AP Calculus BC – Unit 2 Required Practice

Name: answers

2.1 – Required Practice

#1. Find $f'(x)$ for $f(x) = x^2 - x$ ...

...using $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

Find $f'(x)$ for $f(x) = x^2 - x$

...using $\lim\limits_{x \to 2} \dfrac{f(x) - f(2)}{x - 2}$.

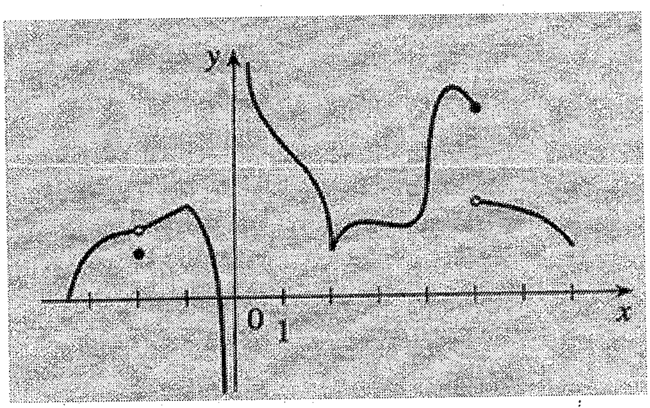
Then plug in to find $f'(2)$ and $f(2)$ and discuss: what does $f'(2)$ and $f(2)$ tell you about $f(x)$?

Finally, sketch $f(x)$ using your calculator, find the equation of a tangent line to $f(x)$ at x = 2 and add it to your sketch.

(work for limit definition of derivative)

tangent line: $(y-2) = 3(x-2)$

#2. Find the x-values for which the derivative does not exist and state why it does not exist:



$x = -2$

$x = 0$

$x = 2$

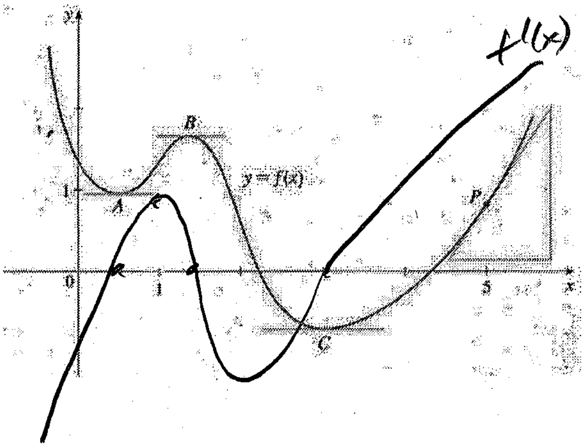
$x = 4$

$x = 5$

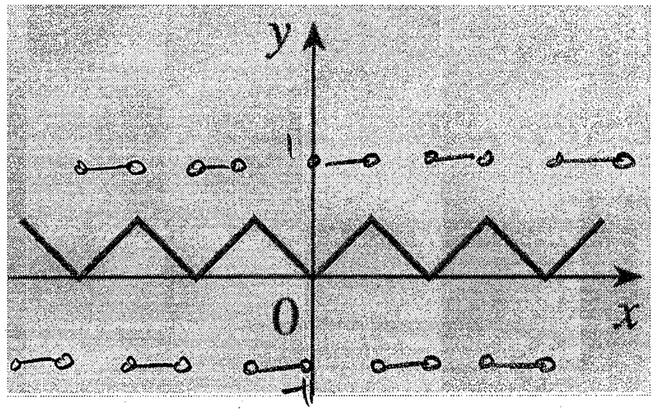
(give reasons)

Given the graph of $f(x)$, sketch the graph of $f'(x)$:

#3.



#4.



