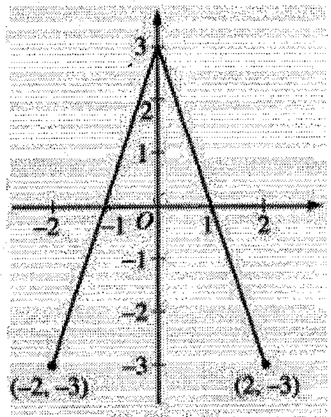
#1



Graph of f

The graph of the function f shown above consists of two line segments. Let g be the function given by

$$g(x) = \int_0^x f(t)dt.$$

Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

$g(-1) = \int_0^{-1} f(t)dt = -\int_{-1}^{0} f(t)dt = -\left[\frac{1}{2}(1)(3)\right] = \boxed{-\frac{3}{2}}$

$g'(x) = \frac{d}{dx}\left[\int_0^x f(t)dt\right] = f(x)$

$g'(-1) = f(-1) = \boxed{0}$

$g''(x) = \frac{d}{dx}(g'(x)) = \frac{d}{dx}[f(x)] = f'(x)$ (is the "slope" of the $f(x)$ curve)

$g''(-1) = f'(-1) = \boxed{3}$

#2

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

$$\int_0^{1,5} (3f'(x) + 4)\,dx = \int_0^{1,5} 3f'(x)\,dx + \int_0^{1,5} 4\,dx$$

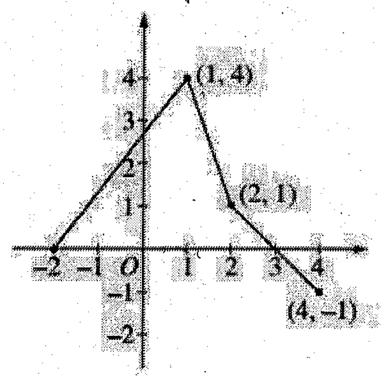
$$= [3f(1,5)] - [3f(0)] + [4x]_0^{1,5}$$

$$= 3(-1) - 3(-7) + 4(1,5) - 4(0)$$

$$= -3 + 21 + 6 - 0$$

$$= \boxed{24}$$

#3



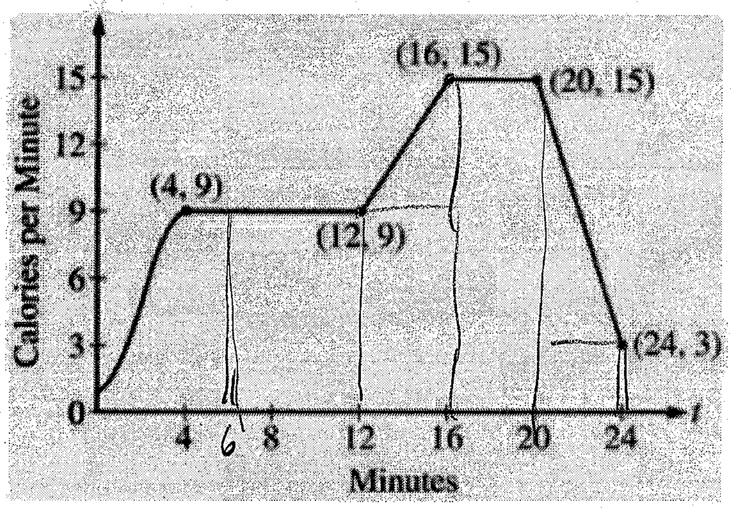
The graph of function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

$g'(x) = \frac{d}{dx}\left[\int_1^x f(t)\,dt\right] = f(x)$

$g'(1) = f(1) = \boxed{4}$

#4



The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f. In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and f is piecewise linear for $4 \le t \le 24$.

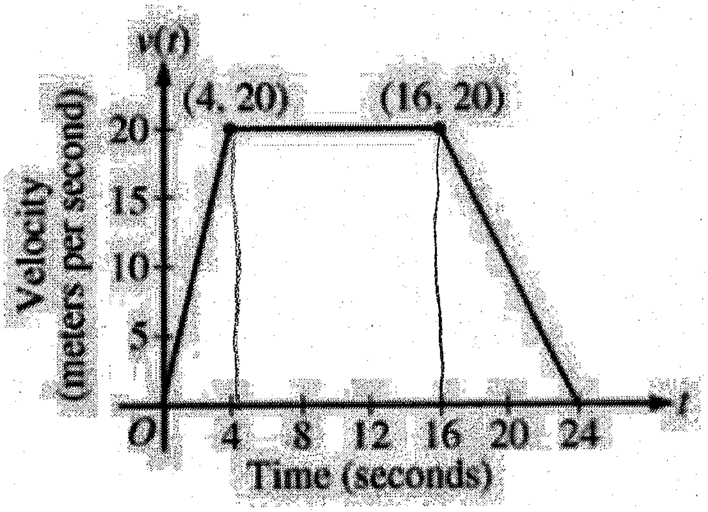
Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

total calories $= \int_6^{18} f(t)\,dt$

$= \int_6^{12} f(t)\,dt + \int_{12}^{16} f(t)\,dt + \int_{16}^{20} f(t)\,dt + \int_{20}^{24} f(t)\,dt$

