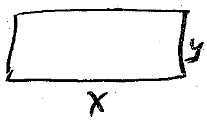
AP Calculus BC – Unit 3, Part 2 Extra Practice

3.5 – Extra Practice

#6b. Find the length and width of a rectangle that has a perimeter of 80 m and will maximize the area enclosed.



objective

max $A = xy$

$A = x(40-x) = 40x - x^2$

$A' = 40 - 2x = 0$

$2x = 40$

$x = 20\text{m}$

$y = 40 - 20 = 20\text{m}$

constraint

$P = 80\text{m}$

$2x + 2y = 80$

$x + y = 40$

$y = 40 - x$

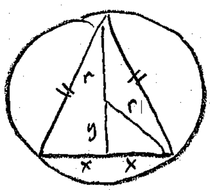
verify max:

$A'' = -2 < 0$

$\therefore A$ is concave down

$\therefore$ this is a max area

#7b. Find the dimensions of the isosceles triangle of largest area that can be inscribed in a circle of radius *r*.



objective

max $A = \frac{1}{2}(2x)(y+r)$

$A = \frac{1}{2}\left(2\sqrt{r^2-y^2}\right)(y+r)$

$A = (r^2-y^2)^{1/2}(y+r)$ (product rule)

$A' = \sqrt{r^2-y^2}\,(1) + (y+r)\frac{1}{2}(r^2-y^2)^{-1/2}(-2y)$

$= \sqrt{r^2-y^2} - \dfrac{y(y+r)}{\sqrt{r^2-y^2}} = 0$

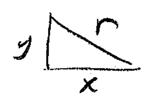
$= \dfrac{\sqrt{r^2-y^2}\sqrt{r^2-y^2}}{\sqrt{r^2-y^2}} - \dfrac{y^2-yr}{\sqrt{r^2-y^2}} = 0$

$= \dfrac{r^2-y^2-y^2-yr}{\sqrt{r^2-y^2}} = 0$

$= \dfrac{r^2-2y^2-yr}{\sqrt{r^2-y^2}} = 0$

constraint

$x^2 + y^2 = r^2$



$x^2 = r^2 - y^2$

$x = \sqrt{r^2-y^2}$

(r is a constant)

$A' = 0$

$-2y^2 - ry + r^2 = 0$ (r is a constant)

$y = \dfrac{r \pm \sqrt{r^2 + 4(2)(r^2)}}{2(-2)}$

$y = \dfrac{r \pm \sqrt{9r^2}}{-4} = \dfrac{r \pm 3r}{-4}$

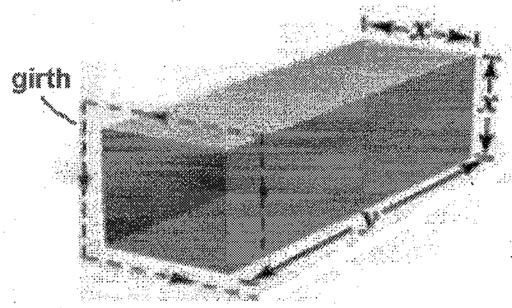
$y = \dfrac{-2r}{-4} = \frac{1}{2}r$ or $y = \dfrac{4r}{-4} = -r$ (not possible)

$y = \frac{1}{2}r$

$x = \sqrt{r^2 - (\frac{1}{2}r)^2} = \sqrt{\frac{3}{4}r^2} = \frac{\sqrt{3}}{2}r$

(skipping verify max on this one :))

#8b. A rectangular package to be sent by a postal service can have a maximum combined length and girth of 108 inches (girth is the distance around all sides of a cross-section). Find the dimension of a package which encloses the maximum possible volume, assuming the cross-section shape is square.



objective

max $V = x^2 y$

constraint

$y + 4x = 108$

$y = 108 - 4x$

$V = x^2(108-4x)$

$V = 108x^2 - 4x^3$

$V' = 216x - 12x^2 = 0$

$x(216-12x) = 0$

$x = 0$ (not possible)

$216 - 12x = 0$

$12x = 216$

$x = \frac{216}{12}$

$x = 18$ in

$y = 108 - 4(18) = 36$ in

verify max:

