AP Calculus BC – Unit 3, Part 1 Required Practice

Name: answers

3.1 – Required Practice

#1. $\lim\limits_{x\to 3^-}\dfrac{x^2+5x+6}{x-3}$

$-\infty$ or DNE

#2. $\lim\limits_{x\to 3^+}\dfrac{x^2+5x+6}{x-3}$

∞ or DNE

#3. $\lim\limits_{x\to 3}\dfrac{x^2+5x+6}{x-3}$

DNE

#4. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{x}$

$= 2$

#5. $\lim\limits_{x\to -\infty}\dfrac{x^2}{e^{-x}}$

$= 0$

#6. $\lim\limits_{x\to \infty}\dfrac{3x^2+x}{4x-7}$

∞ or DNE

#7. $\lim\limits_{x\to\infty} e^{-x}\sqrt{x}$

$= 0$

#8. $\lim_{x \to 0^+} (\sin x)^x$ $= 1$

#9. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

$= e$

#10. $\lim_{x \to 0} \frac{x^2 - 4}{2x - 1}$

$= 4$

#11. $\lim_{x \to 0} \frac{\sin(3x)}{x}$

$= 3$

#12. $\lim_{x\to 0}\frac{\sin(3x)}{x^2}$

DNE

#13. $\lim_{x\to 1}\frac{\cos(\pi x)}{\ln(x)}$

DNE

#14. $\lim_{x\to +\infty} 3^{-x}x^2$

$=0$

#15. $\lim_{x\to +\infty}\frac{2x^3+x^2+1}{5x^2-3x}$

∞

or DNE

#16. $\lim_{x\to +\infty}\frac{2x^3+x^2+1}{5x^3-3x}$

$= \frac{12}{30}$

$\left(= \frac{2}{5}\right)$

#17. $\lim_{x\to +\infty}\frac{2x^3+x^2+1}{5x^4-3x}$

$=0$

Evaluate the limit, using L'Hopital's Rule if necessary.

#18. $\lim_{x\to 3} \frac{x^2-2x-3}{x-3}$

$= 4$

#19. $\lim_{x\to 0} \frac{\sqrt{25-x^2}-5}{x}$

$= 0$

#20. $\lim_{x\to 0} \frac{\sin(3x)}{\sin(5x)}$

$= \frac{3}{5}$

#21. $\lim_{x\to \infty} \frac{x^3}{e^{\left(\frac{1}{2}x\right)}}$

$= 0$

#22. $\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3}$

∞

or

DNE

#23. $\lim_{x\to 0} \frac{\arctan(x)}{\sin(x)}$

$= 1$

#24. $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

$= 1$

#25. $\lim_{x \to 0^+} x^3 \ln(x)$

$= 0$

#26. $\lim_{x \to 1^+} (\ln x)^{x-1}$

$= 1$

3.2 – Required Practice

#1. For $f(x) = \dfrac{-x^2 - 4x - 7}{x+3}$ find the following:

- Critical points and x-intervals where f(x) is increasing/decreasing
- Inflection points and x-intervals where f(x) is concave up/down
- The location of relative (local) extrema (maxima and minima)
- Horizontal and vertical asymptotes

...then sketch the function on the next page.

critical values: $x = -1$, $x = -5$, $x = -3$

f increasing: $(-5, -3) \cup (-3, -1)$

f decreasing: $(-\infty, -5) \cup (-1, \infty)$

f rel. max at $x = -5$ $(-5, 6)$

f rel. min at $x = -1$ $(-1, -1)$

f concave up: $(-\infty, -3)$

f concave down: $(-3, \infty)$

inflection at $x = -3$ (but no point due to asymptote)

vertical asymptote: $x = -3$

horiz. asymptote: (none)

#1 graph:

(graph in your calculator if you want to check)

#2. For $f(x) = 6x^4 + 12x^3 + 20$ find the following:

- Critical points and x-intervals where f(x) is increasing/decreasing
- Inflection points and x-intervals where f(x) is concave up/down
- The location of relative (local) extrema (maxima and minima)
- Horizontal and vertical asymptotes

...then sketch the function on the next page.

