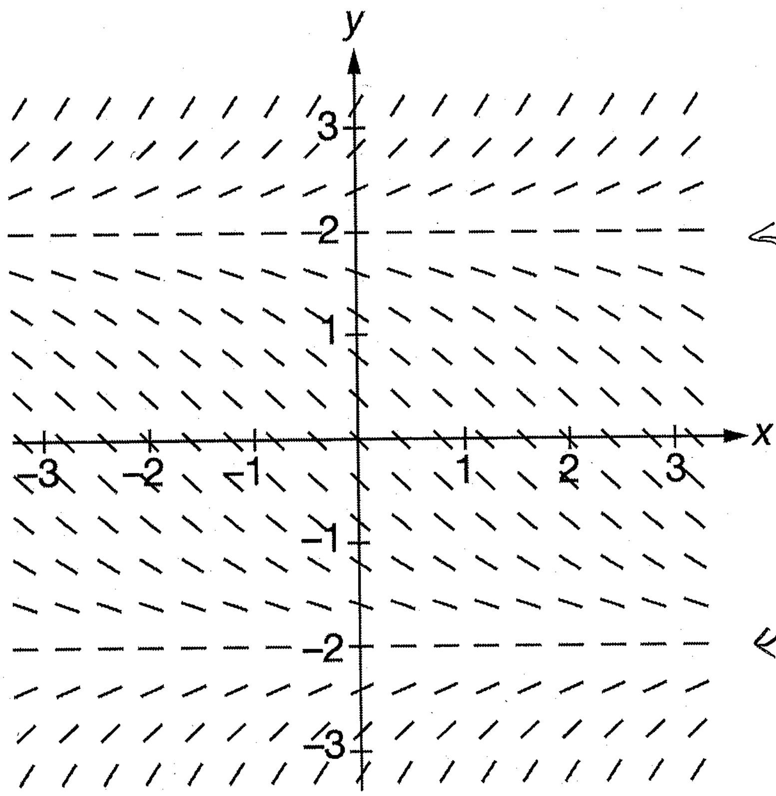
(A) 2.174
(B) 2.739
(C) 13.660
(D) 37.699

4.



$\frac{dy}{dx} = 0$ when $y=2$, $y=-2$

eliminates C & D & A (only $y=2$)

so answer must be B

Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{y-2}{2}$
(B) $\frac{dy}{dx} = \frac{y^2-4}{4}$
(C) $\frac{dy}{dx} = \frac{x-2}{2}$
(D) $\frac{dy}{dx} = \frac{x^2-4}{4}$

5. Let f and g be continuous functions for $a \le x \le b$. If $a < c < b$, $\int_a^b f(x)dx = P$, $\int_c^b f(x)dx = Q$, $\int_a^b g(x)dx = R$, and $\int_c^b g(x)dx = S$, then $\int_a^c (f(x) - g(x))$ dx =

$$\int_a^c (f(x) - g(x))dx = \int_a^c f(x)\,dx - \int_a^c g(x)\,dx$$
$$= \int_a^b f(x)dx + \int_b^c f(x)\,dx - \left(\int_a^b g(x)\,dx + \int_b^c g(x)dx\right)$$
$$-\int_c^b g(x)dx$$
$$P + Q - (R - S) = P+Q-R+S$$

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(A) $P-Q+R-S$
(B) $P-Q-R+S$
(C) $P-Q-R-S$
(D) $P+Q-R-S$
(E) $P+Q-R+S$

6. If the graph of $y=\frac{ax+b}{x+c}$ has a horizontal asymptote y=2 and a vertical asymptote x=-3, then a+c=

(A) -5
(B) -1
(C) 0
(D) 1
(E) 5

HA $\lim_{x\to\infty} \frac{ax+b}{x+c} = 2$

$\underline{a=2}$

VA $\frac{ax+b}{x+c} = \frac{\#}{0}$ at $x=-3$

$-3+c=0$

$\underline{c=3}$

So $a+c=2+3=5$

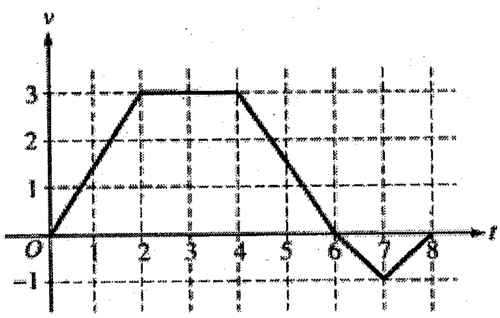
7. Let $y=f(x)$ be the solution to the differential equation $\frac{dy}{dx}=x+y$ with the initial condition $f(1)=2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x=1$ with a step size of 0.5 ?

