AP Calculus BC – Unit 1 Extra Practice

1.1 – Extra Practice

Complete the table. Use the result to estimate the limit. Use your calculator to graph the function to confirm your results.

#9b. $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$ $= 0.5 = \frac{1}{2}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	0.51317	0.50126	0.50013	?	0.49988	0.49876	0.48809

#9c. $\lim_{x \to 4} \frac{x-4}{x^2-5x+4}$ $= 0.33\bar{3} = \frac{1}{3}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
f(x)	0.34483	0.33445	0.33344	?	0.33322	0.33223	0.32258

#9d. $\lim_{x \to 0} \frac{\cos(x)-1}{x}$ $= 0$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	0.04996	0.005	$5 \cdot 10^{-4}$	?	$-5 \cdot 10^{-4}$	-0.005	-0.05

#9e. $\lim_{x \to 0} \frac{\ln(x+1)}{x}$ $= 1$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	1.0536	1.005	1.0005	?	0.9995	0.99503	0.9531

#9f. $\lim_{x \to 0} \frac{e^x-1}{x}$ $= 1$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	0.95163	0.99502	0.9995	?	1.0005	1.005	1.0517

Evaluate the limit using any method. Graph in a calculator to verify your result.

#10b. $\lim_{x \to -4} \frac{x+4}{x^2+9x+20}$

$$= \lim_{x \to -4} \frac{x+4}{(x+4)(x+5)}$$

$$= \lim_{x \to -4} \frac{1}{x+5} = \frac{1}{-4+5} = \frac{1}{1} = 1$$

#11b. $\lim_{x \to -3} \frac{x^3+27}{x+3}$

$$= \lim_{x \to -3} \frac{(x+3)(x^2-3x+9)}{(x+3)} = \lim_{x \to -3} (x^2-3x+9)$$

$$\begin{array}{r|rrrr} -3 & 1 & 0 & 0 & 27 \\ & & -3 & 9 & -27 \\ \hline & 1 & -3 & 9 & \boxed{0} \end{array}$$

x^2-3x+9

$$= (-3)^2-3(-3)+9$$

$$= 9+9+9$$

$$= 27$$

#12b. $\lim_{x\to 0}\frac{\sin(3x)}{x} = 3$

x	$\frac{\sin(3x)}{x}$
$-.1$	2.9552
$-.01$	2.9996
$-.001$	3
$.001$	3
$.01$	2.9996
$.1$	2.9552

$\to 3$

#12c. $\lim_{x\to 0}\frac{5\tan(x)}{\tan(3x)}$

$$= \lim_{x\to 0}\frac{5}{1}\,\frac{\sin(x)}{\cos(x)}\,\frac{\cos(3x)}{\sin(3x)}$$

$$= \lim_{x\to 0}\frac{5}{1}\,\frac{\sin x}{1}\,\frac{3x}{3x}\,\frac{1}{\sin(3x)}\,\frac{\cos(3x)}{1}\,\frac{1}{\cos(x)}$$

$$= \lim_{x\to 0}\frac{5}{1}\,\frac{\sin x}{x}\,\frac{1}{3}\,\frac{3x}{\sin(3x)}\,\frac{\cos(3x)}{1}\,\frac{1}{\cos(x)}$$

$$= (5)(1)\left(\frac{1}{3}\right)(1)\left(\frac{1}{1}\right)\left(\frac{1}{1}\right)$$

$$= \frac{5}{3}$$

(or could use a table or graph if you can use a calculator)

#12d. $\lim_{x\to 0}\frac{\tan(x)}{\tan(2x)}$

$$= \lim_{x\to 0}\frac{\sin x}{\cos x}\,\frac{\cos(2x)}{\sin(2x)}$$

$$= \lim_{x\to 0}\frac{\sin x}{1}\,\frac{2x}{1}\,\frac{1}{2x\sin(2x)}\,\frac{1}{\cos x}\,\frac{\cos}{1}$$

