APCalcBC-HomeworkQuiz-#4

1. If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

$\frac{1}{1+(\cos x)^2}(-\sin x)$

(A) $\frac{-\sin x}{1+\cos^2 x}$
(B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$
(C) $(\operatorname{arcsec}(\cos x))^2$
(D) $\frac{1}{(\arccos x)^2+1}$
(E) $\frac{1}{1+\cos^2 x}$

2. $\int x^2 \cos(x^3)\,dx =$

u-sub $u = x^3$ $\frac{du}{dx} = 3x^2$ $du = 3x^2 dx$ $x^2 dx = \frac{1}{3} du$

$\frac{1}{3}\int \cos(u)\,du$ $\frac{1}{3}\sin u + C$ $\frac{1}{3}\sin(x^3) + C$

(A) $-\frac{1}{3}\sin(x^3) + C$
(B) $\frac{1}{3}\sin(x^3) + C$
(C) $-\frac{x^3}{3}\sin(x^3) + C$
(D) $\frac{x^3}{3}\sin(x^3) + C$
(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

3. $\lim\limits_{x\to\infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1} =$

$\frac{\infty}{\infty}$ multiple L'Hôpital or $\frac{x^3 - \cancel{2x^2}}{4x^3 \cancel{\ldots}} = \frac{1}{4}$

(A) 4
(B) 1
(C) 1/4
(D) 0
(E) -1

4. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

(A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$
(B) $1 - x + \frac{x^2}{2}$
(C) $x - \frac{x^2}{2} + \frac{x^3}{3}$
(D) $-1 + x - \frac{x^2}{2}$
(E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

$P_3 = f(0) + f'(0)(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3$

$f(x) = \ln(1-x)$ $f(0) = \ln(1) = 0$

$f'(x) = \frac{1}{1-x}(-1)$ $f'(0) = \frac{-1}{1-0} = -1$

$f''(x) = \frac{(1-x)(0)-(-1)(-1)}{(1-x)^2} = \frac{-1}{(1-x)^2}$ $f''(0) = -1$

$f'''(x) = \frac{0-(-1)(2(1-x)(-1))}{(1-x)^4} = \frac{-2}{(1-x)^3}$ $f'''(0) = -2$

$P_3 = 0 + (-1)x + \frac{(-1)}{2}x^2 + \frac{(-2)}{3!}x^3 = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3$

5. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

$y = \frac{\ln x}{x}$ $\frac{dy}{dx} = \frac{x(\frac{1}{x}) - \ln x(1)}{x^2} = \frac{1-\ln x}{x^2}$

(A) $\frac{1}{x}$

(B) $\frac{1}{x^2}$

(C) $\frac{\ln x - 1}{x^2}$

(D) $\frac{1-\ln x}{x^2}$

(E) $\frac{1+\ln x}{x^2}$

6. A pizza, heated to a temperature of 350 degrees Fahrenheit (°F) is taken out of an oven and placed in a (75°F) room at time $t = 0$ minutes. The temperature of the pizza is changing at a rate of $-110e^{-0.4t}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time $t = 5$ minutes?

(A) 112°F

(B) 119°F

(C) 147°F

(D) 238°F

(E) 335°F

$\int_0^5 (-110e^{-0.4t})dt = T(5) - T(0)$

$-237.7827771 = T(5) - 350$

$T(5) = 112.217°F$

7. $\int \frac{7x}{(2x-3)(x+2)} dx =$

(A) $\frac{3}{2}\ln|2x-3| + 2\ln|x+2| + C$

(B) $3\ln|2x-3| + 2\ln|x+2| + C$

(C) $3\ln|2x-3| - 2\ln|x+2| + C$

(D) $-\frac{6}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

(E) $-\frac{3}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

Partial fraction expansion: $\frac{7x}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$

$A(x+2) + B(2x-3) = 7x$

$(A+2B)x + (2A-3B) = (7)x + (0)$

$\begin{cases} A+2B=7 \\ 2A-3B=0 \end{cases}$

$-2A - 4B = -14$

$2A - 3B = 0$

$-7B = -14$

$B = 2$

$A + 2(2) = 7$

$A = 3$

$3\int \frac{1}{2x-3}dx + 2\int \frac{1}{x+2}dx$

$u = 2x-3$, $du = 2dx$, $dx = \frac{1}{2}du$ | $u = x+2$

$\frac{3}{2}\int \frac{1}{u}dx + 2\int \frac{1}{u}dx$

$\frac{3}{2}\ln|2x-3| + 2\ln|x+2| + C$

8.

x	2	3	5	8	13
$f(x)$	6	−2	−1	3	9

The function f is continuous on the closed interval $[2, 13]$ and has values as shown in the table above. Using the intervals $[2, 3]$, $[3, 5]$, $[5, 8]$, and $[8, 13]$ what is the approximation of $\int_2^{13} f(x)dx$ obtained from a left Riemann sum?