Critical values: $x=0$, $x=-3/2$

f increasing: $(-\frac{3}{2},0) \cup (0,\infty)$

f decreasing: $(-\infty, -\frac{3}{2})$

f rel min. at $x=-3/2$ $(-3/2, 9.875)$

no relative maxima

f concave up: $(-\infty,-1) \cup (0,\infty)$

f concave down: $(-1,0)$

inflection at $x=-1$ $(-1,14)$

$x=0$ $(0,20)$

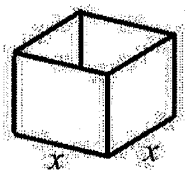
no asymptotes

#2 graph: (graph in calculator to check)

#3. A manufacturer wants to design an open top, square base, box using 108 sq. in. of material. If x is the side length of the base, the volume of the box is given by:
Determine the side length which will give the largest volume.

$$V(x) = 27x - \frac{1}{4}x^3$$

(In Unit 3 part 2 we will practice finding functions like this from word problems)



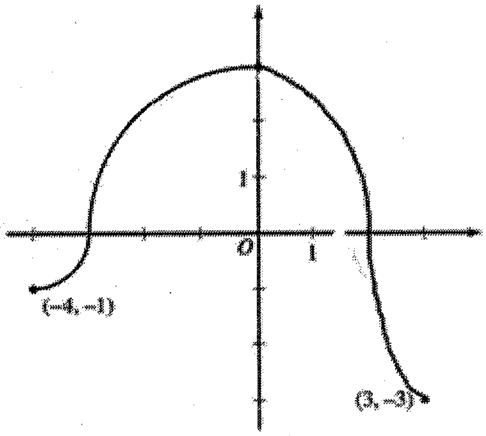
$x = 6$ inches

Working with a graph of a function, $f(x)$

Using the graph, find the following:

- Critical points and intervals where $f(x)$ is increasing/decreasing
- Inflection points and intervals where $f(x)$ is concave up/down
- Relative extrema

#4.



Graph of f

critical pts: $(-4,-1)$, $(0,3)$ and $(3,-3)$

increasing: $(-4,0)$

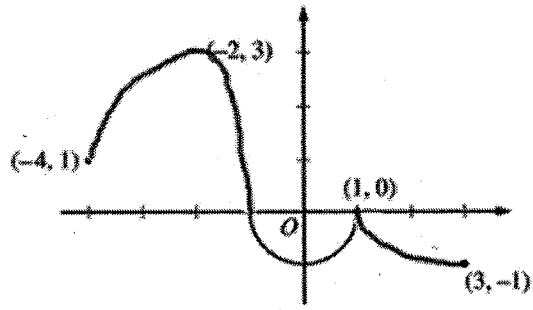
decreasing: $(0,3)$

concave up: $(-4,-3) \cup (2,3)$

concave down: $(-3,2)$

inflection pts $(-3,0)$ and $(2,0)$

#5.



Graph of f

critical pts: $(-2,3)$, $(0,-1)$ and $(1,0)$

increasing: $(-4,-2) \cup (0,1)$

decreasing: $(-2,0) \cup (1,3)$

concave up: $(-1,1) \cup (1,3)$

concave down: $(-4,-1)$

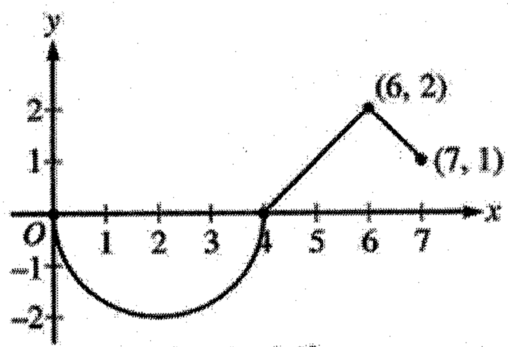
inflection pt $(-1,0)$

Working with a graph of a derivative of a function, $f'(x)$

Using the graph, find the following:

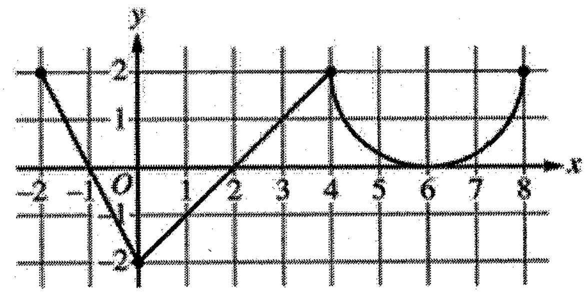
- Critical points and intervals where $f(x)$ is increasing/decreasing
- Inflection points and intervals where $f(x)$ is concave up/down
- Relative extrema

#6.



Graph of f'

#7.



Graph of f'

- critical pts when $f'(x)=0$ or DNE which occurs at $x=0$, $x=4$
- $f(x)$ increasing when $f'(x)>0$ which occurs over $(4,7)$
- $f(x)$ decreasing when $f'(x)<0$ which occurs over $(0,4)$
- $f(x)$ concave up when $f''(x)>0$, when $f'(x)$ increasing which occurs over $(2,6)$
- $f(x)$ concave down when $f''(x)<0$, when $f'(x)$ decreasing which occurs over $(0,2)\cup(6,7)$
- Inflection pts when concavity changes which occurs at $x=2$ and $x=6$.