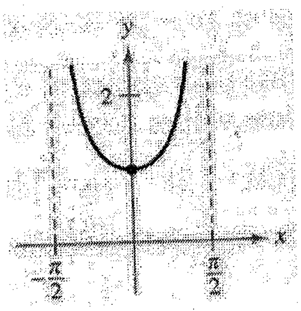
$$= \lim_{x\to 0}\frac{\sin x}{x}\,\frac{1}{2}\,\frac{2x}{\sin(2x)}\,\frac{1}{\cos x}\,\frac{\cos(2x)}{1}$$

$$= (1)\left(\frac{1}{2}\right)(1)\left(\frac{1}{1}\right)\left(\frac{1}{1}\right)$$

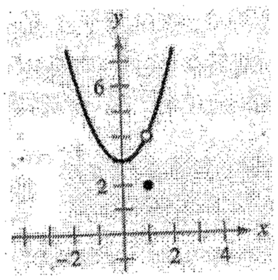
$$= \frac{1}{2}$$

Use the graph to find the limit (if it exists). If the limit does not exist, explain why.

#13b. $\lim_{x\to 0}\sec(x) = 1$



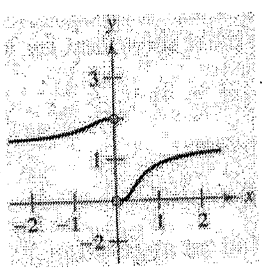
#14b. $\lim_{x\to 1} f(x)$ where $f(x) = \begin{cases} x^2+3 & x \neq 1 \\ 2 & x = 1 \end{cases}$



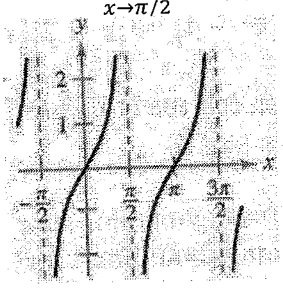
$\lim_{x\to 1} f(x) = 4$

#15b. $\lim_{x\to 0}\frac{4}{2+e^{1/x}}$

DNE because $\lim_{x\to 0^-}\frac{4}{2+e^{1/x}} \neq \lim_{x\to 0^+}\frac{4}{2+e^{1/x}}$



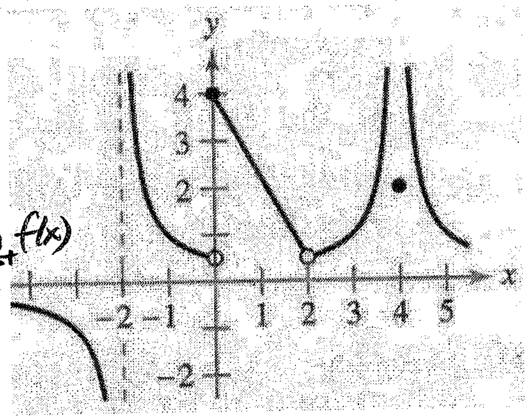
#16b. $\lim_{x\to\pi/2}\tan(x)$



DNE because $\lim_{x\to \frac{\pi}{2}^-}\tan x \neq \lim_{x\to\frac{\pi}{2}^+}\tan x$

Use the graph of the function f to decide whether the value of the quantity exists. If it does, find it. If not, explain why.

#17b.



a. $f(-2)$ DNE

b. $\lim_{x\to 1} f(x) = 2$

c. $f(0) = 4$

d. $\lim_{x\to 0} f(x)$ DNE because $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

e. $f(2)$ DNE

f. $\lim_{x\to 2} f(x) = 1$

g. $f(4) = 2$

h. $\lim_{x\to 4} f(x)$ DNE because $\lim_{x\to 4^-} f(x) \neq \lim_{x\to 4^+} f(x)$ (cannot equate infinities)

#18 (hint) Remember, it doesn't matter what happens at x = c, what matters is whether or not the value being *approached* is the same from both sides.

#19 (hint) Evaluate the value of *f(x)* and x approaches 1. Then plug this value into the sine function. (There is a limit property which allows this: $\lim_{x\to c} f\left(g(x)\right) = f\left(\lim_{x\to c} g(x)\right)$

1.2 – Extra Practice

Find the limit. Use a graphing calculator to verify.