#5. $\lim_{h\to 0}\frac{\sqrt{h+1}-1}{h}$

$f(x)=\sqrt{x}$

at $x=1$

#6. $\lim_{x\to 3\pi}\frac{\cos(x)+1}{x-3\pi}$

$f(x)=\cos x$

at $x=c=3\pi$

#7. Find $f'(x)$ for $f(x)=\frac{1}{\sqrt{x}}$, then find $f(1)$ and $f'(1)$

$f'(x)=\frac{-1}{2x\sqrt{x}}$

$f'(1)=-\frac{1}{2}$

$f(1)=1$

Use the limit definition of the derivative to find the derivative:

#8. $f(x) = x^2 + x - 3$

$f'(x) = 2x + 1$

#9. $v(t) = t^2 - 5$

$v'(t) = 2t$

Use the limit definition of the derivative to find the derivative of f(x), then find f'(x) at the given x-value and write the equation of a tangent line to the function at this point of tangency:

#10. $f(x) = x^3$ at $x = 2$

$(y - 8) = 12(x - 2)$

#11. $f(x) = \sqrt{x}$ at $x = 1$

$(y - 1) = \frac{1}{2}(x - 1)$

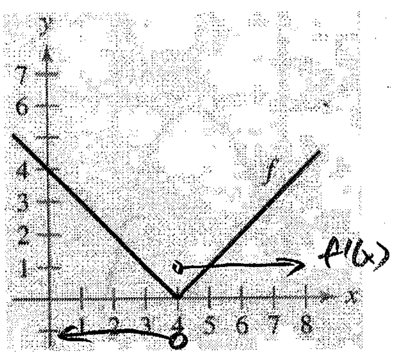
Use the limit definition of the derivative to find the derivative of f(x), then find f'(x) at the given x-value and write the equation of a tangent line to the function at this point of tangency:

#12. $f(x) = x + \dfrac{4}{x}$ at $x = -4$

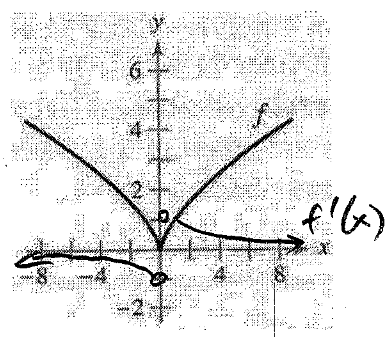
$(y+5) = \frac{3}{4}(x+4)$

Given the graph of f(x), sketch the graph of f'(x) on the same axes:

#13.



#14.



#15. The given limit gives the value of the derivative of a function at a particular x-value. Identify the function and x-value:

$$\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3}+h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$$

$f(x) = \sin x$ at $x = \pi/3$

2.2 – Required Practice

Find the derivative:

#1. $f(x) = x^5$

$f'(x) = 5x^4$

#2. $f(x) = x^{\left(\frac{2}{3}\right)}$

$f'(x) = \frac{2}{3}x^{-1/3}$

#3. $g(t) = \frac{2}{t^5}$

$g'(t) = \frac{-10}{t^6}$

#4. $f(x) = 3x^5 + 4e^x + 7$

$f'(x) = 15x^4 + 4e^x$

#5. $g(x) = \frac{x^4 - 3x^2}{2x}$

$g'(x) = \frac{3}{2}x^2 - \frac{3}{2}$

#6. For what values of a and b is the line $2x + y = b$ tangent to the parabola $y = ax^2$ when $x = 2$?

$a = -1/2$
$b = 2$

#7. Find a cubic function $y = ax^3 + bx^2 + cx + d$ whose graph has horizontal tangents at the points $(-2, 6)$ and $(2, 0)$.

$y = \frac{3}{16}x^3 - \frac{9}{4}x + 3$

Find the derivative:

#8. $f(x) = 8x + \frac{3}{x^2}$

$f'(x) = 8 - \frac{6}{x3}$

#9. $f(x) = \frac{4x^3 + 3x^2}{x}$

$f'(x) = 8x + 3$

#10. $g(x) = x^{\frac{2}{3}} - x^{\frac{1}{3}} + 4$

$g'(x) = \frac{2}{3}x^{-1/3} - \frac{1}{3}x^{-2/3}$

#11. $f(x) = \sqrt{x} - 6\sqrt[3]{x}$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{\sqrt[3]{x^2}} = \frac{1}{2}x^{-1/2} - 2x^{-2/3}$