(A) 6

(B) 14

(C) 28

(D) 32

(E) 50

interval	x_i	$f(x_i) \cdot \Delta x$	= area
[2,3]	2	6 · 1	= 6
[3,5]	3	−2 · 2	= −4
[5,8]	5	−1 · 3	= −3
[8,13]	8	3 · 5	= 15

21
−7
14

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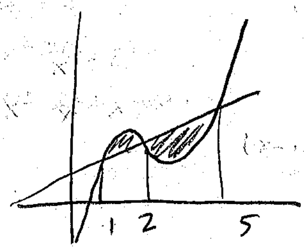
9. (CALCULATOR ALLOWED)

What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667
(B) 11.833
(C) 14.583
(D) 21.333
(E) 32

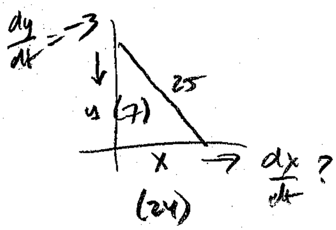


$A = \int_1^2 [(x^3-8x^2+18x-5)-(x+5)]\,dx + \int_2^5 [(x+5)-(x^3-8x^2+18x-5)]\,dx$

.583 + 11.25 = 11.833

10. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute
(B) $-\frac{7}{24}$ feet per minute
(C) $\frac{7}{24}$ feet per minute
(D) $\frac{7}{8}$ feet per minute
(E) $\frac{21}{25}$ feet per minute



(triple: 7-24-25)

$x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(24)\frac{dx}{dt} + 2(7)(-3) = 0$

$\frac{dx}{dt} = \frac{42}{48} = \frac{7}{8}$ ft/min

11. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \le t \le \frac{\pi}{2}$ is given by

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t}\, dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t\sin^2 t + 9\sin^4 t\cos^2 t}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t}\, dt$

arc length $= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \int_0^{\pi/2} \sqrt{(3\cos^2 t(-\sin t))^2 + (3\sin^2 t\cos t)^2}\, dt$

$\Rightarrow \int_0^{\pi/2} \sqrt{9\cos^4 t\sin^2 t + 9\sin^4 t\cos^2 t}\, dt$

$= \int_0^{\pi/2} \sqrt{9(\sin^2 t + \cos^2 t)(\cos^4 t + \sin^4 t)}\, dt$

$= \int_0^{\pi/2} 3\sqrt{\cos^4 t + \sin^4 t}\, dt$

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12. Which of the following series diverge?

I. $\sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$ geometric $r = \frac{\sin 2}{\pi} \approx \frac{0.9}{3.14} < 1$, converges

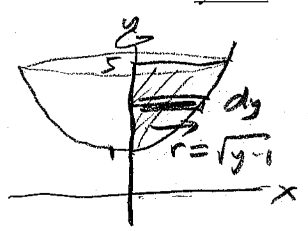
II. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$ p-series $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ w/ $p=1/3$, diverges

III. $\sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$ nth term test $\lim_{n\to\infty} \frac{e^n}{e^n+1} = 1 \neq 0$, diverges

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

13. The region R in the first quadrant is enclosed by the lines x=0 and y=5 and the graph of $y = x^2 + 1$. The volume of the solid generated when R is revolved about the y-axis is

(A) 6π
(B) 8π
(C) 34π/3
(D) 16π
(E) 544π/15



disc $V = \int \pi r^2 \, dh$

$x^2 = y-1$

$x = \pm\sqrt{y-1}$

$= \int_1^5 \pi(\sqrt{y-1})^2 \, dy$

$= \pi\int_1^5 (y-1)\,dy = \pi\left(\frac{1}{2}y^2 - y\right)\Big|_1^5$

$= \pi\left(\frac{25}{2} - 5\right) - \pi\left(\frac{1}{2} - 1\right) = \pi\left[\frac{26}{2} - \right.$

$= \pi(8)$

14. A curve is described by the parametric equations $x = t^2 + 2t$ and $y = t^3 + t^2$. An equation of the line tangent to the curve at the point determined by $t = 1$ is

(A) $2x - 3y = 0$
(B) $4x - 5y = 2$
(C) $4x - y = 10$
(D) $5x - 4y = 7$
(E) $5x - y = 13$

$(x,y) = (3,2)$

$m = \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{3t^2+2t}{2t+2}\Big|_{(t=1)} = \frac{5}{4}$

