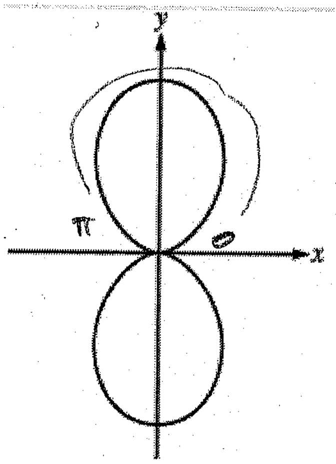
## APCalcBC-HomeworkQuiz-#4

**1.**




$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$

$A = 2\left[\frac{1}{2}\int_0^{\pi} (\sin^2\theta)^2 d\theta\right]$

$A = \int_0^{\pi} \sin^4\theta\, d\theta$

Which of the following expressions gives the total area enclosed by the polar curve $r = \sin^2\theta$ shown in the figure above?

(A) $\frac{1}{2}\int_0^{\pi} \sin^2\theta\, d\theta$

(B) $\int_0^{\pi} \sin^2\theta\, d\theta$

(C) $\frac{1}{2}\int_0^{\pi} \sin^4\theta\, d\theta$

$\boxed{\text{(D)}}$ $\int_0^{\pi} \sin^4\theta\, d\theta$

(E) $2\int_0^{\pi} \sin^4\theta\, d\theta$

**2.** Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u du$

(B) $2\int_1^{4} e^u du$

$\boxed{\text{(C)}}$ $2\int_1^{2} e^u du$

(D) $\frac{1}{2}\int_1^{2} e^u du$

(E) $\int_1^{4} e^u du$

$\int_1^4 e^{\sqrt{x}} \frac{1}{\sqrt{x}} dx$

$\int_1^2 e^u (2du)$

$2\int_1^2 e^u du$

$u = \sqrt{x} = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$du = \frac{1}{2}\frac{1}{\sqrt{x}}dx$

$\frac{1}{\sqrt{x}}dx = 2du$

**3.** If dy/dx = tan x , then $y =$

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$\frac{dy}{dx} = \tan x, \quad y = ?$

(A) $12 \tan^2 x + C$
(B) $\sec^2 x + C$
$\boxed{\text{(C)}}$ $\ln|\sec x| + C$
(D) $\ln|\cos x| + C$
(E) $\sec x \tan x + C$

$y = \int \tan x \, dx$

$y = \int \frac{\sin x}{\cos x} dx$ $\quad u = \cos x$

$y = -\int \frac{1}{u} du$ $\quad \frac{du}{dx} = -\sin x$

$-du = \sin x \, dx$

$y = -\ln|u| + C$

$y = -\ln|\cos x| + C$

$y = \ln|(\cos x)^{-1}| + C$

$y = \ln|\sec(x)| + C$

**4.** $\int_0^{\frac{\pi}{3}} \sin(3x)\, dx =$

(A) -2
$\boxed{\text{(B)}}$ -2/3
(C) 0
(D) 2/3
(E) 2

$u = 3x$
$du = 3dx$

$\frac{1}{3}\int_0^{\pi} \sin(u)\, du$

$-\frac{1}{3}[\cos u]_0^{\pi} = -\frac{1}{3}(\cos\pi - \cos 0)$

$= \frac{1}{3}((-1) - (1))$

$= -\frac{2}{3}$

**5.** The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t)=t^3-3t^2+12t+4$ is

(A) 9
(B) 12
(C) 14
$\boxed{\text{(D)}}$ 21
(E) 40

$a(t) = v'(t) = 3t^2 - 6t + 12$ $\quad$ max $a(t)$ at critical pts or interval ends

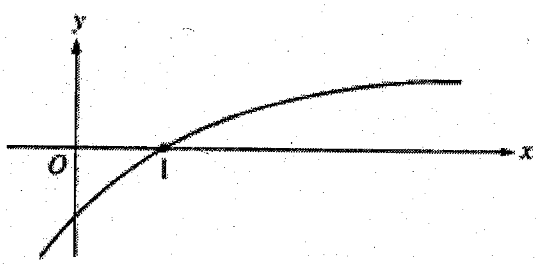
critical pts: $a'(t) = 6t - 6 = 0$

$t = 1$

candidates:

| $t$ | $a(t)$ |
|---|---|
| 1 | $3-6+12 = 9$ |
| 0 | 12 |
| 3 | $27-18+12$ 21 |

**6.**




The graph of a twice-differentiable function $f$ is shown in the figure above. Which of the following is true?