(A) 3
(B) 5
(C) 6
(D) 10
(E) 12

(x,y)	$y_{next} = y_{current} + \left(\frac{dy}{dx}\right)\Delta x$
$(1,2)$	$y=2+(1+2)(0.5) = 2+\frac{3}{2}=3.5$
$(1.5, 3.5)$	$y=3.5+(1.5+3.5)(0.5) = 3.5+2.5=6$
$\underline{(2,6)}$	

$f(2)\approx 6$

8.



A bug begins to crawl up a vertical wire at time t = 0. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

At what value of t does the bug change direction?

when velocity changes sign, at $t=6$

(A) 2
(B) 4
(C) 6
(D) 7
(E) 8

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by parts
$u = x$ $dv = \sec^2 x\,dx$
$\frac{du}{dx} = 1$ $\int dv = \int \sec^2 x\,dx$
$du = dx$ $v = \tan x$
$uv - \int v\,du$
$x\tan x - \int \tan x\,dx$ u-sub
$x\tan x - \int \frac{\sin x}{\cos x}\,dx$ $u = \cos x$ $\frac{du}{dx} = -\sin x$, $du = -\sin x\,dx$
$x\tan x + \int \frac{1}{u}\,du$, $x\tan x + \ln|\cos x| + C$

9. $\int x \sec^2 x \, dx =$

(A) $x \tan x + C$
(B) $\frac{x^2}{2} \tan x + C$
(C) $\sec^2 x + 2\sec^2 x \tan x + C$
(D) $x \tan x - \ln|\cos x| + C$
(E) $x \tan x + \ln|\cos x| + C$

10. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec
(B) 26.447 ft/sec
(C) 32.809 ft/sec
(D) 40.671 ft/sec
(E) 79.342 ft/sec

$AV = \frac{1}{3-0}\int_0^3 (e^t + te^t)\,dt$ math 9

$= 20.086$ ft/sec

11.

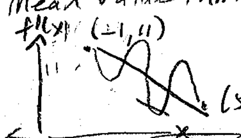
x	-1	0	2	4	5
$f'(x)$	11	9	8	5	2

Let f be a twice-differentiable function. Values of f', the derivative of f, at selected values of x are given in the table above. Which of the following statements must be true?

could be other values between given values

(A) f is increasing for $-1 \leq x \leq 5$.
(B) The graph of f is concave down for $-1 < x < 5$.
(C) There exists c, where $-1 < c < 5$, such that $f'(c) = -\frac{3}{2}$.
(D) There exists c, where $-1 < c < 5$, such that $f''(c) = -\frac{3}{2}$.

Mean Value Thm guarantees D!



$m = \frac{2-11}{5-(-1)} = \frac{-9}{6} = -\frac{3}{2}$

must be c where $f''(c) = m$

f'' (deriv of this curve (not f))

12. Water is pumped into a tank at a rate of $r(t) = 30\left(1 - e^{-0.16t}\right)$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

(A) 380 gallons
(B) 420 gallons
(C) 829 gallons
(D) 1220 gallons
(E) 1376 gallons

$\int_a^b r(t)\,dt = \int_a^b w'(t)\,dt = w(b) - w(a)$

$\int_0^{20} 30(1 - e^{-.16t})\,dt = w(20) - w(0)$
math 9
$420.1429 = w(20) - 800$
$w(20) = 1220.1429$ gallons

13. Which of the following series is conditionally convergent?

Conditionally convegent if $\sum|a_n|$ diverges but $\sum a_n$ converges

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(A) $\sum_{k=1}^{\infty}(-1)^k\frac{5}{k^3+1}$

(B) $\sum_{k=1}^{\infty}(-1)^k\frac{5}{k+1}$

(C) $\sum_{k=1}^{\infty}(-1)^k\frac{5k}{k+1}$

(D) $\sum_{k=1}^{\infty}(-1)^k\frac{5k^2}{k+1}$

for (B) $\sum|a_n| = 5\sum_{k=1}^{\infty}\frac{1}{k+1}$

limit comparison w/ $5\sum_{k=1}^{\infty}\frac{1}{k}$

p-series, w/ $p=1$ diverges

$\lim_{k\to\infty}\left|\frac{\left(\frac{1}{k+1}\right)}{\left(\frac{1}{k}\right)}\right| = \lim_{k\to\infty}\frac{k}{k+1} = 1$

finite, positive

series "linked"

so $5\sum_{k=1}^{\infty}\frac{1}{k+1}$ also diverges ($\sum|a_n|$)

now, Alternating Series Test:

$\sum_{k=1}^{\infty}(-1)^k\frac{5}{k^3+1}$

✓ • $\lim_{n\to\infty} a_n \overset{?}{=} 0$ $\quad \lim_{n\to\infty}\frac{5}{k^3+1}=0$