#12. $g(x) = \sin x + e^x$

$g'(x) = \cos x + e^x$

#13. $f(x) = 4^x + \cos x$

$f'(x) = 4^x(\ln 4) - \sin x$

#14. $v(t) = \frac{4}{t^2} - \frac{3t^2 - t}{t^3}$

$v'(t) = -8t^{-3} + 3t^{-2} - 2t^{-3}$

$= \frac{-8}{t^3} + \frac{3}{t^2} - \frac{2}{t^3}$

#15. $r(t) = \frac{2t^5}{t^3} + \frac{t^2 - 6t}{t^4}$

$r'(t) = 4t - 2t^{-3} + 18t^{-4}$

$= 4t - \frac{2}{t^3} + \frac{18}{t^4}$

2.3 – Required Practice

Find the derivative:

#1. $f(x) = x^3 e^x$

$f'(x) = x^3 e^x + e^x(3x^2)$

#2. $f(x) = 2x^3$

$f'(x) = 6x^2$

#3. $f(x) = 2x^3 e^x$

$f'(x) = 2x^3 e^x + e^x(6x^2)$

#4. $f(x) = \dfrac{2x^5 - x^3}{x^2 + 3x}$

$f'(x) = \dfrac{(x^2+3x)(10x^4-3x^2)-(2x^5-x^3)(2x+3)}{(x^2+3x)^2}$

#5. $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$

$y' = \dfrac{(\sqrt{x}+1)(\frac{1}{2}x^{-1/2}) - (\sqrt{x}-1)(\frac{1}{2}x^{-1/2})}{(\sqrt{x}+1)^2}$

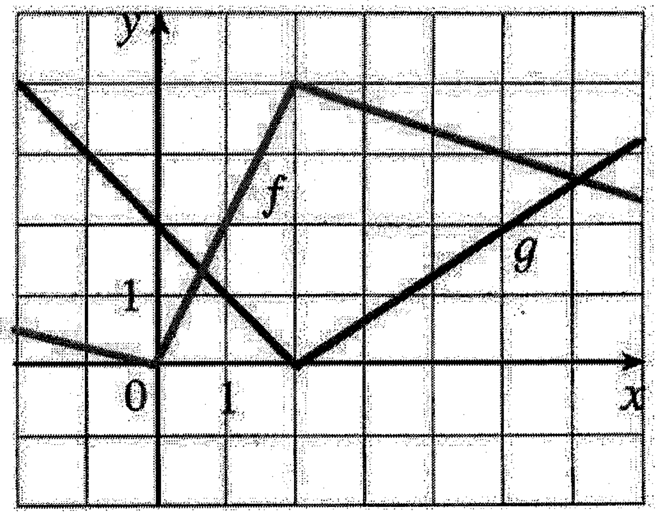
#6. $y = \sqrt{x}\left(x^2 - 2x\right)$

$y' = \sqrt{x}\,(2x-2) + (x^2-2x)\,\frac{1}{2}x^{-1/2}$

#7. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

$y' = \dfrac{\sqrt{x}\,(2x+4) + (x^2+4x+3)\,\frac{1}{2}x^{-1/2}}{(\sqrt{x})^2}$

If f and g are the functions whose graphs are show, let $u(x)=f(x)g(x)$ and $v(x)=f(x)/g(x)$.



#8. Find $u'(1)$ $=0$

#9. Find $v'(5)$ $= \frac{(2)(-\frac{1}{3})-(3)(\frac{2}{3})}{(2)^2} = \frac{-1}{3}$

Find the derivative:

#10. $g(x)=(2x-3)^2(1-5x)^3$

$g'(x) = (2x-3)^2[3(1-5x)^2(-5)]$
$+ (1-5x)^3[2(2x-3)(2)]$

#11. $h(x)=\frac{\sqrt{x}}{x^3+1}$

$h'(x) = \frac{(x^3+1)(\frac{1}{2}x^{-1/2}) - x^{1/2}(3x^2)}{(x^3+1)^2}$

#12. Find $f'(1)$ for $f(x)=\frac{x^2-4}{x-3}$

$f'(1) = \frac{(1-3)(2(1)) - (1^2-4)(1)}{(1-3)^2}$

$= -\frac{1}{4}$

#13. Find $f'\left(\frac{\pi}{2}\right)$ for $f(x)=e^x \sin x$

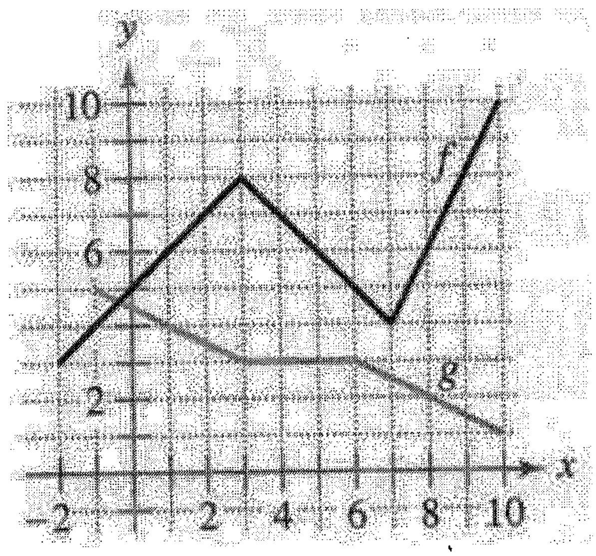
$f'(\frac{\pi}{2}) = e^{\pi/2}(0) + (1)e^{\pi/2} = e^{\pi/2}$

#13. Find (and fully simplify) $f'(x)$ for $f(x) = \dfrac{4-3x-x^2}{x^2-1}$

$f'(x) = \dfrac{3}{(x-1)^2}$

Let $p(x) = f(x)g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$

If the graphs of $f(x)$ and $g(x)$ are:



#14. Find $p'(1)$ $= (6)(-\frac{1}{2}) + (4)(1)$

$= 1$

#15. Find $q'(4)$ $= \dfrac{(3)(-1)-(7)(0)}{(3)^2}$

$= -\frac{1}{3}$

2.4 – Required Practice

Find the derivative:

#1. $y = e^{(x^2+3)}$

$y' = e^{(x^2+3)}(2x)$

#2. $y = \ln\left(\sin\left(e^{x^5}\right)\right)$

$y' = \frac{1}{\sin(e^{x^5})}\cos(e^{x^5})e^{x^5}(5x^4)$

#3. $y = \sin^3\left(x^3 + 7x\right)$

$y' = 3(\sin(x^3+7x))^2\cos(x^3+7x)(3x^2+7)$

#4. $y = \left(x^3 + 4x\right)^5$

$y' = 5(x^3+4x)^4(3x^2+4)$

#5. $y = \sin\left(3x^2 + x\right)$

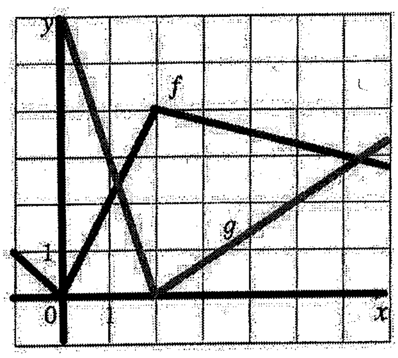
$y' = \cos(3x^2+x)(6x+1)$

If f and g are the functions whose graphs are shown, let $u(x) = f(g(x))$, $v(x) = g(f(x))$, and $w(x) = g(g(x))$.
Find each derivative, if it exists.
If it does not exist, explain why.

#6. $u'(1) = (-\frac{1}{4})(-\frac{1}{3}) = \frac{1}{12}$

#7. $v'(1)$ DNE (reason?)

#8. $w'(1) = (\frac{2}{3})(-\frac{1}{3}) = \frac{-2}{9}$



Find the derivative:

#9. $y=(2x-7)^3$

$y'=3(2x-7)^2(2)$

#10. $g(x)=\sqrt{4-3x^2}$

$g'(x)=\frac{1}{2}(4-3x^2)^{-1/2}(-6x)$

$=\frac{-3x}{\sqrt{4-3x^2}}$

#11. $y=-\frac{3}{(t-2)^4}$

$y'=12(t-2)^{-5}(1)$

$=\frac{12}{(t-2)^5}$

#12. $g(x)=\left(\frac{x+5}{x^2+2}\right)^2$

$g'(x)=2\left(\frac{x+5}{x^2+2}\right)\left[\frac{(x^2+2)(1)-(x+5)(2x)}{(x^2+2)^2}\right]$