#6b. $\lim_{x\to 3}\sqrt{x+1}$

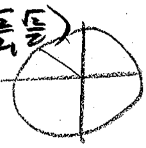
$= \sqrt{3+1}$

$= \sqrt{4} = 2$

#7b. $\lim_{x\to 3}\tan\left(\frac{\pi x}{4}\right)$

$= \tan\left(\frac{3\pi}{4}\right) = \frac{\sin\left(\frac{3\pi}{4}\right)}{\cos\left(\frac{3\pi}{4}\right)} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$



#8b. $\lim_{x\to 1}\frac{e^x}{2x^2-x}$

$= \frac{e^1}{2(1)^2-(1)} = \frac{e}{1} = e$

#9b. $\lim_{x\to 0}\frac{2x}{x^2+4x}$

$= \lim_{x\to 0}\frac{\cancel{x}(2)}{\cancel{x}(x+4)}$

$= \lim_{x\to 0}\frac{2}{x+4} = \frac{2}{0+4} = \frac{1}{2}$

#10b. $\lim_{x\to 2}\frac{x^2+2x-8}{x^2-x-2}$

$= \lim_{x\to 2}\frac{\cancel{(x-2)}(x+4)}{\cancel{(x-2)}(x+1)}$

$= \lim_{x\to 2}\frac{x+4}{x+1} = \frac{2+4}{2+1} = \frac{6}{3} = 2$

#11b. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}$

$= \lim_{x\to 3}\frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$

$= \lim_{x\to 3}\frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3}\frac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$

$= \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{3+1}+2} = \frac{1}{4}$

#12b. $\lim_{x\to 0}\frac{3(1-\cos(x))}{x}$

$= 3\lim_{x\to 0}\frac{1-\cos(x)}{x}$

$= (3)(0)$

$= 0$

#13b. $\lim_{x\to 0}\frac{(\cos(x))(\tan(x))}{x}$

$= \lim_{x\to 0}\frac{\cos x}{1}\cdot\frac{1}{x}\cdot\frac{\sin x}{\cos x}$

$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\cos x}{\cos x}$

$= (1)(1)$

$= 1$

#14b. $\lim_{x\to 0}\frac{\tan^2(x)}{x}$

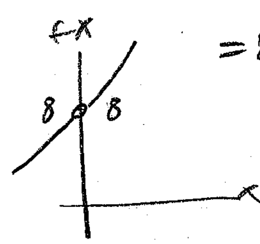
$= \lim_{x\to 0}\frac{\sin x}{\cos x}\cdot\frac{\sin x}{\cos x}\cdot\frac{1}{x}$

$= \frac{\sin x}{1}\cdot\frac{\sin x}{x}\cdot\frac{1}{(\cos x)^2}$

$= (0)(1)\left(\frac{1}{1^2}\right)$

$= 0$

#15b. $\lim_{x\to 0}\frac{4(e^{2x}-1)}{e^x-1}$ (can't use L'Hopital's rule yet, so use a graph or table)



$= 8$ (by graph)

Find the limit analytically, then verify by calculator graph.

#16b. $\lim_{t \to 0} \frac{\sin(3t)}{2t}$

$$= \lim_{t \to 0} \frac{\sin(3t)}{1} \frac{3t}{3t} \frac{1}{2t}$$

$$= \lim_{t \to 0} \frac{\sin(3t)}{3t} \frac{3t}{2t}$$

$$= \lim_{t \to 0} \frac{\sin(3t)}{3t} \frac{3}{2}$$

$$= (1)\left(\frac{3}{2}\right) = \frac{3}{2}$$

#16c. $\lim_{x \to 0} \frac{\cos(x)-1}{2x^2}$

Find $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$. Compare the result to the original function...notice anything?