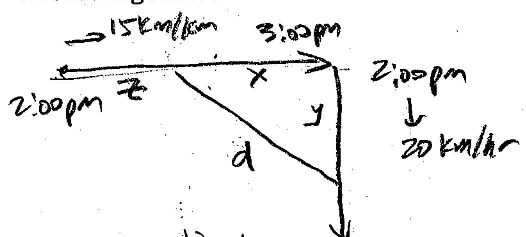
$V'' = 216 - 24x$

$V''(18) = 216 - 24(18) < 0$

$\therefore V$ concave down

$\therefore$ this is a max volume

#9b (extra challenging problem): A boat leave a dock at 2:00 pm and travels due south at a speed of 20 km/hr. Another boar has been heading due east at 15 km/hr and reaches the same dock at 3:00 pm. At what time were the two boats closest together?



Let t be hrs since 2:00pm

At time t, south boat travels $y = 20t$

At time t, east boat travels $z = 15t$

in one hour, east boat travels $z + x = 15\frac{km}{hr}(1\,hr) = 15\,km$

$\therefore x = 15 - z = 15 - 15t$

objective

min $d = \sqrt{x^2+y^2}$

define $S = d^2 = x^2 + y^2$

constraint(s)

$y = 20t, \quad x = 15 - 15t$

min $S = x^2 + y^2$

$S = (15-15t)^2 + (20t)^2 = 225 - 450t + 225t^2 + 400t^2$

$S = 625t^2 - 450t + 225$

$S' = 1250t - 450 = 0$

$1250t = 450$

$t = \frac{450}{1250} = \frac{9}{25} = 0.36$ hrs $\left(\frac{60\,min}{hr}\right) = 21.6$ minutes after 2:00pm

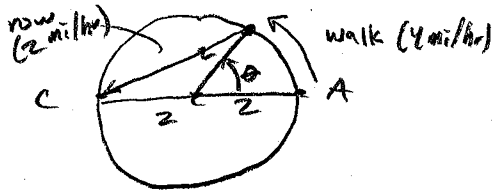
verify min:

$S'' = 1250 > 0$

$\therefore S$ is concave up

$\therefore$ this is min distance

#10b (extra challenging problem): A woman at point A on the shore of a circular lake with radius 2 miles wants to arrive at point C diametrically opposite point A on the other side of the lake in the shortest possible time. She can walk at the rate of 4 miles/hr and row a boat at a rate of 2 miles/hr. How much of the journey should be walking and how much should be boating in order to go from point A to point C in the shortest possible time?



distance = (rate) time

So time = $\frac{\text{dist}}{\text{rate}}$

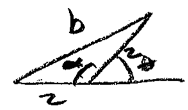
define θ (radians) around to point where she changes from walking to rowing

walk: arc length = $r\theta$

distance = $(2)\theta$

So $t_{walk} = \frac{2\theta}{4}$

boat:



$\alpha = \pi - \theta$

law of cosines:

$b^2 = 2^2 + 2^2 - 2(2)(2)\cos\alpha$

$b^2 = 8 - 8\cos\alpha$

$b = \sqrt{8 - 8\cos\alpha}$

So $t_{boat} = \frac{\sqrt{8-8\cos\alpha}}{2}$

objective

min $t_{(total)} = t_{walk} + t_{boat}$

$t = \frac{2\theta}{4} + \frac{\sqrt{8-8\cos\alpha}}{2}$

need in terms of one variable, let's use α

$\alpha = \pi - \theta$

$\theta = \pi - \alpha$

$t = \frac{2(\pi - \alpha)}{4} + \frac{\sqrt{8-8\cos\alpha}}{2}$

$t = \frac{1}{2}\pi - \frac{1}{2}\alpha + \frac{1}{2}(8 - 8\cos\alpha)^{1/2}$

$t' = -\frac{1}{2} + \frac{1}{4}(8-8\cos\alpha)^{-1/2}(8\sin\alpha)$

$t' = -\frac{1}{2} + \frac{2\sin\alpha}{\sqrt{8-8\cos\alpha}}$

$t' = 0$

$-\frac{1}{2} + \frac{2\sin\alpha}{\sqrt{8-8\cos\alpha}} = 0$

by calculator, zero at

$\alpha = 2.0944$

t' DNE

when $\sqrt{8-8\cos\alpha} = 0$

$8 - 8\cos\alpha = 0$

$\cos\alpha = 1$

$\alpha = 0$

α	$t = \frac{2(\pi-\alpha)}{4} + \frac{\sqrt{8-8\cos\alpha}}{2}$
2.0944	2.2556
0	1.5708 ← min

minimum time of 1.5708 hrs occurs if $\alpha = 0$

(meaning walks all the way around from A to C and never does use the boat)

