AP Calculus BC – Unit 1 Required Practice

Name: answers only

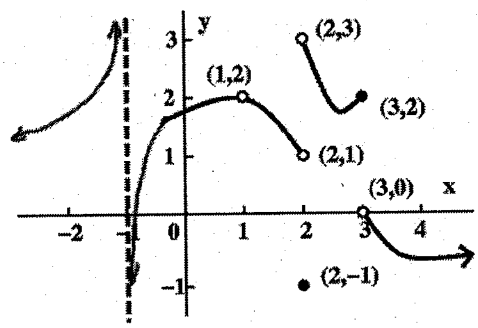
1.1 – Required Practice

#1. $\lim\limits_{x\to 4}\dfrac{x-4}{x^2-5x+4} = \dfrac{1}{3}$

$\lim\limits_{x\to 4^-}\dfrac{x-4}{x^2-5x+4} = \dfrac{1}{3}$ $\qquad$ $\lim\limits_{x\to 4^+}\dfrac{x-4}{x^2-5x+4} = \dfrac{1}{3}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
f(x)	from calculator			?	from	calculator	

#2.



$\lim\limits_{x\to 2^-} f(x) = 1$ $\qquad$ $\lim\limits_{x\to 2} f(x) =$ DNE

$\lim\limits_{x\to 2^+} f(x) = 3$

$f(2) = -1$

$\lim\limits_{x\to -1^+} f(x) = -\infty$ $\qquad$ $\lim\limits_{x\to -1} f(x) =$ DNE

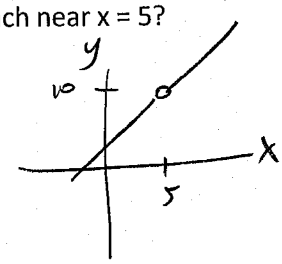
$\lim\limits_{x\to -1^-} f(x) = \infty$

#3. $\lim\limits_{x\to 3}\dfrac{x^2-25}{x-5} = 8$

#4. $\lim\limits_{x\to 5}\dfrac{x^2-25}{x-5} = 10$

Sketch the graph of this function.

What y is approach near x = 5?



#5. $\lim\limits_{x\to 0}\dfrac{2x}{x^2+4x} = \dfrac{1}{2}$

#6. $f(x) = \begin{cases} 3x-1 & x<1 \\ 2 & x=1 \\ 3x & x>1 \end{cases}$

$\lim\limits_{x\to 1} f(x) =$ DNE

#7. $\lim\limits_{x\to 1}\dfrac{x-1}{x^3-1} = \dfrac{1}{3}$

Complete the table. Use the result to estimate the limit. Use your calculator to graph the function to confirm your results.

#9. $\lim\limits_{x \to 0} \frac{\sin(x)}{x}$ $=1$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	from calculator.		1	?	from	calculator	

Evaluate the limit using any method. Graph in a calculator to verify your result.

#10. $\lim\limits_{x \to 1} \frac{x-2}{x^2+x-6}$

$= \frac{1}{4}$

#11. $\lim\limits_{x \to 1} \frac{x^4-1}{x^6-1}$

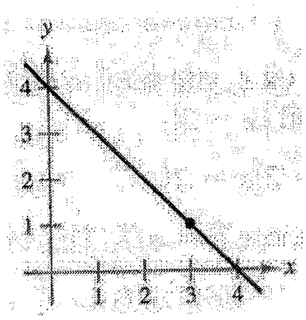
$= \frac{2}{3}$

#12. $\lim\limits_{x \to 0} \frac{\sin(2x)}{x}$

$= 2$

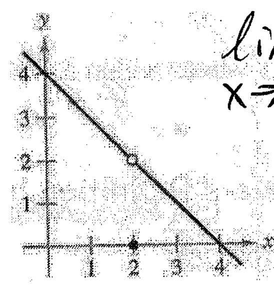
Use the graph to find the limit (if it exists). If the limit does not exist, explain why.