- critical pts when $f'(x)=0$ or DNE which occurs at $x=-1$, $x=2$, and $x=6$
- $f(x)$ increasing when $f'(x)>0$ which occurs over $(-2,-1)\cup(2,6)\cup(6,8)$
- $f(x)$ decreasing when $f'(x)<0$ which occurs over $(-1,2)$
- $f(x)$ concave up when $f''(x)>0$, when $f'(x)$ increasing which occurs over $(0,4)\cup(6,8)$
- $f(x)$ concave down when $f''(x)<0$, when $f'(x)$ decreasing which occurs over $(-2,0)\cup(4,6)$
- Inflection pts when concavity changes which occurs at $x=0$, $x=4$ and $x=6$.

Without using a calculator, find the intervals where the function is increasing and decreasing, and find all relative maxima and minima.

#8. $f(x) = \frac{x^3}{4} - 3x$

increasing: $(-\infty, -2) \cup (2, \infty)$
decreasing: $(-2, 2)$
rel max at $x = -2$ $(-2, 4)$
rel min at $x = 2$ $(2, -4)$

#9. $f(x) = \frac{1}{(x+1)^2}$

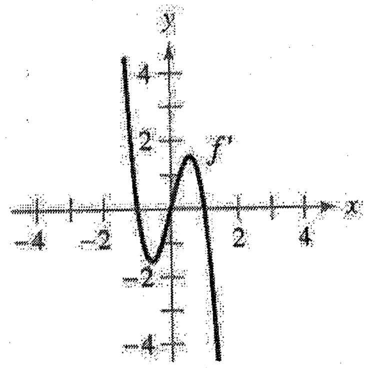
increasing: $(-\infty, -1)$
decreasing: $(-1, \infty)$
critical pt at $x = -1$ but no relative extrema due to vertical asymptote

#10. $f(x) = 3\sin x$ *for* $0 < x < 2\pi$

increasing: $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$
decreasing: $(\frac{\pi}{2}, \frac{3\pi}{2})$
rel. max at $x = \pi/2$ $(\pi/2, 3)$
rel. min at $x = \frac{3\pi}{2}$ $(3\pi/2, -3)$

#11. Use the graph of $f'(x)$ to find:

a) critical numbers of *f*
b) intervals on which *f* is increasing or decreasing
c) for each critical number state whether *f* has a relative maximum, relative minimum, or neither



a) $x=-1,\ x=0,\ x=1$

b) increasing: $(-\infty,-1)\cup(0,1)$
decreasing: $(-1,0)\cup(1,\infty)$

c) $x=-1$ rel. maximum
$x=0$ rel. minimum
$x=1$ rel. maximum

Without using a calculator, find the intervals where the function is concave up and concave down, and find all inflection points.

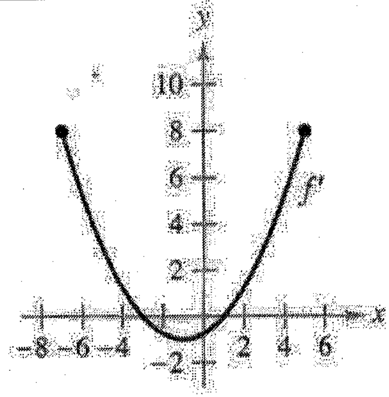
#12. $f(x)=-x^3+6x^2-9x-1$

concave up: $(-\infty,2)$
concave down: $(2,\infty)$
inflection at $x=2$ $(2,-3)$

#13. $f(x)=2x-\tan x \ \ for \ \frac{-\pi}{2}<x<\frac{\pi}{2}$

concave up: $(-\frac{\pi}{2},0)$
concave down: $(0,\frac{\pi}{2})$
inflection at $x=0$ $(0,0)$

#14. Using the graph of the <u>derivative</u> of the function f on the interval [-7, 5]]:



a) On what interval(s) is f increasing / decreasing? Explain.

increasing: $[-7, -3) \cup (1, 5]$ (explanation)

decreasing: $(-3, 1)$ (explanation)

b) On what interval(s) is f concave up / concave down? Explain.

concave up: $(-1, 5)$

concave down: $(-7, -1)$ (explanation)

c) At what x-value(s) does f have relative extrema?

$x = -3, \ x = 1$

d) At what x-value(s) does f have inflection points?

$x = -1$

3.3 – Required Practice

#1. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t (in minutes) is given by a differentiable function C (in ounces). Selected values of $C(t)$ are given in the table:

t (minutes)	0	1	4	9	16
$C(t)$ (ounces)	0	2	4	6	8

a) What is the average rate of change of the amount of coffee over the interval $1 \le t \le 9$?

