#1.

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$h'(x) = \frac{x^2-2}{x}$ for all $x \neq 0$.

Write an equation for the line tangent to the graph of h at $x = 4$.

$h'(4) = \frac{4^2-2}{4} = \frac{14}{4}$

$\boxed{(h+3) = \frac{14}{4}(x-4)}$

#2.

Let f be the function given by $f(x)=3x^4+x^3-21x^2$.

Write an equation of the line tangent to the graph of f at the point $(2, -28)$.

$f'(2) = 24.000025$ (use math 8 on $f(x)$)

$$\boxed{(y+28) = 24.000(x-2)}$$

#3.

Consider the curve defined by $x^2 + xy + y^2 = 27$.

(a) Write an expression for the slope of the curve at any point (x, y).

$$\frac{d}{dx}[x^2] + x\frac{d}{dx}[y] + y\frac{d}{dx}[x] + \frac{d}{dx}[y^2] = \frac{d}{dx}[27]$$

$$2x + x\left(1\frac{dy}{dx}\right) + y(1) + 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$$

$$(x+2y)\frac{dy}{dx} = -2x - y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x-y}{x+2y}}$$

(b) Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

x intercepts occur when $y=0$:

$x^2 + x(0) + (0)^2 = 27$

$x^2 = 27$

$x = \pm\sqrt{27}$

at $(\sqrt{27}, 0)$ and $(-\sqrt{27}, 0)$

$$\left.\frac{dy}{dx}\right|_{(\sqrt{27},0)} = \frac{-2(\sqrt{27})-0}{\sqrt{27}+2(0)} = -2$$

$$\left.\frac{dy}{dx}\right|_{(-\sqrt{27},0)} = \frac{-2(-\sqrt{27})-0}{(-\sqrt{27})+2(0)} = -2$$

yes, the tangent lines at x-intercepts are parallel because $\frac{dy}{dx}$ is equal for both points

(c) Find the points on the curve where the lines tangent to the curve are vertical.

vertical tangents when $\frac{dy}{dx}$ denominator $= 0$: $x+2y=0$

and points must be on the curve, so: $x^2+xy+y^2=27$

$$\begin{cases} x+2y=0 \\ x^2+xy+y^2=27 \end{cases} \rightarrow x=-2y \text{ into } x^2+xy+y^2=27:$$

$(-2y)^2 + (-2y)y + y^2 = 27$

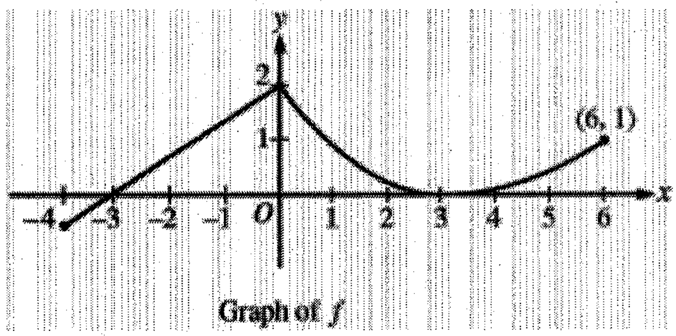
$3y^2 = 27$

$y^2 = 9$

$y = \pm 3$

$y=-3$	$y=3$
$x = 2(-3) = -6$	$x = 2(3) = 6$
$(-6,-3)$	$(6,3)$

#4.



A continuous function f is defined on the closed interval $-4 \leq x \leq 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x=3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

Is f differentiable at $x=0$? Use the definition of the derivative with one-sided limits to justify your answer.

For $x<0$:

$f(x) = \frac{2}{3}x+2$

$$\underset{(x\to 0^-)}{f'(0)} = \lim_{h\to 0} \frac{[\frac{2}{3}(x+h)+2] - (\frac{2}{3}(x)+2]}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{3}x+\frac{2}{3}h+2-\frac{2}{3}x-2}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{3}(h)}{h}$$

$$= \lim_{h\to 0} \frac{2}{3} = \frac{2}{3}$$

$$\underset{(x\to 0^-)}{f'(0)} = \frac{2}{3}$$

For $x>0$:

we don't have a function curve but from the graph:

$$\underset{(x\to 0^+)}{f'(0)} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} < 0$$

because the slope of the tangent line to f at $x=0$ on the right is negative

$$\boxed{\begin{array}{l} \text{Since } \frac{2}{3} \neq \text{(negative slope on the right)} \\ \lim_{x\to 0^-} \frac{f(x+h)-f(x)}{h} \neq \lim_{x\to 0^+} \frac{f(x+h)-f(x)}{h} \\ \therefore f'(x) \text{ DNE at } x=0 \end{array}}$$

~~#6~~ #5

t (minutes)	0	1	2	3	4	5	6
C(t) (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

$$C'(3.5) \approx \frac{C(4)-C(3)}{4-3} = \boxed{\frac{12.8-11.2}{4-3} \ \frac{\text{ounces}}{\text{minute}}}$$

~~#2~~ #6

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2,\ f'(0) = -4, \text{ and } f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answer.

$g'(x) = ae^{ax} + f'(x)$

$g'(0) = ae^{a(0)} + f'(0)$

$g'(0) = a(1) + (-4)$

$\boxed{g'(0) = a - 4}$

$g''(x) = a^2e^{ax} + f''(x)$

$g''(0) = a^2e^{a(0)} + f''(0)$

$g''(0) = a^2(1) + (3)$

$\boxed{g''(0) = a^2 + 3}$

(b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

$\boxed{h'(x) = \cos(kx)\, f'(x) + f(x)(-k\sin(kx))}$

$h'(0) = \cos(0) f'(0) + f(0)(-k\underbrace{(\sin 0)}_{0})$

$h'(0) = (1)(-4) = -4$

$h(0) = \cos(0) f(0)$

$h(0) = (1)(2) = 2$

$\boxed{(h - 2) = -4(x - 0)}$