$= \boxed{(6)(9) + (4)(9) + \frac{1}{2}(4)(6) + (4)(4) + (4)(3) + \frac{1}{2}(4)(12) \text{ calories}}$

#5



A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

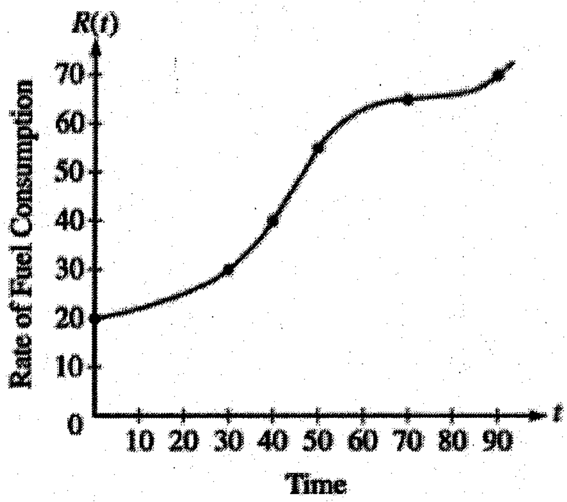
$\int_0^{24} v(t)\,dt = \boxed{\frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) \text{ meters}}$

$\int_0^{24} v(t)dt$ is the displacement (difference is the starting and ending position) in meters, over the time period from 0 to 24 seconds.

Because $v(t)$ is always positive over this time interval, $(v(t) = |v(t)|)$

$\int_0^{24} v(t)dt$ also represents the total distance traveled by the car from $0 \leq t \leq 24$ seconds.

#6

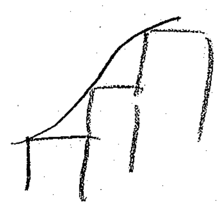


t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, are shown above.

Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

Interval	t_i	$R(t_i)\cdot\Delta t$	
$[0,30]$	0	$20 \cdot 30$	$= 600$
$[30,40)$	30	$30 \cdot 10$	$= 300$
$(40,50)$	40	$40 \cdot 10$	$= 400$
$[50,70)$	50	$55 \cdot 20$	$= 1100$
$(70,90)$	70	$65 \cdot 20$	$= 1300$



$\boxed{\int_0^{90} R(t)\,dt = 3700 \text{ gallons}}$

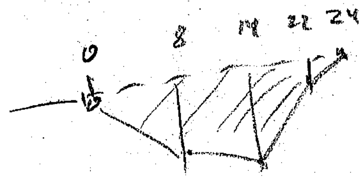
Since $R(t)$ is strictly increasing over $0 \leq t \leq 90$, this numerical approximation is less than the actual value of $\int_0^{90} R(t)\,dt$.

#7

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.



interval	f_L	f_R	$A_{trap} = \frac{1}{2}(f_L+f_R)\Delta x$
$[0,8]$	0	7	$\frac{1}{2}(0+7)(8) =$
$[8,14]$	7	8	$\frac{1}{2}(7+8)(6)$
$[14,22]$	8	2	$\frac{1}{2}(8+2)(8)$
$[22,24]$	2	0	$\frac{1}{2}(2+0)(2)$

$$\text{cross-section area} = \int_0^{24} \text{depth}(x)\,dx$$

$$\approx \boxed{\tfrac{1}{2}(0+7)(8) + \tfrac{1}{2}(7+8)(6) + \tfrac{1}{2}(8+2)(8) + \tfrac{1}{2}(2+0)(2) \ \text{ft}^2}$$

#8

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is 55^{0}F. The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

Use the data in the table to evaluate $\displaystyle\int_0^{20} W'(t)dt$. Using correct units, interpret the meaning of $\displaystyle\int_0^{20} W'(t)dt$ in the context of this problem.

$$\int_0^{20} w'(t)\,dt = W(20) - W(0)$$

$$\int_0^{20} w'(t)dt = 71.0 - 55.0 = \boxed{16.0\ ^{\circ}F}$$

$\int_0^{20} w'(t)dt$ is the accumulated change in water temperature over $0 \le t \le 20$, meaning the water gained 16.0 °F from $t=0$ to $t=20$ minutes.

#9

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

interval	t_i	$W(t_i)$	$\cdot \Delta t$	area
$[0,4)$	0	55	$\cdot 4$	$= 220$
$(4,9)$	4	57.1	$\cdot 5$	$= 285.5$
$[9,15)$	9	61.8	$\cdot 6$	$= 370.8$
$[15,20)$	15	67.9	$\cdot 4$	$= 271.6$
				1147.9

$$\frac{1}{20}\int_0^{20} w(t)\,dt = \frac{1}{20}\left(1147.9\right) = \boxed{57.395\ ^{\circ}F}$$

Since $w(t)$ is strictly increasing over $0 \le t \le 20$, this approximation of $\frac{1}{20}\int_0^{20} w(t)\,dt$ underestimates the average water temperature.

A10

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t=0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t=25$?

Approximating the function W(t) with it's tangent line!

$W(20) = 71.0$

$W'(20) = 0.4\sqrt{20}\cos(0.06(20)) = 0.648205 2569$

$(w - 20) = 0.6482052569(t-20)$

$w(t) \approx 0.6482052569(t-20) + 20$

$\boxed{w(25) \approx 23.241\ °F}$