3.6 – Extra Practice

#6b. If a stone is thrown vertically upward from the surface of the moon with a velocity of 10 meters per second, its height (in meters) after t seconds is $h(t)=10t-0.83t^2$.

a) Find the velocity of the stone after 3 seconds.
b) What is the velocity of the stone after it has risen 25 meters?
c) What is the acceleration of the stone at 4 seconds?

a) $v(t)=h'(t)=10-1.66t$

$v(3)=10-1.66(3)=\boxed{5.02 \text{ m/s}}$

b) $h(t)=25\text{ m}$

$10t-0.83t^2=25$ occurs at $t=3.5402978$ sec (on the way up)

and $t=8.507895$ sec (on the way down)

'risen' might mean only on the way up, but to be safe, we'll report both:

$v(3.5402978)=4.123$ m/s (on the way up)
$v(8.507895)=-4.123$ m/s (on the way down)

c) $a(t)=v'(t)=-1.66$

$\boxed{a(4)=-1.66 \text{ m/s}^2}$

#7b. If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after *t* seconds is given by $h(t) = 80t - 16t^2$.

a) What is the maximum height reached by the ball?
b) What is the velocity of the ball when it is 95 ft above the ground on its way up? ...on its way down?
c) What is the average rate of change of the position of the ball over the interval $1 \le t \le 3$ seconds?
d) What is the instantaneous velocity of the ball at $t = 2$ seconds?

a) max height occurs when $v(t)=0$:

$v(t) = 80 - 32t = 0$

$32t = 80$, $t = \frac{80}{32} = 2.5$ seconds.

$\boxed{h(2.5) = 100 \text{ ft}}$

b) $h(t) = 95$

$80t - 16t^2 = 95$

at $t = 1.9409983$ s (on its way up)

and $t = 3.059017$ s (on its way down)

$\boxed{\begin{array}{l} v(1.940983) = 17.889 \text{ ft/sec (on its way up)} \\ v(3.059017) = -17.889 \text{ ft/sec (on its way down)} \end{array}}$

c) avg rate of change of position $= \frac{h(3)-h(1)}{3-1} = \frac{96-64}{2} = \frac{32}{2} = \boxed{16 \text{ ft/sec}}$

d) $v(2) = 80 - 32(2) = \boxed{16 \text{ ft/sec}}$

#8b. A particle moves according to a law of motion where t is measured in seconds and s in feet:

$$s(t)=\frac{20t}{t^2+1} \text{ for } t\geq 0$$

(Justify all responses)

a) Find the velocity at time t.
b) What is the velocity after 3 seconds?
c) When is the particle at rest?
d) When is the particle moving in the positive direction?
e) Find the acceleration of the particle at t = 4 seconds.

f) Find all the values of t in the interval $0\leq t\leq 8$ for which the speed of the particle is $+1\ \frac{ft}{s}$.

g) At what times in the interval $0\leq t\leq 8$ is the particle changing direction?
h) Is the particle moving the right or the left at time t = 4 seconds?
i) Is the particle speeding up or slowing down at time t = 4 seconds?

a) $v(t)=s'(t)=\frac{(t^2+1)20-20t(2t)}{(t^2+1)^2}=\frac{20t^2+20-40t^2}{(t^2+1)^2}=\frac{-20t^2+20}{(t^2+1)^2}=\boxed{\frac{-20(t^2-1)}{(t^2+1)^2}=v(t)}$

b) $\boxed{v(3)=-4.8 \text{ ft/sec}}$

c) The particle is at rest when $v(t)=0$ which occurs at $\boxed{t=1 \text{ sec}}$

d) The particles moves in positive direction when $v(t)>0$ which occurs for $\boxed{0<t<1 \text{ sec}}$

e) $a(t)=v'(t)=\frac{(t^2+1)^2(-40t)-(-20(t^2-1))2(t^2+1)2t}{(t^2+1)^4}=\frac{(t^2+1)[-40t(t^2+1)+80t(t^2-1)]}{(t^2+1)^3}$

