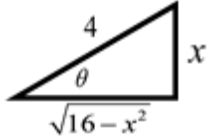


## INTEGRATION TECHNIQUES SUMMARY

Technique	Useful for integrals where...	Examples
Antiderivative shortcuts	(All other procedures' goal is to end with shortcuts)	Memorize entire list, including: $\int \frac{1}{a^2 + u^2} dx = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - u^2}} dx = \arcsin\left(\frac{u}{a}\right) + C$ $\int \frac{1}{u\sqrt{u^2 - a^2}} dx = \operatorname{arcsec}\left(\frac{u}{a}\right) + C$ $\int \sec(x) dx = \ln \sec(x) + \tan(x)  + C$ $\int \csc(x) dx = \ln \csc(x) - \cot(x)  + C$
Algebra	...integrand is a fraction with a common denominator:  ...integrand is a ratio of powers of the variable:  ...integrand is a fraction with non-factorable denominator	$\int \frac{x^3 - 2x}{\sqrt{x}} dx$ $\int \frac{x^5}{x^{1/3}} dx$ $\int \frac{1}{x^2 - 4x + 7} dx = \int \frac{1}{(x-2)^2 + 3} dx$ ( <i>complete the square to arctan form</i> )
u-Substitution	...integrand has a function within another function and the derivative of the inside is also in the integrand	$\int x^3 e^{(3x^4)} dx$ u = inside function, derivative to get dx, substitution into original integral.
by-Parts	...integrand contains a product (two factors multiplied)	$\int x e^{(x)} dx$ Choose u and dv (dx is part of dv), derivative of u to get du, integral of dv to get v, substitute into: $uv - \int v du$
Trig Integrals	...integrand contains trig functions	$\int \sin^3 x \cos^3 x dx, \int \sec^2 x \tan^2 x dx$ ( <i>reserve something for du, convert the rest to u</i> )  $\int \tan x dx, \int \cot x dx$ ( <i>use reciprocal identities: <math>\tan x = \frac{\sin x}{\cos x}</math></i> )  $\int \cos^2(x) dx, \int \sin^2(x) dx$ ( <i>use power-reducing identities</i> )  $\cos^2(x) = \frac{1 + \cos(2x)}{2}, \sin^2(x) = \frac{1 - \cos(2x)}{2}$

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Trig Substitution	...integrand contains a radical of form $\sqrt{u^2 + a^2}$ , $\sqrt{u^2 - a^2}$ or $\sqrt{a^2 - u^2}$	$\int \frac{x^3}{\sqrt{16-x^2}} dx$  $\int \frac{(4\sin\theta)^3}{4\cos\theta} 4\cos\theta d\theta$ $\sin\theta = \frac{x}{4} \quad \cos\theta = \frac{\sqrt{16-x^2}}{4}$ $x = 4\sin\theta \quad \sqrt{16-x^2} = 4\cos\theta$ $dx = 4\cos\theta d\theta$ <p><i>(radical on hypotenuse if plus case, otherwise on a leg, pair the constant side with each other side to write a trig ratio, solve for x and radical, derivative of x to get dx, then substitute into integral)</i></p>
Partial Fractions	...integrand is a fraction with a factorable denominator	$\int \frac{1}{x^2 + x - 6} dx = \int \frac{1}{(x-2)(x+3)} dx$ $\frac{1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} \quad \text{system to find } A, B$
Improper Integrals	...infinite limit of integration:  ...integrand has a vertical asymptote within the interval of integration:	$\int_2^{\infty} x^2 dx = \lim_{b \rightarrow \infty} \int_2^b x^2 dx$ $\int_1^5 \frac{1}{x-3} dx = \lim_{b \rightarrow 3^-} \int_1^b \frac{1}{x-3} dx + \lim_{c \rightarrow 3^+} \int_c^5 \frac{1}{x-3} dx$