AP Calculus BC – Unit 2 Required Practice **Name: ______________________________**

2.1 – Required Practice

#1. Find $f'(x)$ for $f(x) = x^2 - x$ ...

...using $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

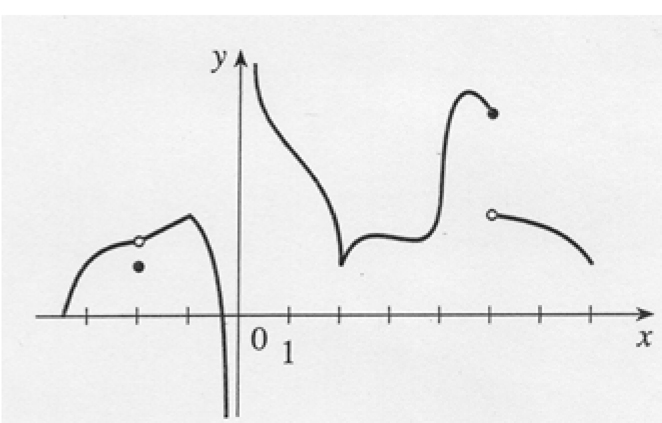
Find $f'(x)$ for $f(x) = x^2 - x$

...using $\lim_{x \to 2} \frac{f(x) - f(2)}{x-2}$.

Then plug in to find $f'(2)$ and $f(2)$ and discuss: what does $f'(2)$ and $f(2)$ tell you about $f(x)$?

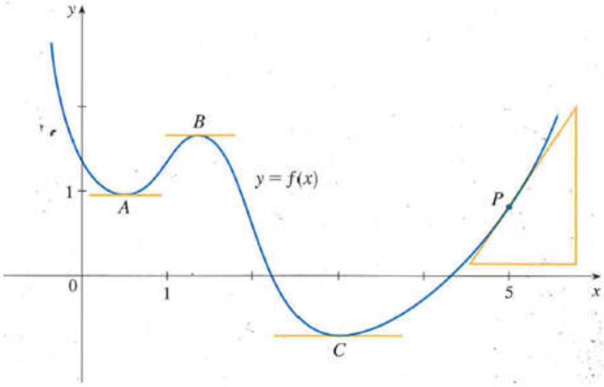
Finally, sketch $f(x)$ using your calculator, find the equation of a tangent line to $f(x)$ at x = 2 and add it to your sketch.

#2. Find the x-values for which the derivative does not exist and state why it does not exist:

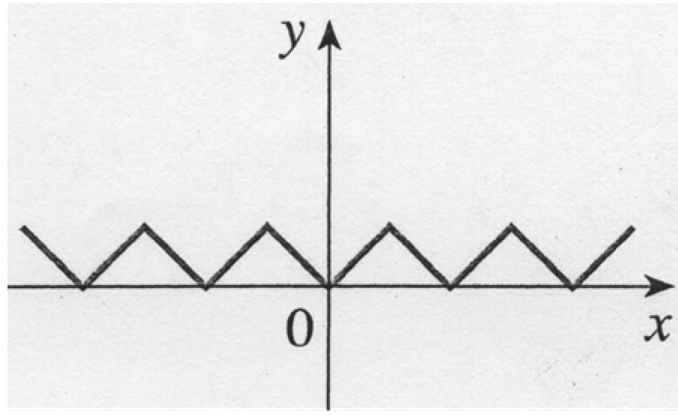


Given the graph of $f(x)$, sketch the graph of $f'(x)$:

#3.



#4.



#5. $\lim_{h \to 0} \frac{\sqrt{h+1}-1}{h}$

#6. $\lim_{x \to 3\pi} \frac{\cos(x)+1}{x-3\pi}$

#7. Find $f'(x)$ for $f(x) = \frac{1}{\sqrt{x}}$, then find $f(1)$ and $f'(1)$

Use the limit definition of the derivative to find the derivative:

#8. $f(x) = x^2 + x - 3$

#9. $v(t) = t^2 - 5$

Use the limit definition of the derivative to find the derivative of f(x), then find f'(x) at the given x-value and write the equation of a tangent line to the function at this point of tangency:

#10. $f(x) = x^3$ at $x = 2$

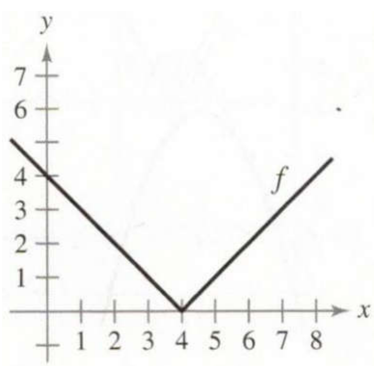
#11. $f(x) = \sqrt{x}$ at $x = 1$

Use the limit definition of the derivative to find the derivative of f(x), then find f'(x) at the given x-value and write the equation of a tangent line to the function at this point of tangency:

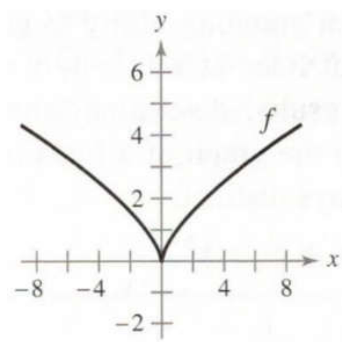
#12. $f(x) = x + \frac{4}{x}$ at $x = -4$

Given the graph of f(x), sketch the graph of f'(x) on the same axes:

#13.



#14.



#15. The given limit gives the value of the derivative of a function at a particular

x-value. Identify the function and x-value:

$$\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{3}+h\right)-\sin\left(\frac{\pi}{3}\right)}{h}$$

2.2 – Required Practice

Find the derivative:

#1. $f(x) = x^5$

#2. $f(x) = x^{\left(\frac{2}{3}\right)}$

#3. $g(t) = \frac{2}{t^5}$

#4. $f(x) = 3x^5 + 4e^x + 7$

#5. $g(x) = \frac{x^4 - 3x^2}{2x}$

#6. For what values of a and b is the line $2x + y = b$ tangent to the parabola $y = ax^2$ when $x = 2$?

#7. Find a cubic function $y = ax^3 + bx^2 + cx + d$ whose graph has horizontal tangents at the points $(-2, 6)$ and $(2, 0)$.

Find the derivative:

#8. $f\left(x\right)=8x+\dfrac{3}{x^2}$

#9. $f\left(x\right)=\dfrac{4x^3+3x^2}{x}$

#10. $g\left(x\right)=x^{\frac{2}{3}}-x^{\frac{1}{3}}+4$

#11. $f\left(x\right)=\sqrt{x}-6\sqrt[3]{x}$

#12. $g\left(x\right)=\sin x+e^x$

#13. $f\left(x\right)=4^x+\cos x$

#14. $v\left(t\right)=\dfrac{4}{t^2}-\dfrac{3t^2-t}{t^3}$

#15. $r\left(t\right)=\dfrac{2t^5}{t^3}+\dfrac{t^2-6t}{t^4}$

2.3 – Required Practice

Find the derivative:

#1. $f\left(x\right)=x^3e^x$

#2. $f\left(x\right)=2x^3$

#3. $f\left(x\right)=2x^3e^x$

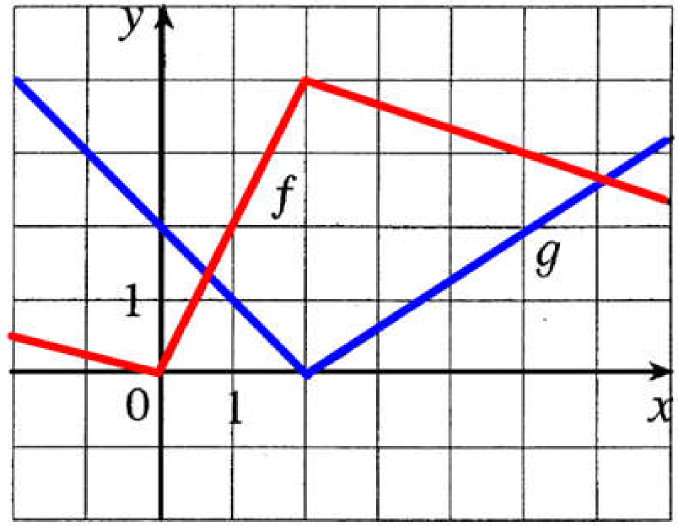
#4. $f\left(x\right)=\dfrac{2x^5-x^3}{x^2+3x}$

#5. $y=\dfrac{\sqrt{x}-1}{\sqrt{x}+1}$

#6. $y=\sqrt{x}\left(x^2-2x\right)$

#7. $y=\dfrac{x^2+4x+3}{\sqrt{x}}$

If f and g are the functions whose graphs are show, let $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$.