$a(t)=\frac{-40t^3-40t+80t^3+80t}{(t^2+1)^3}=\frac{-40t^3+40t}{(t^2+1)^3}=\frac{-40t(t^2-1)}{(t^2+1)^3}=a(t)$

$\boxed{a(4)=-0.489 \text{ ft/sec}^2}$

f) speed $=|v(t)|=-1$ occurs for $\boxed{t=0.050 \text{ sec},\ t=0.904 \text{ sec, and } t=1.106 \text{ sec}}$ (at $v=-1$ too!)

g) The particle changes direction when $v(t)$ changes sign.
This occurs at $\boxed{t=1 \text{ sec}}$

h) $v(4)=-4.152<0$ ∴ $\boxed{\text{the particle is moving left at } t=4 \text{ sec}}$

i) $v(4)=-4.152<0$
$a(4)=-0.489<0$

∴ The particle is speeding up at $t=4$ sec because the signs of $v(4)$ and $a(4)$ are the same.

3.7 – Extra Practice

For #6b and #7b...

a) Find the linearization of the function (find a tangent line) at the given 'easy to compute' x-value.
b) Use the tangent line linearization to approximate the value of the 2nd x-value.
c) Is this approximation an over- or under-estimation of the actual function value?

#6b. $f(x) = x^3 + 3x$ *linearization at* $x = 1$
Approximate $f(1.2)$

a) $f(1) = 1+3 = 4$ $(1,4)$ $m=6$
$f'(x) = 3x^2+3$ $(y-4) = 6(x-1)$
$f'(1) = 3+3 = 6$
b) $y-4 = 6(1.2-1) = 6(0.2) = 1.2$
$y = 5.2$ $\therefore f(1.2) \approx 5.2$
c) $f''(x) = 6x$
$f''(1) = 6 > 0$
$\therefore f$ is concave up in this region
$\therefore$ 5.2 is an under-estimate of $f(1.2)$

#7b. $f(x) = \sin(x)$ *linearization at* $x = \pi$
Approximate $f(3)$

a) $f(\pi) = \sin(\pi) = 0$ $(\pi, 0)$ $m = -1$
$f'(x) = \cos x$ $(y-0) = -(x-\pi)$
$f'(\pi) = \cos\pi = -1$
b) $y = -(3-\pi) = \pi - 3$
$\therefore f(3) \approx \pi - 3$
c) $f''(x) = -\sin x$
$f''(\pi) = -\sin\pi = 0$ (inconclusive)
try $f''(3) = -\sin(3) < 0$ 3.14
$\therefore f$ is concave down near $x=3$
$\therefore \pi - 3$ is an over-estimate of $f(3)$

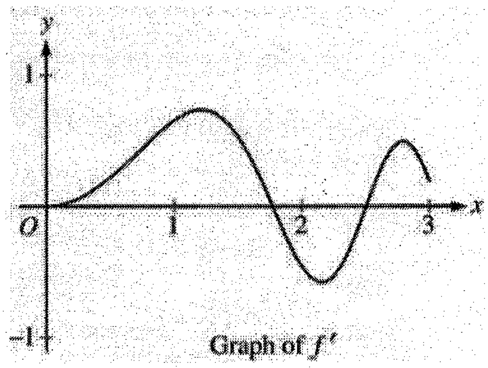
#8b. The function f is twice differentiable with $f(3) = 2$, $f'(3) = -3$, *and* $f''(3) = 5$.

a) What is the value of the approximation of $f(3.2)$ using the line tangent to the graph of f at $x = 3$?
b) Is this approximation an over- or under-estimation of $f(3.2)$? Justify your result.

a) $(3,2)$ $m=-3$ $(y-2) = -3(x-3)$
$y-2 = -3(3.2-3) = -3(0.2) = -0.6$
$\therefore f(3.2) \approx -0.6+2$ (no need to simplify)

b) $f''(3) = 5 > 0$ $\therefore f$ is concave up in this region
$\therefore -0.6+2$ is an under-estimation of $f(3.2)$

#9b. Let f be the function defined for $x \geq 0$, with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{\left(\frac{-x}{4}\right)} \sin\left(x^2\right)$.
The graph of $y = f'(x)$ is shown.