#17b. $f(x) = x^2 - 4x$

$$\lim_{\Delta x \to 0} \frac{[(x+\Delta x)^2 - 4(x+\Delta x)] - [x^2 - 4x]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 4x - 4\Delta x - x^2 + 4x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 4\Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x + 4)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x + 4) = 2x + 4$$

(which is the derivative of $f(x)$)

#18 (hint) You need to use the Squeeze Theorem to solve this problem.

#6b. Let f be a function defined by $f(x)=\begin{cases} x^2-2, & x<1 \\ -x, & x\geq 1 \end{cases}$

Show that f is continuous at $x=1$.

✓ 1) $f(1)=-(1)=-1$

✓ 2) $\lim\limits_{x\to 1} f(x)$

$\lim\limits_{x\to 1^-} x^2-2 = (1)^2-2 = -1$ equal $\lim\limits_{x\to 1^+} -x = -(1) = -1$

$\therefore \lim\limits_{x\to 1} f(x) = 1$

✓ 3) $f(1) = \lim\limits_{x\to 1} f(x)$

$\therefore$ $f(x)$ is continuous at $x=1$

#7b. $R(t)=\begin{cases} 41\sqrt{\frac{t}{20}} & \text{for } 0\leq t<20 \\ \frac{100t^2+1000}{t^2+600} & \text{for } t\geq 20 \end{cases}$

Is the function R continuous at $t=20$? Justify your answer.

✓ 1) $R(20) = \dfrac{100(20)^2+1000}{(20)^2+600} = 41$

✓ 2) $\lim\limits_{t\to 20} R(t)$

$\lim\limits_{t\to 20^-} 41\sqrt{\frac{t}{20}} = 41\sqrt{\frac{20}{20}} = 41$

$\lim\limits_{t\to 20^+} \dfrac{100t^2+1000}{t^2+600} = \dfrac{100(20)^2+1000}{(20)^2+600} = 41$

equal

$\therefore \lim\limits_{t\to 20} R(t) = 41$

✓ 3) $R(20) = \lim\limits_{t\to 20} R(t)$

$\therefore$ $R(t)$ is continuous at $t=20$.

#8b.

$$f(x) = \begin{cases} \frac{x^2+kx-2}{3x^2+4x+3} & \text{for } x < -2 \\ x^3+2 & \text{for } -2 \le x < 0 \\ 0 & \text{for } x = 0 \\ \frac{2e^x}{2-e^x} & \text{for } x > 0 \end{cases}$$

Let f be the function defined above, where k is a constant.

(a) For what value of k, if any, is f continuous at $x = -2$? Justify your answer.

$$\lim_{x \to -2^-} \frac{x^2+kx-2}{3x^2+4x+3} \overset{\text{must}}{=} \lim_{x \to -2^+} x^3+2$$

$$\frac{4+k(-2)-2}{3(-2)^2+4(-2)+3} \qquad (-2)^3+2$$

$$-8+2$$

$$\frac{2-2k}{7} = -6$$

$$2-2k=-42$$

$$-2k=-44$$

$$k=22$$

(b) What type of discontinuity does f have at $x = 0$? Give a reason for your answer.

$$\lim_{x \to 0^-} x^3+2 \qquad \lim_{x \to 0^+} \frac{2e^x}{2-e^x}$$

$$0^3+2$$

$$2 = 2$$

but $f(0)=0$

x	$\frac{2e^x}{2-e^x}$
0.1	2.4701
0.01	2.0406
0.001	2.004
0.0001	2.0004
↓	↓
0	2

$\therefore$ $f(x)$ has a removable discontinuity at $x=0$

#9b.

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	−6.1	−5.0	−4.4	−3.8	−3.5

An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r, where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \le t \le 12$.

Is there a time t, $0 \le t \le 3$, for which $r'(t) = -6$? Justify your answer.

Since $r(t)$ is differentiable, it, and $r'(t)$, are continuous over $[0,3]$

$r'(0) = -6.1$

$r'(3) = -5.0$

Because $-6.1 \le -6 \le -5.0$, the Intermediate Value Theorem guarantees c, $0 \le c \le 3$, such that $r'(c) = -6$.

1.4 – Extra Practice ** No extra practice for section 1.4 **