#13. $\lim\limits_{x \to 3} (4 - x)$ $= 1$

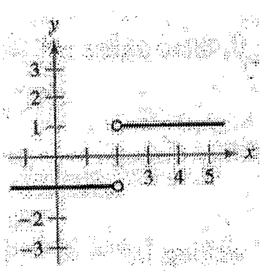


#14. $\lim\limits_{x \to 2} f(x)$ where $f(x) = \begin{cases} 4 - x & x \neq 2 \\ 0 & x = 2 \end{cases}$

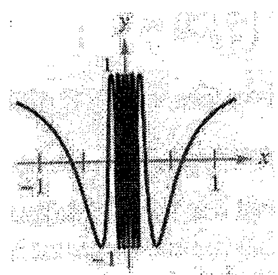
$\lim\limits_{x \to 2} f(x) = 2$



#15. $\lim\limits_{x \to 2} \frac{|x-2|}{x+3}$ DNE

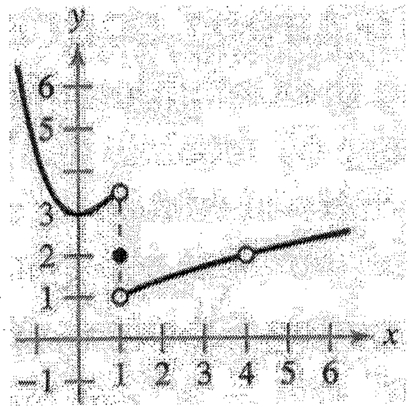


#16. $\lim\limits_{x \to 0} \cos\left(\frac{1}{x}\right)$ DNE (reason)



Use the graph of the function f to decide whether the value of the quantity exists. If it does, find it. If not, explain why.

#17.

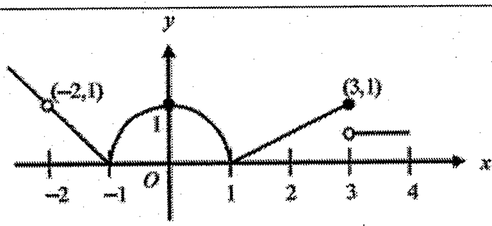


a. $f(1)$ = 2

b. $\lim_{x\to 1} f(x)$ DNE (explanation)

c. $f(0)$ = 3

d. $\lim_{x\to 4} f(x)$ = 2



Graph of f

#18. The graph of a function f is shown above. For which of the following values of c does $\lim_{x\to c} f(x) = 1$?

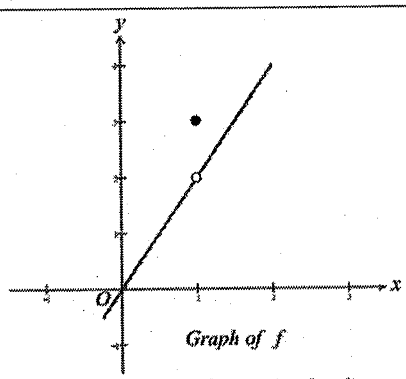
(A) 0 only

(B) 0 and 3 only

(C) −2 and 0 only

(D) −2 and 3 only

(E) −2, 0, and 3



Graph of f

#19. The graph of the function f is shown in the figure above. The value of $\lim_{x\to 1} \sin(f(x))$ is

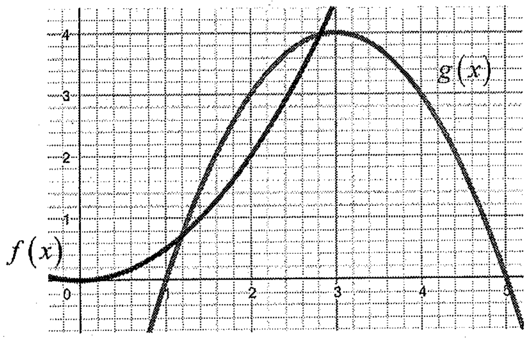
(A) 0.909
(B) 0.841
(C) 0.141
(D) −0.416
(E) nonexistent

1.2 – Required Practice

#1. Evaluate the limit

$$\lim_{x\to 2}\frac{2f(x)+1}{4-g(x)}$$

$= \frac{7}{2}$



#2. $\lim_{x\to 9}\frac{x^2-81}{\sqrt{x}-3}$ $=(9+9)(\sqrt{9}+3)=108$

#3. $\lim_{x\to 0}\frac{\cos(x)\tan(x)}{x} = 1$

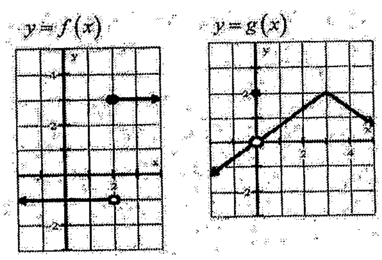
#4. Use the Squeeze Theorem to show that $\lim_{x\to 0} x^2\cos 20\pi x = 0$.