a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.
b) Write an equation for the line tangent to the graph of f at $x = 0$.
c) Does the line tangent to the graph of f at $x = 0$ lie above or below the graph of f for $0 < x < 1$? Why?

a) For $1.7 < x < 1.9$ the graph of $f'(x)$ is decreasing which means $f''(x) < 0$, therefore the graph of $f(x)$ is concave down in this interval.

b) $f(0) = 5$ $\quad (0,5)\ m = 0$
$f'(0) = 0$ $\quad (y-5) = 0(x-0)$
$y - 5 = 0$
$y = 5$

c) the graph of $f'(x)$ is increasing over $0 < x < 1$, which means $f''(x) > 0$ and $f(x)$ is concave up near $x = 0$

∴ The line tangent to the graph of f at $x=0$ lies below the graph of f.

3.8 – Extra Practice

#7b. A spherical balloon is inflated with gas at the rate 800 cubic centimeters per minute. How fast is the radius of the the ballon increasing at the instant when the radius is 60 cm? (The volume of a sphere is $V_{sphere} = \frac{4}{3}\pi r^3$)

$V = \frac{4}{3}\pi r^3$

Find $\frac{dr}{dt}$

$\frac{dv}{dt} = +800 \text{ cm}^3/\text{min}$

$\frac{d}{dt}[v] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$

$1\,\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$(800) = 4\pi(60)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{800}{4\pi(60)^2}$ cm/min

#8b. An air traffic controller spots two planes at the same altitude converging on a point as they fly at right angles to each other. One plane is 225 miles from the point moving at 450 miles per hour. The other plane is 300 miles from the point moving at 600 miles per hour. At what rate is the distance between the planes decreasing?

$x^2 + y^2 = s^2$

$\frac{d}{dt}[x^2] + \frac{d}{dt}[y^2] = \frac{d}{dt}[s^2]$

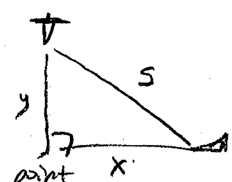
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = s\frac{ds}{dt}$

$(225)(-450) + (300)(-600) = (375)\frac{ds}{dt}$

$\frac{ds}{dt} = \frac{(225)(-450) + (300)(-600)}{375}$ miles/hr

$= -750$ miles/hr



Find $\frac{ds}{dt}$

$\frac{dx}{dt} = -450$ mi/hr

$(x = 225 \text{ mi})$

$\frac{dy}{dt} = -600$ mi/hr

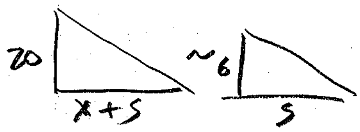
$(y = 300 \text{ mi})$

$s = \sqrt{225^2 + 300^2} = 375$

#9b. A 6-foot tall man walks at a rate of 5 feet per second toward a lamppost that is 20 feet tall.
a) When he is 10 feet from the base of the light, at what rate is the tip of his shadow moving?
b) When he is 10 feet from the base of the light, at what rate is the length of his shadow changing?

do b) 1st!

Find $\frac{ds}{dt}$, $\frac{dx}{dt} = -5$ ft/sec



$\frac{x+s}{s} = \frac{20}{6} = \frac{10}{3}$

$3(x+s) = 10s$

$3x + 3s = 10s$

$3x = 7s$

$\frac{d}{dt}[3x] = \frac{d}{dt}[7s]$

$3\frac{dx}{dt} = 7\frac{ds}{dt}$

$3(-5) = 7\frac{ds}{dt} \rightarrow \frac{ds}{dt} = \frac{-15}{7}$ ft/sec

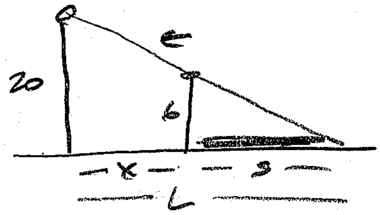
a) Find $\frac{dL}{dt}$, $\frac{dx}{dt} = -5$ ft/sec