#13. $y=\frac{2}{e^x+e^{-x}}$

$y'=-2(e^x+e^{-x})^{-2}(e^x+e^{-x}(-1))$

$=\frac{-2(e^x-e^{-x})}{(e^x+e^{-x})^2}$

#14. $f(x)=\ln\left(\frac{x}{x^2+1}\right)$

$f'(x)=\frac{1}{\left(\frac{x}{x^2+1}\right)}\left[\frac{(x^2+1)(1)-x(2x)}{(x^2+1)^2}\right]$

$=\frac{-x^2+1}{x(x^2+1)}$

#15. $y=\ln|\sin(x)|$

$y'=\frac{1}{\sin x}\cos x=\cot x$

#16. Write an equation for the tangent line to

$y=\sqrt{x^2+8x}$ at $x=1$

$(y-3)=\frac{5}{3}(x-1)$

#17. Write an equation for the tangent line to

$f(x) = \tan^2(x)$ at $x = \frac{\pi}{4}$

$(y-1) = 4(x - \frac{\pi}{4})$

#18. Find $f''(x)$ for $f(x) = 5(2-7x)^4$

$f''(x) = -420(2-7x)^2(-7)$

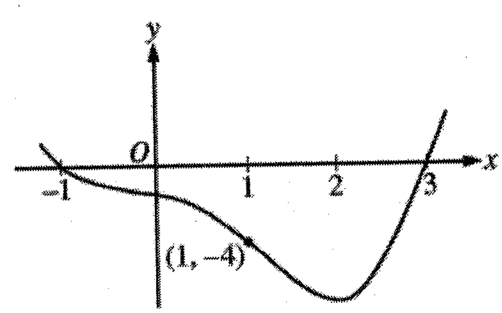
$= -2940(2-7x)^2$

#19. Find $f''(x)$ for $f(x) = \sec^2(\pi x)$

$f''(x) = 2\pi\,(\sec(\pi x))^2 \sec^2(\pi x)(\pi) + \tan(\pi x)(4\pi(\sec(\pi x))\sec(\pi x)\tan(\pi x))\,\pi$

#20. Let *f* be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$.

The graph of *f′(x)*, the derivative is shown. Define $g(x) = e^{f(x)}$.
Write an equation for the line tangent to the graph of *g* ant x = 1.



Graph of f'

$(y-2) = -4e^2(x-1)$

2.5 – Required Practice

#1. Find the slope of the line tangent to this curve $x^3 + xy + y^3 = 5$ at the point (-1, 2).

$$m = \frac{dy}{dx} = \frac{-3x^2 - y}{x + 3y^2}$$

#2. Find $\frac{dy}{dx}$ if $x^2y + y^2x = -2$

$$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$$

#3. Find $\frac{dy}{dx}$ if $y = \frac{\left(x^2+3x\right)^4\left(5x^3\right)}{\left(2x^4-4x\right)^2}$

$$\frac{dy}{dx} = \left[\frac{4(2x+3)}{x^2+3x} + \frac{15x^2}{5x^3} - \frac{2(8x^3-4)}{2x^4-4x}\right]\frac{(x^2+3x)^4(5x^3)}{(2x^4-4x)^2}$$

#4. Find $\frac{dy}{dx}$ if $y = x^{\left(5x^3+2x\right)}$

$$\frac{dy}{dx} = \left[\frac{5x^3+2x}{x} + \ln(x)(15x^2+2)\right]x^{(5x^3+2x)}$$

#5. Find $\frac{dy}{dx}$ if $y = x^{\left(\frac{2}{x}\right)}$

$$\frac{dy}{dx} = \left[\frac{2}{x^2} - \frac{2\ln x}{x^2}\right]x^{\left(\frac{2}{x}\right)}$$

#6. Find $\frac{dy}{dx}$ if $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 16$

$$\frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}} = \frac{-\sqrt{y}}{\sqrt{x}}$$