#8. Find $u'(1)$

#9. Find $v'(5)$

Find the derivative:

#10. $g(x) = (2x-3)^2(1-5x)^3$

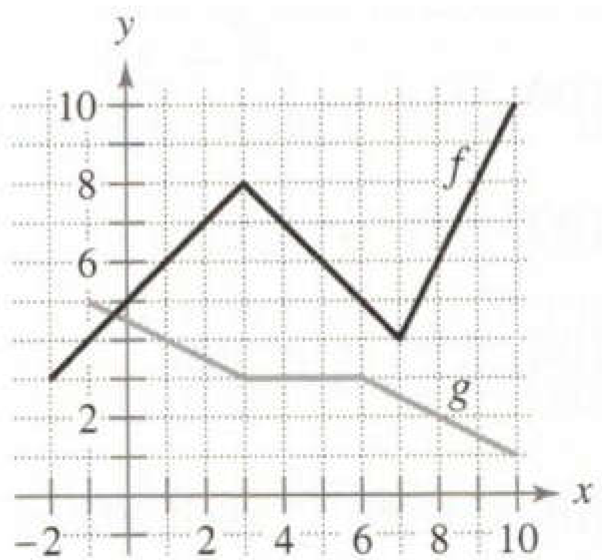
#11. $h(x) = \dfrac{\sqrt{x}}{x^3+1}$

#12. Find $f'(1)$ for $f(x) = \dfrac{x^2-4}{x-3}$

#13. Find $f'\left(\dfrac{\pi}{2}\right)$ for $f(x) = e^x \sin x$

#13. Find (and fully simplify) $f'(x)$ for $f(x)=\dfrac{4-3x-x^2}{x^2-1}$

Let $p(x)=f(x)g(x)$ and $q(x)=\dfrac{f(x)}{g(x)}$ If the graphs of $f(x)$ and $g(x)$ are:



#14. Find $p'(1)$

#15. Find $q'(4)$

2.4 – Required Practice

Find the derivative:

#1. $y = e^{\left(x^2+3\right)}$

#2. $y = \ln\left(\sin\left(e^{x^5}\right)\right)$

#3. $y = \sin^3\left(x^3 + 7x\right)$

#4. $y = \left(x^3 + 4x\right)^5$

#5. $y = \sin\left(3x^2 + x\right)$

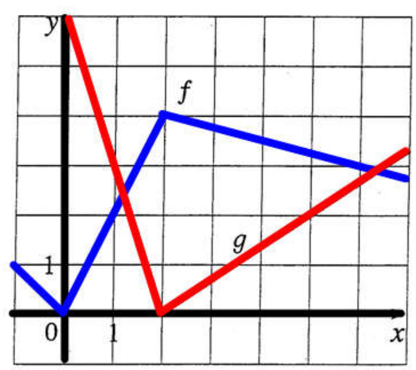
If f and g are the functions whose graphs are shown, let $u(x) = f(g(x))$, $v(x) = g(f(x))$, and $w(x) = g(g(x))$.

Find each derivative, if it exists. If it does not exist, explain why.

#6. $u'(1)$

#7. $v'(1)$

#8. $w'(1)$



Find the derivative:

#9. $y=(2x-7)^3$

#10. $g(x)=\sqrt{4-3x^2}$

#11. $y=-\dfrac{3}{(t-2)^4}$

#12. $g(x)=\left(\dfrac{x+5}{x^2+2}\right)^2$

#13. $y=\dfrac{2}{e^x+e^{-x}}$

#14. $f(x)=\ln\left(\dfrac{x}{x^2+1}\right)$

#15. $y=\ln\left|\sin(x)\right|$

#16. Write an equation for the tangent line to

$y=\sqrt{x^2+8x}$ at $x=1$

#17. Write an equation for the tangent line to

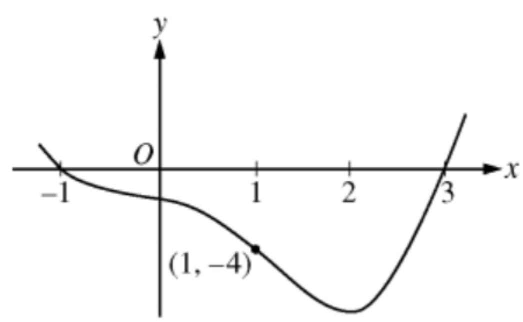
$$f(x) = \tan^2(x) \ \ at \ \ x = \frac{\pi}{4}$$

#18. Find $f''(x)$ for $f(x) = 5(2-7x)^4$

#19. Find $f''(x)$ for $f(x) = \sec^2(\pi x)$

#20. Let *f* be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$.