Illustrate by graphing the functions

$f(x)=-x^2$, $g(x)=x^2\cos 20\pi x$, and $h(x)=x^2$ (a graph and explanation)

on the same screen.

#5. Evaluate $\lim_{x\to 3} f(g(x))$



$=-1$

hint: think about what side $g(x)$ is approaching its value

Find the limit. Use a graphing calculator to verify.

#6. $\lim_{x \to -3} x^2 + 3x$ $= 0$

#7. $\lim_{x \to 0} \sec(2x)$ $= 1$

#8. $\lim_{x \to 1} \ln(3x) + e^x$ $\Rightarrow \ln(3) + e$

#9. $\lim_{x \to 0} \frac{x}{x^2 - x}$ $= -1$

#10. $\lim_{x \to 4} \frac{x^2 - 5x + 4}{x^2 - 16}$ $= \frac{3}{8}$

#11. $\lim_{x \to 4} \frac{\sqrt{x+5} - 3}{x - 4}$ $= \frac{1}{6}$

#12. $\lim_{x \to 0} \frac{\sin(x)}{5x}$ $= \frac{1}{5}$

#13. $\lim_{x \to 0} \frac{(\sin(x))(1 - \cos(x))}{x^2}$ $= 0$

#14. $\lim_{x \to 0} \frac{\sin^2(x)}{x}$ $= 0$

#15. $\lim_{x \to 0} \frac{1 - e^{-x}}{e^x - 1}$ $= 1$ (can't use L'Hopital's yet, so use a graph or table)

Find the limit analytically, then verify by calculator graph.

#16. $\lim_{x \to 0} \frac{\sin(2x)}{\sin(3x)}$ $= \frac{2}{3}$

Find $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$. Compare the result to the original function...notice anything?

#17. $f(x) = 3x - 2$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = 3$

(which is the derivative of $f(x)$)

#18. Let g and h be the functions defined by $g(x) = \sin\left(\frac{\pi}{2}x\right) + 4$ and $h(x) = -\frac{1}{4}x^3 + \frac{3}{4}x + \frac{9}{2}$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for $-1 < x < 2$, what is $\lim_{x \to 1} f(x)$?

(A) 4

(B) $\frac{9}{2}$

(C) 5

(D) The limit cannot be determined from the information given.

1.3 – Required Practice

#1. Is g(x) continuous at x=2? Is g(x) continuous at x=3?

$$g(x)=\begin{cases} x^2 & x<2 \\ -3 & x=2 \\ 3x & x>2 \end{cases}$$

g(x) is discontinuous at x=2
(a jump discontinuity)

g(x) is continuous at x=3

#2. Find the constant c that makes g continuous on $(-\infty, \infty)$

$$g(x)=\begin{cases} x^2-c^2 & \text{if } x<4 \\ cx+20 & \text{if } x\geq 4 \end{cases}$$

c = -2

#3. Use the Intermediate Value Theorem to show that the following polynomial has a zero in the interval [0,1].

$f(x)=x^3+2x-1$

(specific format wording required)

#4. The height of an object changes with time as described by the function
$h(t)=2t-3t^2+10$
Can you guarantee that there is a time value, t in the interval [0,2] at which the height of the object is 4? Explain.

yes, (Intermediate Value Theorem wording)

#5. Find the limit and any asymptotes: $\lim_{x\to 5}\frac{x+2}{x-5}$

vertical asymptote : x=5
horizontal asymptote : y=1 (both sides)

#6. Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

Show that f is continuous at $x = 0$.