#7. Find $\frac{dy}{dx}$ if $x^3 - xy + y^2 = 7$

$$\frac{dy}{dx} = \frac{y-3x^2}{2y-x}$$

#8. Find $\frac{dy}{dx}$ if $e^{xy} + x^2 - y^2 = 10$

$$\frac{dy}{dx} = \frac{-2x-ye^{xy}}{xe^{xy}-2y}$$

#9. Find $\frac{dy}{dx}$ if $y^2 = \ln x$

$$\frac{dy}{dx} = \frac{1}{2xy}$$

#10. Find $\frac{d^2y}{dx^2}$ $(\textit{the 2nd derivative})$ if $x^2 + y^2 = 4$

$$\frac{d^2y}{dx^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

#11. Find $\frac{dy}{dx}$ if $y=(x-2)^{x+1}$

$\frac{dy}{dx} = \left[\frac{x+1}{x-2} + \ln(x-2) \right] (x-2)^{x+1}$

#12. Find $\frac{dy}{dx}$ if $y=x^{\ln(x)}$

$\frac{dy}{dx} = \left(\frac{2\ln x}{x} \right) x^{\ln(x)}$

2.6 – Required Practice

#1. If $f(x) = x^3 + x$ and $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

$g'(2) = \frac{1}{4}$

#2. Find $f'(x)$ and $f''(x)$ if $f(x) = \dfrac{\sin x}{1+\cos x}$

$f'(x) = \dfrac{2}{(1+\cos x)^2}$

$f''(x) = \dfrac{4\sin x}{(1+\cos x)^3}$

#3. Find the equation of the tangent line to the curve $y = x + \cos x$ *at* $(0,1)$

$(y-1) = 1(x-0)$

#4. Find y' if $y = \ln(t^2+4) - \frac{1}{2}\arctan\left(\frac{t}{2}\right)$

$y' = \dfrac{1}{t^2+4}(2t) - \dfrac{1}{2}\,\dfrac{1}{1+\left(\frac{t}{2}\right)^2}\left(\frac{1}{2}\right)$

Find the derivative:

#5. $f(x) = \arcsin(x+1)$

$f'(x) = \dfrac{1}{\sqrt{1-(x+1)^2}}(1)$

#6. $g(x) = \dfrac{\arcsin(3x)}{x}$

$g'(x) = \dfrac{x\dfrac{1}{\sqrt{1-(3x)^2}}(3) - \arcsin(3x)(1)}{x^2}$

Find the derivative:

#7. $y = 2x\arccos(x) - 2\sqrt{1-x^2}$

$y' = 2\arccos(x)$

Find an equation of the tangent line to the graph of the function at the x-value:

#8. $y = 2\arcsin(x)$ $\quad x = \frac{1}{2}$

$(y - \frac{\pi}{3}) = \frac{4}{\sqrt{3}}(x - \frac{1}{2})$

Unit 2 Test Review

Find the derivative:

#1. $f(x) = 3x^5 + 5e^{4x} + \sin(x^3 + 2x) + \ln(x^4)$

$$\boxed{f'(x) = 15x^4 + 5e^{4x}(4) + \cos(x^3+2x)(3x^2+2) + \frac{1}{x^4}(4x^3)}$$

labels required by AP

#2. $f(x) = (3x+4)^2 \tan(x^3)$ (product rule)

$$\boxed{f'(x) = (3x+4)^2 \sec^2(x^3)(3x^2) + \tan(x^3)(2(3x+4)(3))}$$

don't forget to label what you are doing

#3. $f(x) = \dfrac{\ln(x^5)}{\sin(e^{2x})}$ (quotient rule)

$$\boxed{f'(x) = \frac{\sin(e^{2x})\frac{1}{x^5}(5x^4) - \ln(x^5)\cos(e^{2x})e^{2x}(2)}{[\sin(e^{2x})]^2}}$$

#4. $f(t) = \cos(\ln(e^{3t}))$

$$\boxed{f'(t) = -\sin(\ln(e^{3t}))\frac{1}{e^{3t}}e^{3t}(3)}$$

#5. $y = 2^{\sin(\pi x)}$

$$\boxed{y' = 2^{\sin(\pi x)}\ln(2)\cos(\pi x)\pi}$$

#6. $y = x^{(x^3+5x+7)}$ (requires logarithmic diff)

$\ln y = \ln(x^{x^3+5x+7})$

$\ln y = (x^3+5x+7)\ln(x)$ (product rule)

$\frac{1}{y}\frac{dy}{dx} = (x^3+5x+7)\frac{1}{x} + \ln(x)(3x^2+5)$

$\frac{dy}{dx} = \left[\frac{x^3+5x+7}{x} + \ln(x)(3x^2+5)\right]y$ ← must substitute this

$$\boxed{\frac{dy}{dx} = \left[\frac{x^3+5x+7}{x} + \ln(x)(3x^2+5)\right]x^{(x^3+5x+7)}}$$