$L = x + s$

$\frac{d}{dt}[L] = \frac{d}{dt}[x] + \frac{d}{dt}[s]$

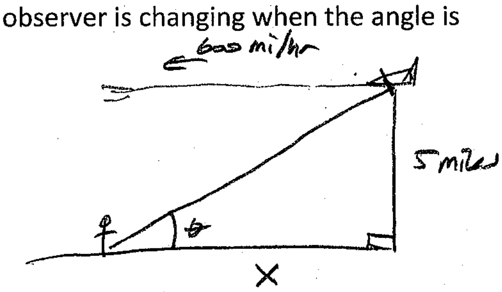
$\frac{dL}{dt} = \frac{dx}{dt} + \frac{ds}{dt}$

$\frac{dL}{dt} = (-5) + \left(-\frac{15}{7}\right)$ ft/sec



$\frac{dx}{dt} = -5 \frac{ft}{sec}$

#10b. An airplane flies at an altitude of 5 miles toward a point directly over an observer. The speed of the plane if 600 miles per hour. Find the rate at which the angle of elevation of the plan from the observer is changing when the angle is $\frac{\pi}{3}$ radians.



Find $\frac{d\theta}{dt}$

$\frac{dx}{dt} = -600 \frac{miles}{hr}$

(instant)

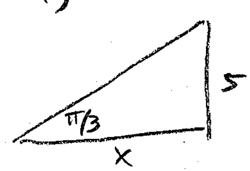
$\tan\theta = \frac{5}{x}$

$\frac{d}{dt}[\tan\theta] = \frac{d}{dt}[5x^{-1}]$

$\sec^2\theta \frac{d\theta}{dt} = -5x^{-2}\frac{dx}{dt} = \frac{-5}{x^2}\frac{dx}{dt}$

$\left(\sec\left(\frac{\pi}{3}\right)\right)^2 \frac{d\theta}{dt} = \frac{-5}{\left(\frac{5}{\tan\pi/3}\right)}(-600)$

$\frac{d\theta}{dt} = \frac{(-5)(-600)}{\left(\frac{5}{\tan\pi/3}\right)\left(\sec\left(\frac{\pi}{3}\right)\right)^2}$ radians/hr ← could leave like this on an AP FRQ :)

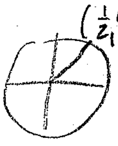


$\tan\frac{\pi}{3} = \frac{5}{x}$

$x = \frac{5}{\tan\pi/3}$

...but if on MCQ, might need to simplify:

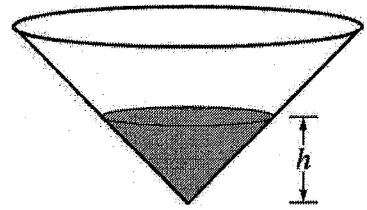
$\frac{d\theta}{dt} = \frac{3000}{\left(\frac{5}{\sqrt{3}}\right)(2)^2} = 150\sqrt{3} \approx 259.808$ radians/hr



$\tan\pi/3 = \frac{\frac{\sqrt{3}}{2}}{1/2} = \sqrt{3}$, $\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

Additional, more challenging, related rates problems...

#11. The height of the water in a conical storage tank is modeled by a differentiable function h, where $h(t)$ is measured in meters and t is measured in hours.



At time $t = 0$, the height of the water in the tank is 25 meters. The height is changing at at the rate $h'(t) = 2 - \frac{24e^{-0.025t}}{t+4}$ meters per hour for $0 \le t \le 24$

When the height of the water in the tank is h meters, the volume of water is $V = \frac{1}{3}\pi h^3$.

At what rate is the volume of water changing at time $t = 0$?

Find $\frac{dv}{dt}$

$$\left.\frac{dh}{dt}\right|_{t=0} = h'(0) = 2 - \frac{24(1)}{4} = 2-6$$

$$= -4 \text{ m/hr}$$

$$V = \frac{1}{3}\pi h^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{1}{3}\pi h^3\right]$$

$$\frac{dv}{dt} = \pi h^2 \frac{dh}{dt}$$

$$\frac{dv}{dt} = \pi (25)^2(-4) \ \frac{m^3}{hr}$$

#12. A man starts walking north at 4 ft/s from a point P. Five minutes later, a woman starts walking south at 5 ft/s from a point 500 due east of P. At what rate are the people moving apart 15 minutes after the woman starts walking?