0.5 $\frac{\text{ounces}}{\text{min}}$

b) Estimate the instantaneous rate of change in the amount of coffee at $t = 10$ *minutes*.

0.286 $\frac{\text{ounces}}{\text{min}}$

c) If the function is found to be $C(t) = 2\sqrt{t}$, find the instantaneous rate of change in the amount of coffee at $t = 10$ *minutes*.

0.316 $\frac{\text{ounces}}{\text{min}}$

#2. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t (in minutes) is given by a differentiable function C (in ounces). Selected values of $C(t)$ are given in the table:

t (minutes)	0	1	4	9	16
$C(t)$ (ounces)	0	2	4	6	8

Is there a time, $t = k$, in the interval $0 \le t \le 16$ when the coffee cup definitely contains 5 ounces of coffee? Justify your answer.

c(t) is differentiable, and therefore also continuous over [0, 16]

since $0 \le 5 \le 8$, the Intermediate Value Theorem guarantees k, $0 \le k \le 16$,

such that c(6) = 5 ounces.

#3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t (in minutes) is given by a differentiable function C (in ounces). Selected values of $C(t)$ are given in the table:

t (minutes)	0	1	4	9	16
$C(t)$ (ounces)	0	2	4	6	8

a) Is there a time, $t=k$, in the interval $0 \le t \le 16$ when the rate of change of the amount of coffee equals the average rate of change of the amount of coffee? Justify your answer.

$c(t)$ is differentiable, and therefore also continuous over $[0,16]$

$\therefore$ the Mean Value Theorem does guarantee k, $0 \le k \le 16$ such that $c'(k)$ equals the average rate of change of $c(t)$ over $[0,16]$.

b) If you are further told that $C(t)=2\sqrt{t}$, find k.

$$\text{avg rate of change} = \frac{c(16)-c(0)}{16-0} = \frac{8-0}{16} = \frac{1}{2} \ \frac{\text{ounces}}{\text{min}}$$

$$c(t) = 2t^{1/2}$$

$$c'(t) = t^{-1/2} = \frac{1}{\sqrt{t}}$$

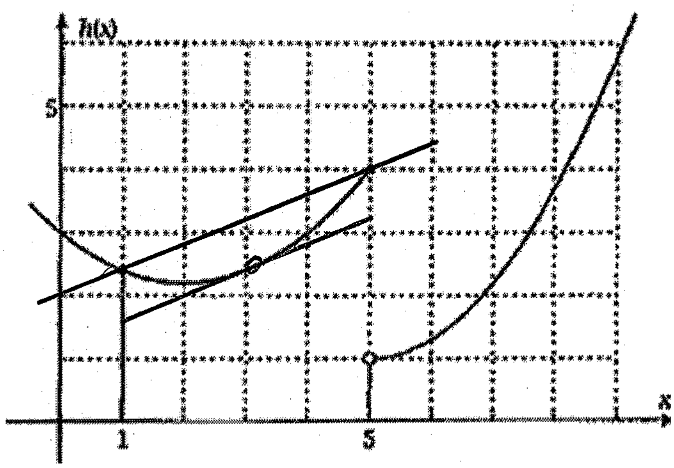
$$c'(k) = \frac{1}{\sqrt{k}} = \frac{1}{2} \text{ when } \sqrt{k} = 2$$

$$k = 4 \text{ minutes}$$

#4. The graph below is function h from Problem 5.

- Draw a secant line through $(1, h(1))$ and $(5, h(5))$.
- Show that there is a point $x = c$ in $(1, 5)$ where $h'(c)$ equals the slope of the secant line.
- Is h differentiable on $(1, 5)$? yes
- Explain why h is continuous on $[1, 5]$, even though there is a step discontinuity at $x = 5$.

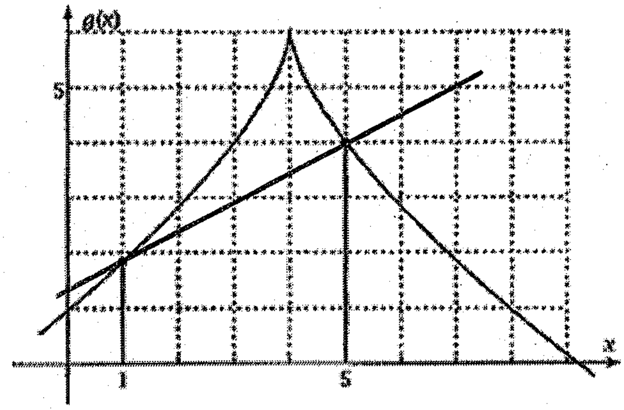
(explanation)



#5. For $g(x) = 6 - 2(x - 4)^{2/3}$, graphed below,

- Draw a secant line through $(1, g(1))$ and $(5, g(5))$.
- Is g differentiable on $(1, 5)$? no
- Is g continuous on $[1, 5]$? yes
- Tell why there is *no* value of $x = c$ between $x = 1$ and $x = 5$ at which $g'(c)$ equals the slope of the secant line.