(A) $f(1)<f'(1)<f''(1)$
(B) $f(1)<f''(1)<f'(1)$
(C) $f'(1)<f(1)<f''(1)$
$\boxed{\text{(D)}}$ $f''(1)<f(1)<f'(1)$
(E) $f''(1)<f'(1)<f(1)$

$f(1) = 0$

$f'(1) > 0$

$f''(1) < 0$

$f''(1) < f(1) < f'(1)$

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**7.**

| $X$ | –2 | 0 | 3 | 5 | 6 |
|---|---|---|---|---|---|
| $f'(x)$ | 3 | 1 | 4 | 7 | 5 |

Let $f$ be a polynomial function with values of $f'(x)$ at selected values of $x$ given in the table above. Which of the following must be true for $-2 < x < 6$?

(A) The graph of $f$ is concave up. NO ( f' decreases for x=-2 to 0)

$\boxed{\text{(B)}}$ The graph of $f$ has at least two points of inflection. true because f'(x) changes from (–) (3→1) to (+) (1→7) to (–) (5–6)

(C) $f$ is increasing. not necessarily, f'(x) could be <0 in between

(D) $f$ has no critical points. must have some due to f'(x) sign change and IVT

(E) $f$ has at least two relative extrema. not necessarily; f'(x) is never <0 in indicated points, may not ever be decreasing.

**8.** Let $f$ be the function given by $f(x) = 2xe^x$ The graph of $f$ is concave down when

$\boxed{\text{(A)}}$ $x < -2$

(B) $x > -2$

(C) $x < -1$

(D) $x > -1$

(E) $x < 0$

$f'(x) = 2xe^x + e^x 2 = 2xe^x + 2e^x$

$f''(x) = 2xe^x + 2e^x + 2e^x = 2xe^x + 4e^x = 0$

$2e^x(x+2) = 0 \quad x = -2$

$-\infty \quad f''(-3) < 0 \quad -2 \quad f''(0) \; 4 \quad \infty$

**9.** What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$ converges?

(A) $-3 \le x \le 3$

(B) $-3<x<3$

(C) $-1 < x \le 5$

(D) $-1 \le x \le 5$

$\boxed{\text{(E)}}$ $-1 \le x < 5$

ratio test: $\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n \cdot 3^n}{(x-2)^n} \right|$

$\lim_{n\to\infty} \left| \frac{(x-2)(x-2)^n \, n \, 3^n}{(n+1)\, 3 \, 3^n \, (x-2)^n} \right|$

$\lim_{n\to\infty} \frac{n}{n+1} \left| \frac{x-2}{3} \right| = (1) \left| \frac{x-2}{3} \right| < 1$

$-1 < \frac{x-2}{3} < 1$

$-3 < x-2 < 3$

$-1 < x < 5$ endpts:

$\boxed{-1 \le x < 5}$

$x=-1$: $\sum_{n=1}^{\infty} \frac{(-1-2)^n}{n 3^n}$, $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{-3}{3}\right)^n = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$

Alt series test: $\lim_{n\to\infty} \frac{1}{n} = 0$, $\frac{1}{n+1} < \frac{1}{n}$ converge

$x=5$: $\sum_{n=1}^{\infty} \frac{(5-2)^n}{n 3^n} = \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{3}{3}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n}$ p-series w/ p=1 diverge

**10.** What is the $x$-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$ ?

(A) 5

(B) 0

(C) $-\frac{10}{3}$

$\boxed{\text{(D)}}$ -5

(E) -10

$y' = x^2 + 10x$

$y'' = 2x + 10 = 0$

$2x = -10$

$x = -5$

**11.** The point on the curve $x^2 + 2y = 0$ that is nearest the point$(0, -1/2)$ occurs where $y$ is nearest = minimum distance.

objective: $f = \sqrt{(x-0)^2+(y+\frac{1}{2})^2}$

constraint

$x^2+2y=0$

$2y=-x^2$

$y=-\frac{1}{2}x^2$

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$f=\sqrt{x^2+(-\frac{1}{2}x^2+\frac{1}{2})^2}$