$(y-2) = \frac{5}{4}(x-3)$

$4y - 8 = 5x - 15$

$5x - 4y = 7$

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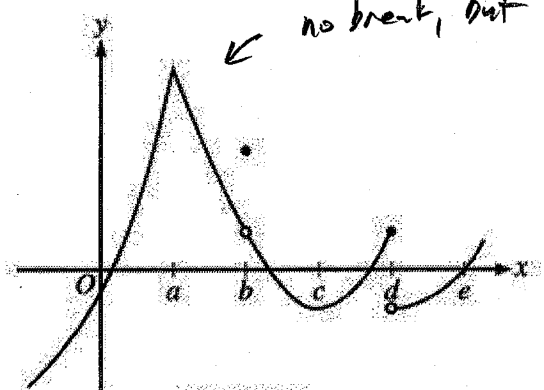
15. no break, but 'corner'



Graph of f

The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a
(B) b
(C) c
(D) d
(E) e

16. $\int_1^{\infty} \frac{x}{(1+x^2)^2} dx$ is $= \lim_{b\to\infty} \int_1^b \frac{x}{(1+x^2)^2} dx$

u-Sub: $u = 1+x^2$
$\frac{du}{dx} = 2x$
$du = 2x\,dx$, $x\,dx = \frac{1}{2} du$
$\frac{1}{2}\int u^{-2} du$, $\frac{1}{2}\frac{u^{-1}}{-1} = \frac{-1}{2(1+x^2)}$

(A) $-\frac{1}{2}$
(B) $-\frac{1}{4}$
(C) $\frac{1}{4}$
(D) $\frac{1}{2}$
(E) divergent

$= \lim_{b\to\infty} \left[\frac{-1}{2(1+b^2)}\right] - \left[\frac{-1}{2(1+1^2)}\right]$
$0 + \frac{1}{4} = \frac{1}{4}$

17. If $y = \cos^2 3x$, then dy/dx=

$y = (\cos(3x))^2$
$\frac{dy}{dx} = 2(\cos(3x))^1(-\sin(3x))\cdot 3$
$= -6\sin(3x)\cos(3x)$

(A) -6 sin $3x$ cos $3x$
(B) -2 cos $3x$
(C) 2 cos $3x$
(D) 6 cos $3x$
(E) 2 sin $3x$ cos $3x$

18. A rectangular area is to be enclosed by a wall on one side and fencing on the other three sides. If 18 meters of fencing are used, what is the maximum area that can be enclosed?

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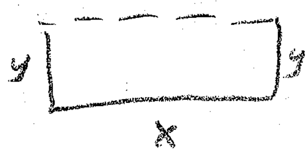


objective

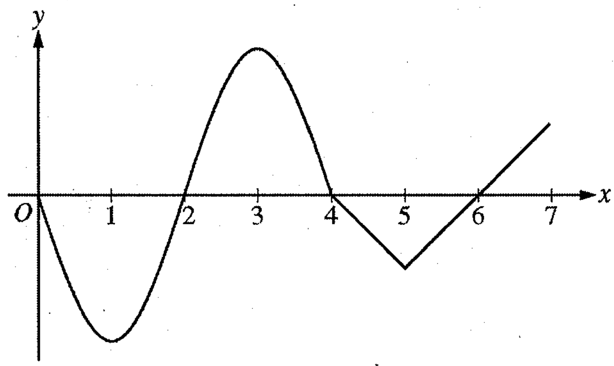
max
$A = xy$
$A = (18-2y)y$
$A = 18y - 2y^2$
$A' = 18 - 4y = 0$
$4y = 18, \ y = \frac{18}{4} = \frac{9}{2}$
$A_{max} = \frac{81}{2} m^2$

constraint

$F = x + 2y = 18$
$x = 18 - 2y$
$x = 18 - 2(\frac{9}{2}) = 9$

(A) $\frac{9}{2}\, m^2$
(B) $\frac{81}{4}\, m^2$
(C) $27\, m^2$
(D) $40\, m^2$
(E) $\frac{81}{2}\, m^2$

19.



Graph of f'

The graph of f', the derivative of the function f is shown above. On which of the following intervals is f decreasing? when $f' < 0$ $(0,2) \cup (4,6)$

(A) [2, 4] only
(B) [3, 5] only
(C) [0, 1] and [3, 5]
(D) [2, 4] and [6, 7]
(E) [0, 2] and [4, 6] (really should be open intervals)

20. The velocity vector of a particle moving in the xy-plane has components given by $\frac{dx}{dt} = \sin\left(t^2\right)$ and $\frac{dy}{dt} = e^{\cos t}$. At time $t = 4$, the position of the particle is (2, 1). What is the y-coordinate of the position vector at time $t = 3$?