(explanation)

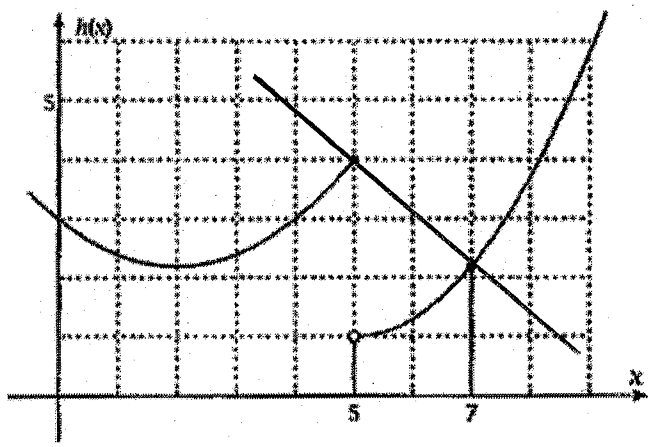


#6. Piecewise function h is defined by

$$h(x) = \begin{cases} 0.2(x-2)^2 + 2.2, & \text{if } x \le 5 \\ 0.3(x-5)^2 + 1, & \text{if } x > 5 \end{cases}$$

- Draw a secant line through $(5, h(5))$ and $(7, h(7))$.
- Is h differentiable on $(5, 7)$? yes
- Is h continuous on $[5, 7]$? no
- Why is there *no* value $x = c$ in $(5, 7)$ for which $h'(c)$ equals the slope of the secant line?

(explanation)



#7.

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

a) Explain why there must be at least one time, t, for $2 < t < 10$, such that $H'(t) = 2$.

(hint: use the interval [3,5] which has average rate of change = 2)

(this is about $H'(t)$ so must use the Mean Value Theorem)

b) Explain why there must be at least one time, t, for $2 < t < 10$, such that $H(t) = 4$.

(this is about $H(t)$ so must use Intermediate Value Theorem)

Determine whether the Mean Value Theorem or Rolle's Theorem can be applied for the specified function and interval, and if it can be applied, find all *x*-values in the interval where the instantaneous rate of change equals the average rate of change:

#8. $f(x) = -x^2 + 5 \quad over \quad [-1, 2]$

(MVT wording)

$c = 1/2$

#9. $f(x) = x\log_2(x) \quad over \quad [1, 2]$

(MVT wording)

$c = e^{(2\ln 2 - 1)} = 1.472$

#10. $f(x) = \dfrac{x^2 - 1}{x} \quad over \quad [-1, 1]$

(there is a discontinuity in the interval)

#11. $f(x) = -x^2 + 3x \quad over \quad [0, 3]$

(Rolle's wording)

$c = \frac{3}{2}$

3.4 – Required Practice

Without using a calculator, find the absolute maximum and absolute minimum value of the function over the given interval:

#1. $f(x)=\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{3}{2}x^2 \quad over \quad [-4,3]$

abs max $= \frac{99}{4}$ (at $x=3$)

abs min $= \frac{-135}{12}$ (at $x=-3$)

#2. $f(x)=x^2-4x+1 \quad over \quad [-3,5]$

abs max $= 22$ (at $x=-3$)

abs min $= 3$ (at $x=2$)

Without using a calculator, find the absolute maximum and absolute minimum value of the function over the given interval:

#3. $f(x) = 3x^{\frac{2}{3}} - 2x \quad over \quad [-1,1]$

abs max = 5 (at $x = -1$)

abs min = 0 (at $x = 0$)

#4. $f(x) = \sin x \quad over \quad \left[\frac{5\pi}{6}, \frac{11\pi}{6}\right]$

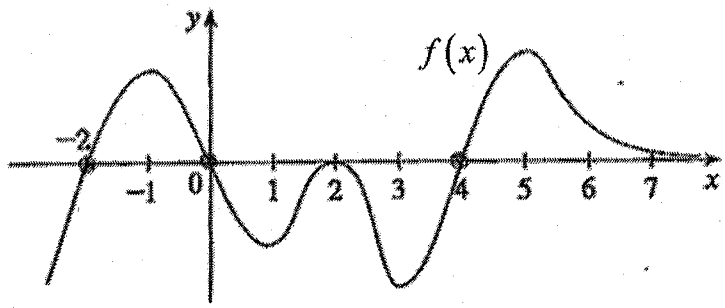
abs max = $\frac{1}{2}$ (at $x = 5\pi/6$)

abs min = -1 (at $x = \frac{3\pi}{2}$)