$f=(x^2+(\frac{1}{4}x^4-\frac{1}{2}x^2+\frac{1}{4})^{1/2}$

$f=(\frac{1}{2}x^2+\frac{1}{4}x^4+\frac{1}{4})^{1/2}$

$f'=\frac{1}{2}(\frac{1}{2}x^2+\frac{1}{4}x^4+\frac{1}{4})^{-1/2}(x+x^3)=0$

$\frac{x+x^3}{2\sqrt{\frac{1}{2}x^2+\frac{1}{4}x^4+\frac{1}{4}}}=0$ when $x+x^3=0$

$x(1+x^2)=0$

$x=0, x=1, x=-1$

| $x$ | $y$ | $f=\sqrt{x^2+(y+\frac{1}{2})^2}$ |
|---|---|---|
| 0 | 0 | 1/2 |
| 1 | $-\frac{1}{2}$ | 1 |
| $-1$ | $-\frac{1}{2}$ | 1 |

min at $x=0, y=0$

(A) 1/2
(B) 0
(C) −1/2
(D) -1
(E) none of the above

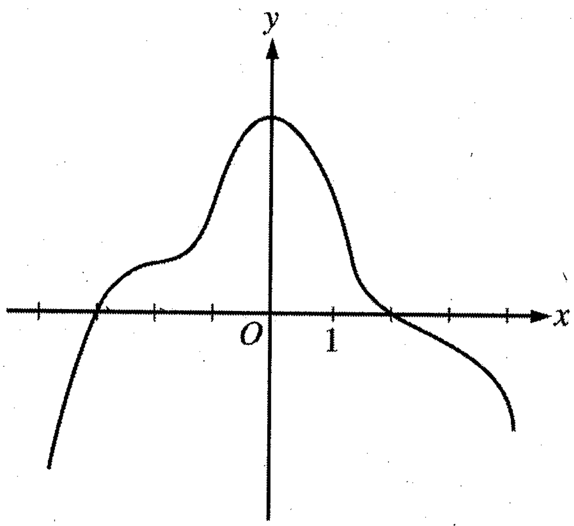
**12.** A curve $C$ is defined by the parametric equations $x = t^2 - 4t + 1$ and $y = t^3$. Which of the following is an equation of the line tangent to the graph of $C$ at the point $(-3, 8)$? ← when $t=2$

(A) $x = -3$
(B) $x = 2$
(C) $y = 8$
(D) $y = -\frac{27}{10}(x+3)+8$
(E) $y = 12(x+3)+8$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t-4}\Big|_{t=2} \Rightarrow \frac{12}{0}$ vertical tangent

$x=-3$

**13.**




Graph of $f'$

The graph of $f'$, the derivative of the function $f$, is shown above. Which of the following statements must be true?

✓ I. $f$ has a relative minimum at $x = -3$. $f'$: − + true

✗ II. The graph of $f$ has a point of inflection at $x = -2$. $f'$ increasing — $f'$ increasing, $f''>0$ — $f''>0$ no sign change false

✓ III. The graph of $f$ is concave down for $0 < x < 4$. $f$ concave down when $f''<0$ when $f'$ decreases true

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

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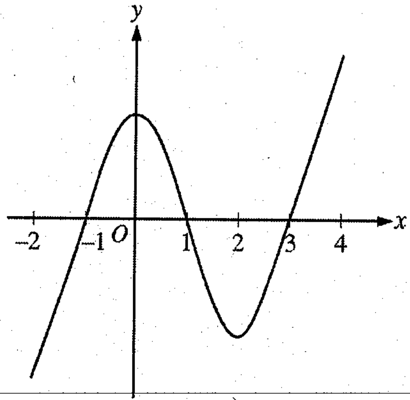
**14.**




Graph of $f''$

The graph of $f''$, the second derivative of $f$, is shown above for $-2 \le x \le 4$. What are all intervals on which the graph of the function $f$ is concave down? where $f'' < 0$

$-2 \le x < -1$

$-1 < x < 3$

(A) $-1 < x < 1$

(B) $0 < x < 2$

(C) $1 < x < 3$ only

(D) $-2 < x < -1$ only

(E) $-2 < x < -1$ and $1 < x < 3$

**15.** If $a \neq 0$, then $\lim_{x \to a} \frac{x^2-a^2}{x^4-a^4}$ is

$\lim_{x \to a} \frac{x^2-a^2}{(x^2)^2-(a^2)^2} = \lim_{x \to a} \frac{(x^2-a^2)}{(x^2-a^2)(x^2+a^2)}$