Analyze man's movement:

$5\,\text{min}\left(\frac{60\,\text{sec}}{\text{min}}\right) = 300\,\text{sec}$

$(15\,\text{min} = 900\,\text{sec})$

when women starts walking, man has already walked

$300\,\text{sec}\left(\frac{4\,\text{ft}}{\text{sec}}\right) = 1200\,\text{ft}$

north of P.

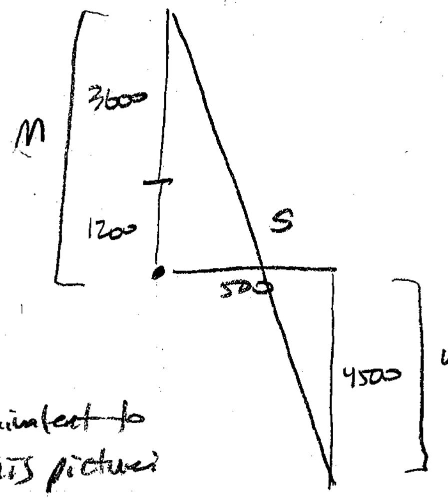
He continues for 15 more minutes:

$900\,\text{sec}\left(\frac{4\,\text{ft}}{\text{sec}}\right) = 3600$ more ft north

analyze women's movement:

In 15 minutes, the women walks:

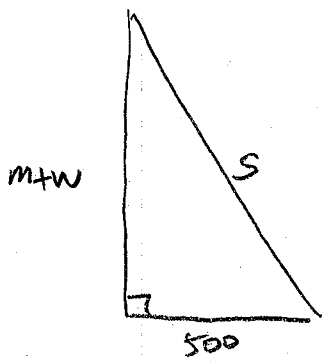
$900\,\text{sec}\left(\frac{5\,\text{ft}}{\text{sec}}\right) = 4500\,\text{ft}$ South



$\frac{dm}{dt} = +4$ ft/sec

$\frac{dw}{dt} = +5$ ft/sec

equivalent to this picture:



$$s^2 = (m+w)^2 + 500^2$$

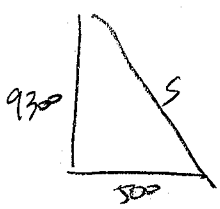
$$s^2 = m^2 + 2mw + w^2 + 500^2$$

$$\frac{d}{dt}[s^2] = \frac{d}{dt}[m^2] + 2m\frac{d}{dt}[w] + w\frac{d}{dt}[2m] + \frac{d}{dt}[w^2] + \frac{d}{dt}[500^2]$$

$$2s\frac{ds}{dt} = 2m\frac{dm}{dt} + 2m\frac{dw}{dt} + 2w\frac{dm}{dt} + 2w\frac{dw}{dt} + 0$$

(drop the 2s)

(instant)



$s = \sqrt{9300^2 + 500^2}$

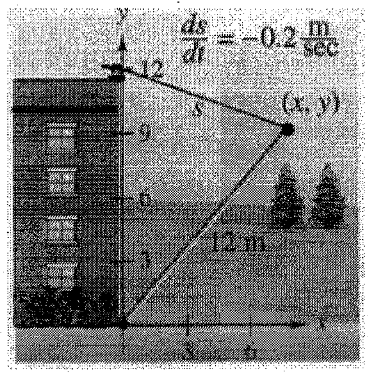
$s = 9313.431162$

$$(9313.431162)\frac{ds}{dt} = (4800)(4) + (4800)(5) + (4500)(4) + (4500)(5)$$

$$(9313.431162)\frac{ds}{dt} = 83700$$

$$\frac{ds}{dt} = \frac{83700}{9313.431162} = 8.987 \text{ ft/sec}$$

#13. A winch at the top of a 12-meter building pulls a pipe of the same length to a vertical position (see figure). The winch pulls in rope at a rate of -0.2 meters per second. Find the rate of vertical change and the rate of horizontal change at the end of the pipe when $y = 6$.