✓ • $a_{n+1} \overset{?}{\le} a_n$

$\frac{5}{(k+1)^3+1} = \frac{5}{k^3+1}$

converges ($\sum a_n$)

so conditionally convergent

14. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t)=t^3-3t^2+12t+4$ is

(A) 9

(B) 12

(C) 14

(D) 21

(E) 40

$a(t)=v'(t)=3t^2-6t+12$ $\quad$ max accel when $a'(t)=0$ (or DNE)

$a'(t)=6t-6=0$, when $t=1$ local extrema

max could be at $t=1$ or at interval endpts

t	$a(t)$
1	$a(1)=3(1)^2-6(1)+12=9$
0	$a(0)=3(0)^2-6(0)+12=12$
3	$a(3)=3(3)^2-6(3)+12=27-18+12=21$

15. In the xy-plane, a particle moves along the parabola $y=x^2-x$ with a constant speed of $2\sqrt{10}$ units per second. If $\frac{dx}{dt}>0$, what is the value of $\frac{dy}{dt}$ when the particle is at the point $(2, 2)$?

(A) $\frac{2}{3}$

(B) $\frac{2\sqrt{10}}{3}$

(C) 3

(D) 6

(E) $6\sqrt{10}$

$\text{speed} = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} = 2\sqrt{10}$ $\qquad \frac{dy}{dx}=2x-1\Big|_{(2,2)}=2(2)-1=3$

$\frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=3$ so $3\frac{dx}{dt}=\frac{dy}{dt}$, $\frac{dx}{dt}=\frac{1}{3}\frac{dy}{dt}$

$\sqrt{\left(\frac{1}{3}\frac{dy}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=2\sqrt{10}$, $\sqrt{\frac{1}{9}\left(\frac{dy}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}=2\sqrt{10}$, $\sqrt{\frac{10}{9}\left(\frac{dy}{dt}\right)^2}=2\sqrt{10}$, $\frac{\sqrt{10}}{3}\frac{dy}{dt}=2\sqrt{10}$

$\frac{dy}{dt}=6$

16. If $f(x)=e^{\tan^2 x}$, then $f'(x)=$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

(D) $2\tan x\sec^2 x\, e^{\tan^2 x}$

(E) $2\tan x\, e^{\tan^2 x}$

$f'(x)=e^{(\tan^2 x)}\,2(\tan x)'\sec^2 x$

17. $\int\frac{x}{x^2-4}\,dx=$

$u=x^2-4$

$\frac{du}{dx}=2x$

$du=2x\,dx$

$x\,dx=\frac{1}{2}du$

$\frac{1}{2}\int\frac{1}{u}\,du$

$\frac{1}{2}\ln|x^2-4|+C$

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(A) $\frac{-1}{4(x^2-4)^2}+C$

(B) $\frac{1}{2(x^2-4)}+C$

(C) $\frac{1}{2}\ln\left|x^2-4\right|+C$

(D) $2\ln\left|x^2-4\right|+C$

(E) $\frac{1}{2}\arctan(\frac{x}{2})+C$

18. A differentiable function f has the property that $f(5)=3$ and $f'(5)=4$. What is the estimate for $f(4.8)$ using the local linear approximation for f at x= 5 ?

(A) 2.2
(B) 2.8
(C) 3.4
(D) 3.8
(E) 4.6

tangent line: $(y-3)=4(x-5)$

$y-3=4x-20$

$f(x)=y=4x-17$

$f(4.8)=4(4.8)-17$

$4.8 \times 4 = 19.2$

$19.2 - 17 = 2.2$

19. $\displaystyle\int_1^2 \frac{x-4}{x^2}\,dx$

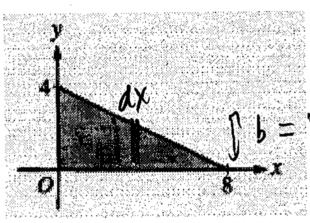
(A) $-\frac{1}{2}$
(B) ln2-2
(C) ln2
(D) 2
(E) ln2+2

$\frac{x-4}{x^2}=\frac{x}{x^2}-\frac{4}{x^2}=\frac{1}{x}-4x^{-2}$

$\int_1^2\left(\frac{1}{x}-4x^{-2}\right)dx=\ln|x|+4x^{-1}=\left[\ln|x|+\frac{4}{x}\right]_1^2$

$\left(\ln 2+\frac{4}{2}\right)-\left(\ln 1+\frac{4}{1}\right)$

$\ln 2+2-4=\ln(2)-2$

20. 