$= \lim_{x \to a} \frac{1}{x^2+a^2}$

$= \frac{1}{(a)^2+a^2} = \frac{1}{2a^2}$

(A) $\frac{1}{a^2}$

(B) $\frac{1}{2a^2}$

(C) $\frac{1}{6a^2}$

(D) 0

(E) nonexistent

**16.** If $x = t^2 + 1$ and $y = t^3$, then $d^2y/dx^2 =$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t}$

$\frac{dy}{dx} = \frac{3}{2}t$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{(dx/dt)} = \frac{\left(\frac{3}{2}\right)}{2t} = \frac{3}{2} \cdot \frac{1}{2t} = \frac{3}{4t}$

(A) 3/4t

(B) 3/2t

(C) 3$t$

(D) 6$t$

(E) 3/2

**17.** The Taylor series for $\ln x$, centered at $x=1$, is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$. Let $f$ be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

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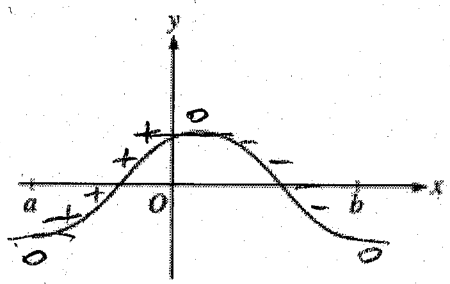
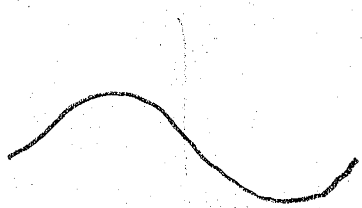
**17.**




The graph of $f$ is shown in the figure above. Which of the following could be the graph of derivative of $f$?



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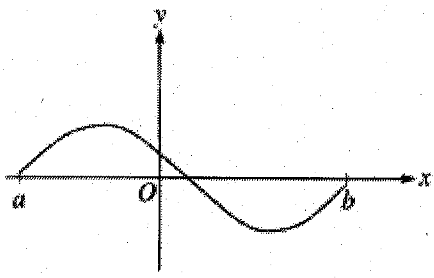
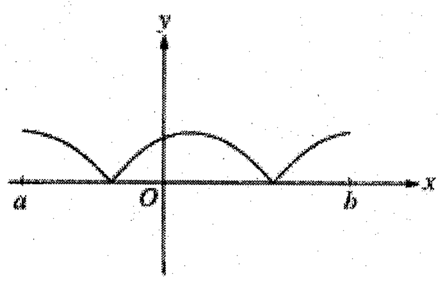
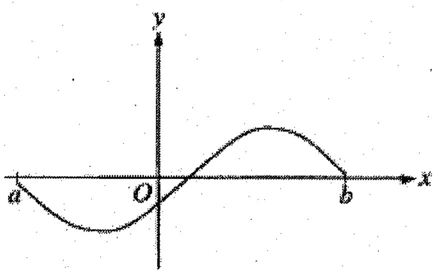
(A)



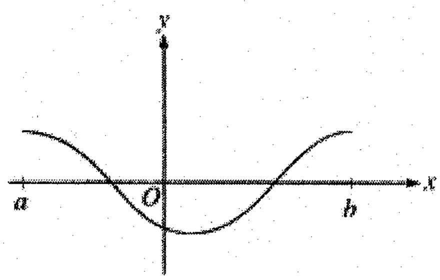

(B)



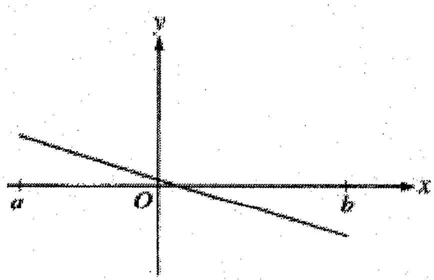

(C)




(D)




(E)




**18.** For what values of $t$ does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

vertical tangent when $\frac{dy/dt}{dx/dt} = 0$

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$\frac{dx}{dt} = 3t^2 - 2t = 0$

$t(3t-2) = 0$

$t = 0 \quad t = \frac{2}{3}$ as long as dy/dt is not also zero...

