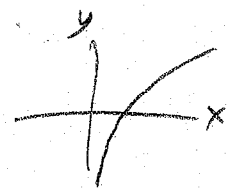
#1.

Let f be the function given by the $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by

$f'(x) = \frac{1 - \ln x}{x^2}$.

Find $\lim_{x \to 0^+} f(x)$.



$\lim_{x \to 0^+} \ln x = -\infty$

$\lim_{x \to 0^+} x = 0$

uncancelled zero in denominator, $\therefore$ $f(x)$ has a vertical asymptote at $x=0$

$\therefore$ $\boxed{\lim_{x \to 0^+} f(x) \text{ DNE}}$

#2.

Let F be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

Write an equation of the horizontal asymptote for the graph of F.

$$\lim_{x \to \infty} 2xe^{-x} = \lim_{x \to \infty} \frac{2x}{e^x}$$

$$\lim_{x \to \infty} 2x = \infty$$

$$\lim_{x \to \infty} e^x = \infty$$

but e^x increases more quickly than $2x$

$\therefore \lim_{x \to \infty} 2xe^{-x} = 0$

$\therefore$ $\boxed{\text{horizontal asymptote is } y=0}$

#3.

Let f be the function defined by $f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$

Is f continuous at $x = 3$? Explain why or why not.

1) $f(3) = \sqrt{3+1} = 2$

2) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \sqrt{x+1} = \sqrt{3+1} = 2$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (5-x) = 5-3 = 2$

$\therefore \lim_{x \to 3} f(x) = 2$

3) $\lim_{x \to 3} f(x) = f(3)$

$(2 = 2)$

all this work is required

$\boxed{\therefore\ f(x)\ \underline{\text{is}}\ \text{continuous at } x=3}$

#4.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

For $0<t<60$, must there be a time t when $v(t)=-5$? Justify your answer.

Yes. $v(t)$ is continuous on $[0,60]$, and
$v(0)=-20$, $v(60)=10$, and $-20 \le -5 \le 10$,
$\therefore$ the Intermediate Value Theorem guarantees a
time $t=c$, $0 \le c \le 60$, such that $v(c)=-5$.

#5.

Let f be the function given by $f(x) = \frac{x}{\sqrt{x^2-4}}$.

Write an equation for each vertical asymptote to the graph of f.

(vertical asymptotes occur at uncanceled zeros in denominator)

$\sqrt{x^2-4} = 0$

$x^2-4 = 0$

$(x-2)(x+2) = 0$

$\boxed{\begin{array}{l} x = 2 \\ x = -2 \end{array}}$