$b=4-\frac{1}{2}x$

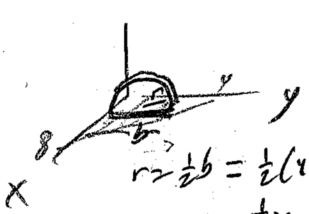
$x+2y=8$

$2y=8-x$

$y=4-\frac{1}{2}x$

The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566
(B) 14.661
(C) 16.755
(D) 67.021
(E) 134.041



$r=\frac{1}{2}b=\frac{1}{2}(4-\frac{1}{2}x)$

$r=2-\frac{1}{4}x$

$A_{cross}=\frac{1}{2}\pi r^2$

$=\frac{1}{2}\pi(2-\frac{1}{4}x)^2$

$\int_0^8\frac{1}{2}\pi(2-\frac{1}{4}x)^2dx=16.755$

(math 9)

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21. What are all values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2}{x^2+1}\right)^n$ converges?

(A) $-1 < x < 1$
(B) $x > 1$ only
(C) $x \geq 1$ only
(D) $x < -1$ and $x > 1$ only
(E) $x \leq -1$ and $x \geq 1$

geometric, converges if
$\frac{2}{x^2+1} < 1$
$2 < x^2+1$
$x^2+1 > 2$
$x^2 > 1$
$-1 \quad 0 \quad 1$

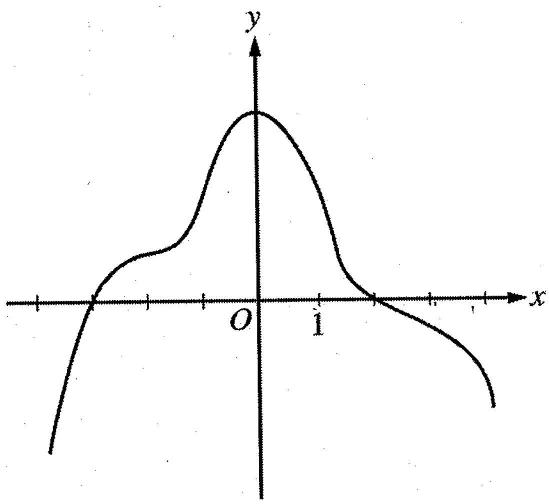
22. At $x = 3$, the function given by $f(x) = \begin{cases} x^2 & , x < 3 \\ 6x-9 & , x \geqq 3 \end{cases}$ is

(A) undefined
(B) continuous but not differentiable
(C) differentiable but not continuous
(D) neither continuous nor differentiable
(E) both continuous and differentiable

continuous if
1) $f(3)$ exists
$f(3) = 6(3) - 9 = 9$
2) $\lim_{x\to 3} f(x)$ exists ✓ $= 9$
$\lim_{x\to 3^-} x^2$ $\quad$ $\lim_{x\to 3^+} 6x-9$
3^2 $\quad$ $6(3)-9$
$9 = 9$
3) $f(3) = \lim_{x\to 3} f(x)$
$9 = 9$ ✓
continuous

differentiable if
$f'(x)_{(x<3)} = f'(x)_{(x\geq 3)}$
(slopes equal, no corner)
$2x \stackrel{?}{=} 6$
at $x = 3$
$2(3) = 6$ ✓
differentiable

23.



Graph of f'

The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

True I. f has a relative minimum at $x = -3$. rel min when $f'(x) = 0$ or DNE ($x = -3$ & $x = 2$) $-\ +$ -3 2 min

False II. The graph of f has a point of inflection at $x = -2$. when f' changes incr to decr. (not at $x = -2$)

True III. The graph of f is concave down for $0 < x < 4$. when f' decreasing, $0 < x < 4$

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(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

24. The length of a curve from $x = 1$ to $x = 4$ is given by $\int_1^4 \sqrt{1 + 9x^4}dx$. If the curve contains the point $(1, 6)$, which of the following could be an equation for this curve?

(A) $y = 3 + 3x^2$
(B) $y = 5 + x^3$
(C) $y = 6 + x^3$
(D) $y = 6 - x^3$
(E) $y = \frac{16}{5} + x + \frac{9}{5}x^5$

arclength $= \int_a^b \sqrt{1+(f'(x))^2}\,dx$

so $(f'(x))^2 = 9x^4$ so $f'(x) = 3x^2$

$f(x) = \int 3x^2 dx = x^3 + c$

$6 = (1)^3 + c$, $c = 5$

$f(x) = x^3 + 5$

25. The $\lim\limits_{h \to 0} \frac{\tan 3(x+h) - \tan(3x)}{h}$ is

(A) 0
(B) $3\sec^2(3x)$
(C) $\sec^2(3x)$
(D) $3\cot(3x)$
(E) nonexistent

limit definition of the derivative

if $f(x) = \tan(3x)$

so $f'(x) = 3\sec^2(3x)$