Test Review for Unit 3 Part 1 Test
(recommend you also review required and extra practice for additional review)
NO Calculators on this test

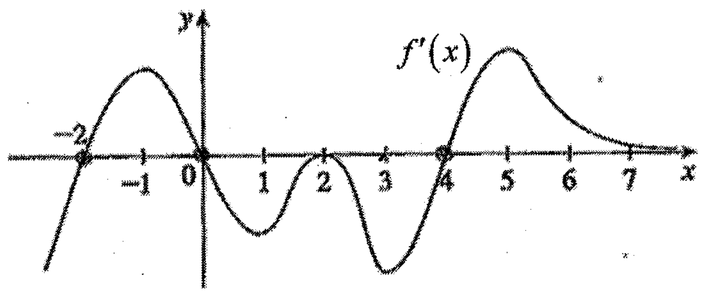
#1. The figure shows the graph of $f(x)$.
Find the following (approximate coordinates to the nearest 0.5):

a) All critical point x-values.
b) x-intervals where $f(x)$ is increasing/decreasing.
c) All inflection point x-values.
d) x-intervals where $f(x)$ is concave up/down.
e) All relative maxima.



#2. The figure shows the graph of $f'(x)$, the <u>derivative</u> of $f(x)$.
Find the following (approximate coordinates to the nearest 0.5):

a) All critical point x-values.
b) x-intervals where $f(x)$ is increasing/decreasing.
c) All inflection point x-values.
d) x-intervals where $f(x)$ is concave up/down.
e) All relative maxima.



For the function given, find the following (without a calculator):
a) critical x-values and intervals where the function is increasing/decreasing.
b) inflection x-values and intervals where the function is concave up/down.
c) the coordinates of all relative maximum and minimum points.
d) the absolute maximum and absolute minimum over the interval.

#3. $f(x)=x^2e^{-x}$ for the interval $[0,3]$.

For the function given,

a) Find the average rate of change over the interval $[1,4]$

b) Show that the Mean Value Theorem (or Rolle's Theorem) guarantees a time value within the interval $[1,4]$ where the instantaneous rate of change equals the average rate of change.

c) Find the time value where the instantaneous rate of change equals the average rate of change over the interval $[1,4]$.

#4. $g(t)=\dfrac{t}{t+2}$

Evaluate the limit:

#5. $\displaystyle\lim_{x\to\infty}\frac{\ln(\ln x)}{\ln x}$

#6. $\displaystyle\lim_{x\to 1} x^{\left(\frac{1}{1-x}\right)}$

#7. $\displaystyle\lim_{x\to 0}\frac{\ln(1-x)+x+\frac{1}{2}x^2}{x^3}$

#8. $\displaystyle\lim_{x\to -1}\frac{2x^2+3x-2}{5x^2+x+1}$

#9. $\displaystyle\lim_{x\to\infty} e^{-x}\ln(x)$

#10. $\displaystyle\lim_{x\to\infty}\frac{2x^2+3x-2}{5x^2+x+1}$

Test Review for Unit 3 Part 1 Test – Solutions

#1. a) critical pts when $f'(x)=0$ or DNE: $x=-1$, $x=1$, $x=2$, $x=3$, $x=5$

b) f increasing: $(-\infty,-1)\cup(-1,2)\cup(3,5)$

(no arrow shown, but I'll assume the function continues)

f decreasing: $(-1,1)\cup(2,3)\cup(5,\infty)$

c) $x=0$, $x=1.5$, $x=2.5$, $x=4$, $x=5.5$

d) f concave up: $(0,1.5)\cup(2.5,4)\cup(5.5,\infty)$

f concave down: $(-\infty,0)\cup(1.5,2.5)\cup(4,5.5)$

e) f rel. max at $x=-1$, $x=2$, $x=5$

f rel. min at $x=1$, $x=3$

#2 a) critical pts when $f'(x)=0$ or DNE: $x=-2$, $x=0$, $x=2$, $x=4$

b) $f(x)$ increasing when $f'(x)>0$: $(-2,0)\cup(4,\infty)$

$f(x)$ decreasing when $f'(x)<0$: $(-\infty,-2)\cup(0,2)\cup(2,4)$

d) $f(x)$ concave up when $f''(x)>0$, which is when $f'(x)$ increasing, this occurs for $(-\infty,-1)\cup(1,2)\cup(3,5)$

$f(x)$ concave down when $f''(x)<0$, which is when $f'(x)$ decreasing, this occurs for $(-1,1)\cup(2,3)\cup(5,\infty)$

c) inflection when sign of $f''(x)$ changes, when $f'(x)$ curve changes direction: $x=-1$, $x=1$, $x=2$, $x=3$, $x=5$

e) f rel. max /\ when $f'(x)$ goes from + to −; This happens at $x=0$

f rel. min \/ when $f'(x)$ goes from − to +: This happens at $x=-2$ and $x=4$.