$f(x)$ is continuous at $x=0$

(show the 3 steps)

#7. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5 - x & \text{for } 3 < x \leq 5. \end{cases}$$

Is f continuous at $x = 3$? Explain why or why not.

$f(x)$ is continuous at $x=3$

(show the 3 steps)

#8.
$$f(x)=\begin{cases}\frac{2x^2+5x-3}{x^2+4x+3} & \text{for } x<-3\\ kx+\frac{1}{2} & \text{for } -3\le x\le 0\\ \frac{2^x}{3^x-1} & \text{for } x>0\end{cases}$$

Let f be the function defined above, where k is a constant.

(a) For what value of k, if any, is f continuous at $x=-3$? Justify your answer.

$$\lim_{x\to -3^-}\frac{2x^2+5x-3}{x^2+4x+3} \underset{=}{\text{must}} \lim_{x\to 3^+} kx+\frac{1}{2}$$

x	$f(x)$
-3.1	3.4286
-3.01	3.4925
-3.001	3.4995
$\downarrow$	$\downarrow$
-3	$3.5 = 7/2$

$$\frac{7}{2} = k(-3)+\frac{1}{2}$$
$$7=-6k+1$$
$$-6k=6$$
$$k=-1$$

(b) What type of discontinuity does f have at $x=0$? Give a reason for your answer.

$$\lim_{x\to 0^-} kx+\frac{1}{2} \qquad \lim_{x\to 0^+}\frac{2^x}{3^x-1}$$

$$\frac{1}{2} \qquad \infty$$

x	$\frac{2^x}{3^x-1}$
0.1	9.2296
0.01	91.154
0.001	910.370

$f(x)$ has an infinite discontinuity at $x=0$

#9.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

For $0<t<60$, must there be a time t when $v(t)=-5$? Justify your answer.

yes (conditions and conclusion wording for the Intermediate Value Theorem)

1.4 – Required Practice

#1. $\lim_{r\to\infty} \frac{r^4 - r^2 + 1}{r^5 + r^3 - r}$

$= 0$

#2. $\lim_{x\to-\infty} \frac{3x^3 - x}{x^2 + 2x + 1}$

$\to -\infty$
(or DNE)

#3. $\lim_{t\to-\infty} \frac{6t^2 + 5t}{(1-t)(2t-3)}$

$= -3$

#4. $\lim_{x\to\infty} e^{-x^2}$

$= 0$

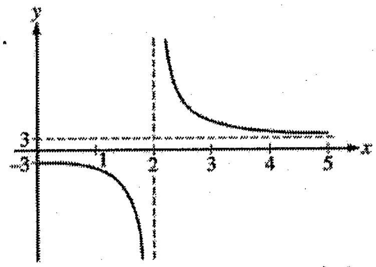
#5. Find the horizontal and vertical asymptotes of the curve. Check by graphing.

$$y = \frac{x^2 + 4}{x^2 - 1}$$

vertical asymptotes: $x = 1$, $x = -1$

horizontal asymptote: $y = 1$
(both sides)

#6.



The function f is given by $f(x) = \frac{ax^2+12}{x^2+b}$.

The figure above shows a portion of the graph of f.

What are the values of the constants a and b?

$a = 3$

$b = -4$

#7. The continuous function f is positive and has domain $x > 0$. If the asymptotes of the graph of f are $x = 0$ and $y = 2$, which of the following statements must be true?

(A) $\lim_{x \to 0^+} f(x) = \infty$ and $\lim_{x \to 2} f(x) = \infty$

(B) $\lim_{x \to 0^+} f(x) = 2$ and $\lim_{x \to \infty} f(x) = 0$

$\boxed{\text{(C)}}$ $\lim_{x \to 0^+} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 2$

(D) $\lim_{x \to 2} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 2$

#8. The graph of which of the following functions has exactly one horizontal asymptote and no vertical asymptotes?

$\boxed{\text{(A)}}$ $y = \frac{1}{x^2+1}$

(B) $y = \frac{1}{x^3+1}$

(C) $y = \frac{1}{e^x-1}$

(D) $y = \frac{1}{e^x+1}$

#9. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x \to 2} f(x) = \infty$

$\boxed{\text{(E)}}$ $\lim_{x \to \infty} f(x) = 2$