#7. $y=\sqrt[3]{x^2+2}\left(x^2+1\right)$ (product rule)

$y=(x^2+2)^{1/3}(x^2+1)$

$$\boxed{y'=(x^2+2)^{1/3}(2x)+(x^2+1)\tfrac{1}{3}(x^2+2)^{-2/3}(2x)}$$

#8. $\sqrt{x}+x\sqrt{y}=1$ (implicit diff)

$x^{1/2}+xy^{1/2}=1$ (product rule req'd)

$\frac{d}{dx}[x^{1/2}]+x\frac{d}{dx}[y^{1/2}]+y^{1/2}\frac{d}{dx}[x]=\frac{d}{dx}[1]$

$\frac{1}{2}x^{-1/2}+x\left(\frac{1}{2}y^{-1/2}\frac{dy}{dx}\right)+y^{1/2}(1)=0$ must solve for $\frac{dy}{dx}$ (but no need to simplify)

$\frac{1}{2}xy^{-1/2}\frac{dy}{dx}=-\frac{1}{2}x^{-1/2}-y^{1/2}$

$$\boxed{\frac{dy}{dx}=\frac{-\frac{1}{2}x^{-1/2}-y^{1/2}}{\frac{1}{2}xy^{-1/2}}}$$

#9. Find y' and y'' for $x^4-y^4=16$

(implicit) $\frac{d}{dx}[x^4]-\frac{d}{dx}[y^4]=\frac{d}{dx}[16]$

$4x^3-4y^3\frac{dy}{dx}=0$

$-4y^3\frac{dy}{dx}=-4x^3$

$$\boxed{y'=\frac{dy}{dx}=\frac{-4x^3}{-4y^3}=\frac{x^3}{y^3}}$$

(helpful to simplify)

$y''=\frac{d^2y}{dx^2}=\frac{d}{dx}\left[\frac{x^3}{y^3}\right]$ (quotient rule)

$=\frac{y^3\frac{d}{dx}[x^3]-x^3\frac{d}{dx}[y^3]}{[y^3]^2}$

must substitute this

$$\boxed{y''}=\frac{y^3(3x^2)-x^3\left(3y^2\boxed{\frac{dy}{dx}}\right)}{y^6}=\boxed{\frac{y^3(3x^2)-x^3\left(3y^2\left(\frac{x^3}{y^3}\right)\right)}{y^6}}$$

#10. Find the equation of the tangent line to the curve at the given point: $y=\sin\left(\sin\left(x\right)\right)$ $(\pi,0)$ (know unit circle values)

$m=y'=\cos(\sin(x))\cdot\cos(x)\Big|_{x=\pi}=\cos(\sin(\pi))\cos(\pi)=\cos(0)\cos(\pi)=(1)(-1)=-1$

tangent line: $\boxed{(y-0)=-(x-\pi)}$

#11. If $y=e^{x^2}$, find $\frac{d^2y}{dx^2}$

$\frac{dy}{dx}=e^{(x^2)}(2x)$ (now, product rule)

$$\boxed{\frac{d^2y}{dx^2}=e^{(x^2)}(2)+2x\left(e^{(x^2)}2x\right)}$$

#12. Find an equation of the tangent line to the curve at the given point: $y^4 + xy = x^3 - x + 2$ at $(1,1)$

(find $\frac{dy}{dx}$ implicitly)

$\frac{d}{dx}[y^4] + x\frac{d}{dx}[y] + y\frac{d}{dx}[x] = \frac{d}{dx}[x^3] - \frac{d}{dx}[x] + \frac{d}{dx}[2]$

$4y^3\frac{dy}{dx} + x(1\frac{dy}{dx}) + y(1) = 3x^2 - 1 + 0$

$(4y^3 + x)\frac{dy}{dx} = 3x^2 - 1 - y$

$m = \frac{dy}{dx} = \frac{3x^2-1-y}{4y^3+x}\Big|_{(1,1)} = \frac{3(1)^2-1-(1)}{4(1)^3+(1)} = \frac{1}{5}$

tangent line:

$\boxed{(y-1) = \frac{1}{5}(x-1)}$

#13. Find $\frac{dy}{dx}$ for $e^{x-y} = 2x^2 - y^2$ (implicit)

$\frac{d}{dx}[e^{(x-y)}] = \frac{d}{dx}[2x^2] - \frac{d}{dx}[y^2]$

$e^{(x-y)}\frac{d}{dx}[x-y] = 4x - 2y\frac{dy}{dx}$

$e^{(x-y)}[\frac{d}{dx}[x] - \frac{d}{dx}[y]] = 4x - 2y\frac{dy}{dx}$

$e^{(x-y)}(1 - 1\frac{dy}{dx}) = 4x - 2y\frac{dy}{dx}$

$(e^{(x-y)} + 2y)\frac{dy}{dx} = 4x - e^{(x-y)}$

$\boxed{\frac{dy}{dx} = \frac{4x - e^{(x-y)}}{e^{(x-y)} + 2y}}$

#14. Find $\frac{dy}{dx}$ for $y - x^2y^2 = 6$ (implicit)

$\frac{d}{dx}[y] - (x^2\frac{d}{dx}[y^2] + y^2\frac{d}{dx}[x^2]) = \frac{d}{dx}[6]$

$1\frac{dy}{dx} - (x^2(2y\frac{dy}{dx}) + y^2(2x)) = 0$

$1\frac{dy}{dx} - 2x^2y\frac{dy}{dx} - 2xy^2 = 0$

$(1 - 2x^2y)\frac{dy}{dx} = 2xy^2$

$\boxed{\frac{dy}{dx} = \frac{2xy^2}{1-2x^2y}}$

#15. Find $\frac{d^2y}{dx^2}$ for $x^2 + y^2 = 6$

implicit: $\frac{d}{dx}[x^2] + \frac{d}{dx}[y^2] = \frac{d}{dx}[6]$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

(now quotient rule)

$\frac{d^2y}{dx^2} = \frac{y\frac{d}{dx}[-x] - (-x)\frac{d}{dx}[y]}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y(-1) + x\boxed{\frac{dy}{dx}}}{(y^2)^2}$ ← substitute this

$\boxed{\frac{d^2y}{dx^2} = \frac{-y + x(\frac{-x}{y})}{(y^2)^2}}$

#16. Find $\frac{dy}{dx}$ for $\operatorname{arcsec}(5x^2) = \ln(y)$ (2 methods)

(implicit)

$\frac{d}{dx}[\operatorname{arcsec}(5x^2)] = \frac{d}{dx}[\ln(y)]$

$\frac{1}{5x^2\sqrt{(5x^2)^2-1}}(10x) = \frac{1}{y}\frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{10xy}{5x^2\sqrt{(5x^2)^2-1}}}$

$\boxed{\frac{dy}{dx} = \frac{2y}{x\sqrt{25x^4-1}}}$

-or- (solve for y)

$y = e^{(\operatorname{arcsec}(5x^2))}$

$\frac{dy}{dx} = \underbrace{e^{(\operatorname{arcsec}(5x^2))}}_{\text{this is } y}\frac{1}{5x^2\sqrt{(5x^2)^2-1}}(10x)$

$\boxed{\frac{dy}{dx} = \frac{10xy}{5x^2\sqrt{(5x^2)^2-1}}}$

$\boxed{\frac{dy}{dx} = \frac{2y}{x\sqrt{25x^4-1}}}$

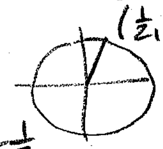
#17. *Evaluate* $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{3}+h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$

$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$ is derivative $f'(x)$

if $f(x) = \sin x$

and $x = \frac{\pi}{3}$

$(\frac{1}{2}, \frac{\sqrt{3}}{2})$



so $f(x) = \sin(x)$

$f'(x) = \cos(x)$

$f'(x) = \cos(\frac{\pi}{3}) = \frac{1}{2}$

$\therefore \boxed{\lim_{h\to 0}\frac{\sin(\frac{\pi}{3}+h) - \sin(\frac{\pi}{3})}{h} = \frac{1}{2}}$

* MEMORIZE ALL SHORTCUTS AND RULES *