The graph of *f'(x)*, the derivative is shown. Define $g(x) = e^{f(x)}$.
Write an equation for the line tangent to the graph of *g* ant x = 1.



Graph of f'

2.5 – Required Practice

#1. Find the slope of the line tangent to this curve $x^3 + xy + y^3 = 5$ at the point (-1, 2).

#2. Find $\frac{dy}{dx}$ if $x^2 y + y^2 x = -2$

#3. Find $\frac{dy}{dx}$ if $y=\frac{\left(x^2+3x\right)^4\left(5x^3\right)}{\left(2x^4-4x\right)^2}$

#4. Find $\frac{dy}{dx}$ if $y=x^{\left(5x^3+2x\right)}$

#5. Find $\frac{dy}{dx}$ if $y=x^{\left(\frac{2}{x}\right)}$

#6. Find $\frac{dy}{dx}$ if $x^{1/2} + y^{1/2} = 16$

#7. Find $\frac{dy}{dx}$ if $x^3 - xy + y^2 = 7$

#8. Find $\frac{dy}{dx}$ if $e^{xy} + x^2 - y^2 = 10$

#9. Find $\frac{dy}{dx}$ if $y^2 = \ln x$

#10. Find $\frac{d^2y}{dx^2}$ $(\textit{the 2nd derivative})$ if $x^2 + y^2 = 4$

#11. Find $\frac{dy}{dx}$ if $y=(x-2)^{x+1}$

#12. Find $\frac{dy}{dx}$ if $y=x^{\ln(x)}$

2.6 – Required Practice

#1. If $f(x) = x^3 + x$ and $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

#2. Find $f'(x)$ and $f''(x)$ if $f(x) = \dfrac{\sin x}{1 + \cos x}$

#3. Find the equation of the tangent line to the curve

$y = x + \cos x \quad at \ (0,1)$

#4. Find y' if $y = \ln\left(t^2 + 4\right) - \frac{1}{2}\arctan\left(\frac{t}{2}\right)$

Find the derivative:

#5. $f(x) = \arcsin(x+1)$

#6. $g(x) = \dfrac{\arcsin(3x)}{x}$

Find the derivative:

#7. $y = 2x\arccos(x) - 2\sqrt{1-x^2}$

Find an equation of the tangent line to the graph of the function at the x-value:

#8. $y = 2\arcsin(x) \qquad x = \frac{1}{2}$

Unit 2 Test Review

Find the derivative:

#1. $f(x) = 3x^5 + 5e^{4x} + \sin\left(x^3 + 2x\right) + \ln\left(x^4\right)$

#2. $f(x) = (3x+4)^2 \tan\left(x^3\right)$

#3. $f(x) = \dfrac{\ln\left(x^5\right)}{\sin\left(e^{2x}\right)}$

#4. $f(t) = \cos\left(\ln\left(e^{3t}\right)\right)$

#5. $y = 2^{\sin(\pi x)}$

#6. $y = x^{\left(x^3+5x+7\right)}$

#7. $y=\sqrt[3]{x^2+2}\left(x^2+1\right)$

#8. $\sqrt{x}+x\sqrt{y}=1$

#9. Find y' and y'' for $x^4-y^4=16$

#10. Find the equation of the tangent line to the curve at the given point: $y=\sin\left(\sin\left(x\right)\right)$ $\left(\pi,0\right)$

#11. If $y=e^{x^2}$, find $\dfrac{d^2y}{dx^2}$

#12. Find an equation of the tangent line to the curve at the given point: $y^4 + xy = x^3 - x + 2 \quad at \ (1,1)$

#13. Find $\frac{dy}{dx}$ for $e^{x-y} = 2x^2 - y^2$

#14. Find $\frac{dy}{dx}$ for $y - x^2y^2 = 6$

#15. Find $\frac{d^2y}{dx^2}$ for $x^2 + y^2 = 6$

#16. Find $\frac{dy}{dx}$ for $\operatorname{arcsec}\left(5x^2\right) = \ln\left(y\right)$

#17. *Evaluate* $\lim_{h\to 0} \frac{\sin\left(\frac{\pi}{3}+h\right) - \sin\left(\frac{\pi}{3}\right)}{h}$