(no relative extrema at $x=2$ b/c no sign change)

#3 $f(x) = x^2 e^{-x}$

a) $f'(x) = x^2 e^{-x}(-1) + e^{-x}(2x)$

$= -x^2 e^{-x} + 2xe^{-x}$

$= -xe^{-x}(x-2)$

$f'(x) = 0$ $\quad$ $f'(x)$ DNE

$x = 0,\ x = 2$

$-\infty \quad 0 \quad 2 \quad \infty$

$f'(-1)$: $(-)(-)(+)(-)$ $\searrow$

$f'(1)$: $(-)(+)(+)(-)$ $\nearrow$

$f'(3)$: $(-)(+)(+)(+)$ $\searrow$

$\boxed{\begin{array}{l}\text{critical values: } x=0,\ x=2 \\ f \text{ increasing: } (0,2) \\ f \text{ decreasing: } (-\infty,0) \cup (2,\infty)\end{array}}$

b) $f''(x) = (-x^2)(-e^{-x}) + e^{-x}(-2x) + (2x)(-e^{-x}) + e^{-x}(2)$

$= x^2e^{-x} - 2xe^{-x} - 2xe^{-x} + 2e^{-x}$

$= x^2e^{-x} - 4xe^{-x} + 2e^{-x}$

$= e^{-x}(x^2 - 4x + 2)$

$f''(x) = 0$

$x^2 - 4x + 2 = 0$

$x = \dfrac{4 \pm \sqrt{16 - 4(1)(2)}}{2(1)}$

$= \dfrac{4 \pm \sqrt{8}}{2}$

$= \dfrac{4}{2} \pm \dfrac{2\sqrt{2}}{2}$

$= 2 \pm \sqrt{2}$

$-\infty \quad 2-\sqrt{2}\ (2-1.4) \quad 2+\sqrt{2} \quad \infty$

$f''(-10)$: $(+)(100+40+2)$, $(+)(+)$ $\cup$

$f''(1)$: $(+)(1-4+2)$, $(+)(-)$ $\cap$

$f''(10)$: $(+)(100-40+2)$, $(+)(+)$ $\cup$

$\boxed{\begin{array}{l}\text{inflection values: } x = 2-\sqrt{2},\ x = 2+\sqrt{2} \\ f \text{ concave up: } (-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty) \\ f \text{ concave down: } (2-\sqrt{2}, 2+\sqrt{2})\end{array}}$

c) $\boxed{f \text{ rel. max}}$ at $x = 2$ $\quad f(2) = (2)^2 e^{-2}$: $\boxed{(2, 4e^{-2})}$

$\boxed{f \text{ rel. min}}$ at $x = 0$ $\quad f(0) = 0$ $\quad \boxed{(0,0)}$

d)

x	$f(x) = x^2e^{-x}$
0	0 abs min
2	$(2)^2e^{-2} = 4e^{-2} = 0.54134$ abs max
3	$(3)^2e^{-3} = 9e^{-3} = 0.44808$

$\boxed{\begin{array}{l} f \text{ has abs max of } 4e^{-2} \text{ (at } x=2) \\ f \text{ has abs min of } 0 \text{ (at } x=0)\end{array}}$

on the test the problem is easier so you won't need a calculator to find values :)

(4) $g(t) = \frac{t}{t+2}$

a) avg rate of change $= \frac{g(4)-g(1)}{4-1} = \frac{\frac{4}{6} - \frac{1}{3}}{3} = \frac{\frac{4}{6} - \frac{2}{6}}{3} = \frac{\frac{1}{3}}{3} = \frac{1}{9}$

b) $g(t)$ has a discontinuity at $t = -2$ but this is not in the interval, so $g(t)$ is continuous over $[1,4]$ and differentiable over $(1,4)$
$\therefore$ the Mean Value Theorem guarantees at least one t, $1 \le t \le 4$, such that $g'(t) = \frac{1}{9}$ (the average rate of change).

c) $g'(t) = \frac{(t+2)(1) - t(1)}{(t+2)^2} = \frac{t+2-t}{(t+2)^2} = \frac{2}{(t+2)^2}$