$\int_3^4 e^{\cos t}\, dt = y(4) - y(3)$
$0.409707596 = 1 - y(3)$
$y(3) = 1 - .409707596 = 0.59029$

(A) 0.410
(B) 0.590
(C) 0.851
(D) 1.410

21. $\int xe^{2x}dx=$

by parts: $u = x \quad dv = e^{2x}dx \qquad uv - \int v\,du$
$\frac{du}{dx} = 1 \quad \int dv = \int e^{2x}dx \qquad \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx$
$du = dx \quad v = \frac{1}{2}e^{2x} \qquad \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

(A) $xe^{2x}/2 - e^{2x}/4 + C$
(B) $xe^{2x}/2 - e^{2x}/2 + C$
(C) $xe^{2x}/2 + e^{2x}/4 + C$
(D) $xe^{2x}/2 + e^{2x}/2 + C$
(E) $x^2e^{2x}/4 + C$

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$\int 4y\,dy = \int x\cos(x^2)dx$ $u=x^2$ $du=2x$

$2y^2 = \frac{1}{2}\int \cos u\,du$

$2y^2 = \frac{1}{2}\sin(x^2)+c$

$y^2 = \frac{1}{4}\sin(x^2)+c$

$y = \pm\sqrt{\frac{1}{4}\sin(x^2)+c}$

22. What is the general solution to the differential equation $\frac{dy}{dx} = \frac{x\cos(x^2)}{4y}$ for $y > 0$?

(A) $y = \frac{1}{2}\sqrt{\sin(x^2)} + C$

(B) $y = \sqrt{\frac{1}{4}\sin(x^2) + C}$

(C) $y = \frac{1}{8}\sin(x^2) + C$

(D) $y = Ce^{\frac{1}{8}\sin(x^2)}$

23. Let f be a differentiable function such that $f'(x) \geq 1$ for all x. If $a < b$, which of the following statements could be false?

(A) $\frac{f(b)-f(a)}{b-a} \geq 1$ true

(B) $f(b) > f(a)$ true

(C) There is a value c in the open interval (a, b) such that $f(c) = 0$. (f(a) & f(b) might both be positive or both negative)

(D) There is a value c in the open interval (a, b) such that $f(c) = \frac{f(a)+f(b)}{2}$. guaranteed by I.V.T.

24. The Taylor series for $\ln x$, centered at x=1, is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \leq x \leq 1.7$ is

$n=3$

(A) 0.030

(B) 0.039

(C) 0.145

(D) 0.153

(E) 0.529

$n=1$ $n=2$ $n=3$ $n=4$

$\underbrace{(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3}_{f(x)}, -\frac{1}{4}(x-1)^4$

$f(x) = \ln x$

$f'(x) = \frac{1}{x} = x^{-1}$

$f''(x) = -x^{-2} = \frac{-1}{x^2}$

$f'''(x) = 2x^{-3} = \frac{2}{x^3}$

error for $n=3$ $= \frac{\text{max of } f'''(x) \text{ in interval}}{4!}(x-1)^4 = \frac{\left(\frac{2}{(0.3)^3}\right)}{4!}(0.3-1)^4$

$= .741$?? hmm..

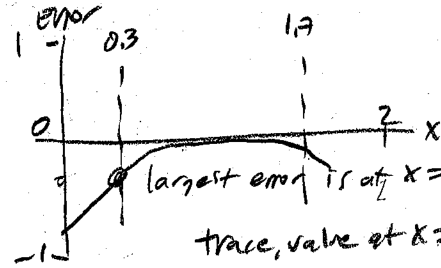
this is also an alt series so error should be less than 1st neglected term; $-\frac{1}{4}(x-1)^4$ which over interval is no larger than $\frac{1}{4}(.7)^4 = .060025$, also not helpful

★ However, if in a calculator you enter:

$Y1 = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

$Y2 = \ln x$

$Y3 = Y2 - Y1$ (the error) and just graph Y3:



largest error is at $x = 0.3$

trace, value at $x = .03$, error $= -.14463$

25. For what value of k, if any, will $y = ke^{-2x} + 4\cos(3x)$ be a solution to the differential equation $y'' + 9y = 26e^{-2x}$?

(A) 2

(B) $\frac{13}{5}$

(C) 26

(D) There is no such value of k.

$y' = -2ke^{-2x} - 12\sin(3x)$

$y'' = 4ke^{-2x} - 36\cos(3x)$

$[4ke^{-2x} - 36\cos(3x)] + 9[ke^{-2x} + 4\cos 3x] = 26e^{-2x}$

$(4k+9k)e^{-2x} = 26e^{-2x}$

$13k = 26$

$k = 2$