$\frac{dy}{dt} = 4t^3 + 4t - 8$

$t = 0, \; dy/dt = -8$

$t = 2/3 \quad \frac{dy}{dt} = 4(\frac{2}{3})^3 + 4(\frac{2}{3}) - 8 =$

$\frac{32}{27} + \frac{8}{3} - \frac{8}{1} \neq 0$

(A) 0 only

(B) 1 only

(C) 0 and $\frac{2}{3}$ only

(D) 0, $\frac{2}{3}$, and 1

(E) No value

**19.** The sum of the infinite geometric series $\frac{3}{2} + \frac{9}{16} + \frac{27}{128} + \frac{81}{1{,}024} + \ldots$ is

$r = \frac{3}{8} \quad r = \frac{3}{8}$

$\sum_{n=0}^{\infty} \frac{3}{2}\left(\frac{3}{8}\right)^n \quad S = \frac{a}{1-r} \quad a = \frac{3}{2} \quad r = \frac{3}{8}$

$S = \frac{(\frac{3}{2})}{1-\frac{3}{8}} = 2.4$

(A) 1.60

(B) 2.35

(C) 2.40

(D) 2.45

(E) 2.50

**20.** For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function $f$. Which of the following statements must be true?

either $\lim_{x\to\infty} f(x) = 2$ or $\lim_{x\to-\infty} f(x) = 2$ (or both)

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x\to 2} f(x) = \infty$

(E) $\lim_{x\to\infty} f(x) = 2$

**21.** Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

$y = \int 2\sin x \, dx = -2\cos x + C$

$1 = -2\cos(\pi) + C$

$1 = 2 + C, \quad C = -1$

$y = -2\cos(x) - 1$

(A) $y = 2\cos x + 3$

(B) $y = 2\cos x - 1$

(C) $y = -2\cos x + 3$

(D) $y = -2\cos x + 1$

(E) $y = -2\cos x - 1$

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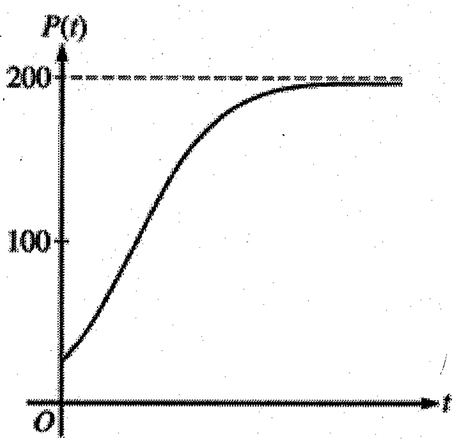
22.




$L=200$

$\frac{dP}{dt} = kP\left(1-\frac{P}{L}\right)$

$\frac{dP}{dt} = kP - \frac{kP^2}{200}$

$0.2 \quad - .001$ works

Which of the following differential equations for a population $P$ could model the logistic growth shown in the figure above?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$ → $\frac{dP}{dt} = 0.2P(1 - .005P)$

$\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{200}\right)$ ✓

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$

23. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$ ?

$= \sum_{n=1}^{\infty} 2 \cdot \frac{2^n}{3^n} = \sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^n$ Geometric so sum $= \frac{a}{1-r}$

$a = \frac{2^2}{3^1} = \frac{4}{3} \quad r = \frac{2}{3}$

$S = \frac{\left(\frac{4}{3}\right)}{1-\frac{2}{3}} = \frac{\left(\frac{4}{3}\right)}{\left(\frac{1}{3}\right)} = 4$

(A) 1

(B) 2

(C) 4

(D) 6

(E) The series diverges.

24. The velocity, in ft/sec, of a particle moving along the $x$-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$ ?

$V_{avg} = \frac{1}{3-0} \int_0^3 (e^t + te^t)\,dt = 20.0855$

(A) 20.086 ft/sec

(B) 26.447 ft/sec

(C) 32.809 ft/sec

(D) 40.671 ft/sec

(E) 79.342 ft/sec

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**25.** f(x)={x2−3x+9 for x≤2

kx+1 for x>2

The function *f* is defined above. For what value of $k$, if any, is *f* continuous at $x = 2$ ?

(A) 1

(B) 2

(C) 3

(D) 7

(E) No value of $k$ will make *f* continuous at $x = 2$ .

must at $x = 2$

$x^2 - 3x + 9 = kx + 1$

$(2)^2 - 3(2) + 9 = k(2) + 1$

$4 - 6 + 9$

$7 = 2k + 1$

$2k = 6$

$k = 3$