2 triangles:

$$x^2 + (12-y)^2 = s^2$$

$$\frac{d}{dt}[x^2] + \frac{d}{dt}[(12-y)^2] = \frac{d}{dt}[s^2]$$

$$2x\frac{dx}{dt} + 2(12-y)(-1)\frac{dy}{dt} = 2s\frac{ds}{dt}$$

(this one first)

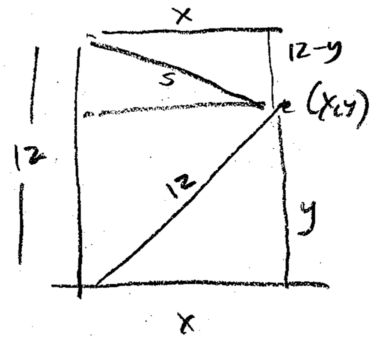
$$x^2 + y^2 = 12^2$$

$$\frac{d}{dt}[x^2] + \frac{d}{dt}[y^2] = \frac{d}{dt}(12^2)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

(substitute)



$$2x\frac{dx}{dt} - 2(12-y)\left(-\frac{x}{y}\frac{dx}{dt}\right) = 2s\frac{ds}{dt}$$

$$2(\sqrt{108})\frac{dx}{dt} - 2(12-6)\left(\frac{-\sqrt{108}}{6}\right)\frac{dx}{dt} = 2(12)(-0.2)$$

$$2\sqrt{108}\frac{dx}{dt} + 2\sqrt{108}\frac{dx}{dt} = 24(-0.2)$$

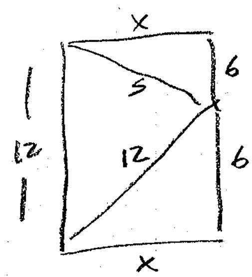
$$4\sqrt{108}\frac{dx}{dt} = -4.8 \quad \boxed{\frac{dx}{dt} = \frac{-4.8}{4\sqrt{108}} = -0.115 \text{ m/sec}}$$

then $\boxed{\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = \frac{-(\sqrt{108})}{6}\left(\frac{-4.8}{4\sqrt{108}}\right) = 0.2 \text{ m/sec}}$

Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$

$\frac{ds}{dt} = -0.2$ m/s

(instant):



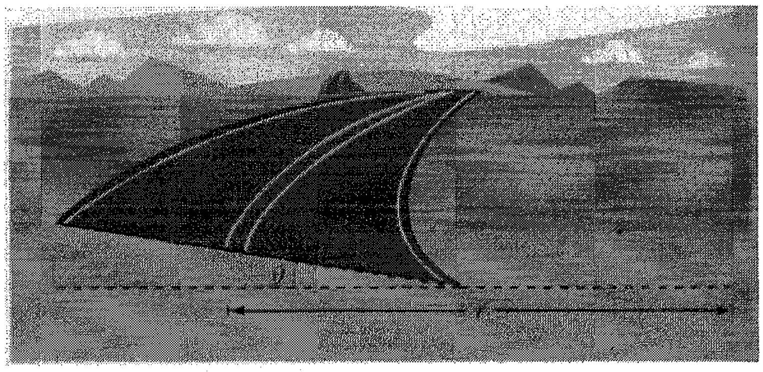
$$x^2 + 6^2 = 12^2$$

$$x = \sqrt{12^2 - 6^2} = \sqrt{108}$$

$$s^2 = 6^2 + (\sqrt{108})^2$$

$$s = \sqrt{36 + 108} = 12$$

#14. Cars on a certain roadway travel of a circular arc of radius r. In order not to rely on friction alone to overcome the centrifugal force, the road is banked at an angle of magnitude θ from the horizontal (see figure).



The banking angle must satisfy the equation $rg\tan(\theta) = v^2$, where v is the velocity of the cars and g = 32 ft/s^2 is the acceleration due to gravity.

Find the relationship between the related rates $\frac{dv}{dt}$ and $\frac{d\theta}{dt}$.

(for any specific point on the curve, r is just a constant)
($\&$ g is also a constant)

$$rg\tan(\theta) = v^2$$

$$\frac{d}{dt}[rg\tan(\theta)] = \frac{d}{dt}[v^2]$$

$$rg\sec^2\theta \frac{d\theta}{dt} = 2v\frac{dv}{dt}$$

$g = 32$ ft/s^2
and $v = \sqrt{rg\tan\theta}$

so $$\frac{dv}{dt} = \frac{rg\sec^2\theta}{2v}\frac{d\theta}{dt}$$

$$\boxed{\frac{dv}{dt} = \frac{r(32)\sec^2\theta}{2\sqrt{r32\tan\theta}}\frac{d\theta}{dt}}$$