$g'(t) = \frac{2}{(t+2)^2} = \frac{1}{9}$

$18 = (t+2)^2$

$t+2 = \sqrt{18}$

$t = \sqrt{18} - 2$

#5 $\lim_{x\to\infty} \frac{\ln(\ln x)}{\ln x}$ $\quad \lim_{x\to\infty} \ln(\ln x) \to \infty$ $\left(\frac{\infty}{\infty}\right)$ use L'Hopital's Rule

$\lim_{x\to\infty} \ln x \to \infty$

$$= \lim_{x\to\infty} \frac{\left(\frac{1}{\ln x} \cdot \frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{1}{\ln x} = \boxed{0}$$

#6 $\lim_{x\to 1} x^{\left(\frac{1}{1-x}\right)}$ function raised to a function requires $\ln$

$y = \lim_{x\to 1} x^{\left(\frac{1}{1-x}\right)}$

$\ln y = \ln\left(\lim_{x\to 1} x^{\left(\frac{1}{1-x}\right)}\right)$

$\ln y = \lim_{x\to 1}\left[\ln\left(x^{\left(\frac{1}{1-x}\right)}\right)\right]$ don't need to show these steps

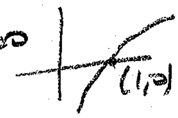
$\ln y = \lim_{x\to 1}\left[\left(\frac{1}{1-x}\right)\ln(x)\right]$

$\ln y = \lim_{x\to 1} \frac{1}{1-x} \lim_{x\to 1} \ln(x)$ $\quad \lim_{x\to 1} \frac{1}{1-x}$ uncancelled zero in denom, vert. asymptote

$(0.9) \quad \frac{(+)}{(+)} \to \infty$

$(\infty)(0)$

(rearrange) $\quad \lim_{x\to 1} \ln(x) = 0$



$\ln y = \lim_{x\to 1} \frac{\ln x}{1-x}$ $\left(\frac{0}{0}\right)$ use L'Hopital's rule

$$\ln y = \lim_{x\to 1} \frac{\left(\frac{1}{x}\right)}{(-1)} = \frac{\left(\frac{1}{1}\right)}{-1} = -1$$

$y = e^{-1} = \frac{1}{e}$ $\quad \therefore \boxed{\lim_{x\to 1} x^{\left(\frac{1}{1-x}\right)} = \frac{1}{e}}$

#7 $\lim_{x\to 0} \frac{\ln(1-x)+x+\frac{1}{2}x^2}{x^3}$ $\lim_{x\to 0}(\ln(1-x)+x+\frac{1}{2}x^2)=0$ $(\frac{0}{0})$ use L'Hopital's rule

$\lim_{x\to 0} x^3=0$

$=\lim_{x\to 0} \frac{\frac{1}{1-x}(-1)+1+x}{3x^2}$ $(\frac{0}{0})$

$=\lim_{x\to 0} \frac{-(1-x)^{-1}+1+x}{3x^2}$ use L'Hop.

$=\lim_{x\to 0} \frac{(1-x)^{-2}(-1)+1}{6x}$ $(\frac{0}{0})$

$=\lim_{x\to 0} \frac{2(1-x)^{-3}(-1)}{6} = \frac{-2}{6} = \boxed{\frac{-1}{3}}$

#8 $\lim_{x\to -1} \frac{2x^2+3x-2}{5x^2+x+1} = \frac{2(-1)^2+3(-1)-2}{5(-1)^2+(-1)+1} = \frac{2-3-2}{5} = \boxed{\frac{-3}{5}}$

compare to #10... (only difference is $x\to -1$ vs $x\to\infty$)

#10 $\lim_{x\to\infty} \frac{2x^2+3x-2}{5x^2+x+1}$ $\lim_{x\to\infty}(2x^2+3x-2)\to\infty$ $(\frac{\infty}{\infty})$ use L'Hopital's Rule

$\lim_{x\to\infty}(5x^2+x+1)\to\infty$

$=\lim_{x\to\infty} \frac{4x+3}{10x+1}$ $(\frac{\infty}{\infty})$

$=\lim_{x\to\infty} \frac{4}{10} = \frac{4}{10} = \boxed{\frac{2}{5}}$

#9 $\lim_{x\to\infty} e^{-x}\ln(x)$ $\lim_{x\to\infty} e^{-x} = 0$ $(0)(\infty)$ rearrange...

$\lim_{x\to\infty} \ln(x) \to \infty$

$= \lim_{x\to\infty} \frac{\ln x}{e^x}$ $\lim_{x\to\infty} \ln x \to \infty$ $\left(\frac{\infty}{\infty}\right)$ use L'Hopital's Rule

$\lim_{x\to\infty} e^x \to \infty$

$= \lim_{x\to\infty} \frac{\left(\frac{1}{x}\right)}{e^x} \to \frac{0}{\infty} = \boxed{0}$