AP Calculus BC – Unit 3, Part 2 Required Practice **Name:** answers

3.5 – Required Practice

#1. A rectangular storage container with an open top is to have a volume of 10 m³. The length of its base is twice the width. Material for the base costs $10 per m² and material for the sides costs $6 per m². Find the cost of materials for the cheapest container.

minimum cost = $163.541

#2. Find the point on the parabola $x+y^2=0$ that is closest to the point $(0,-3)$.

point closest is $(-1,-1)$

#3. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 245,000 square meters, and no fencing is needed along the one side which is along the river. What dimensions will require the least amount of fencing?

700m X 350m

#4. Find two positive numbers where the sum of the first number plus three times the second number is a minimum, if the product of the numbers must be 147.

21, 7

#5. The rate (in mg carbon/m³/hr) at which photosynthesis takes place for a species of phytoplankton is modeled by the function: $P = \dfrac{100I}{I^2 + I + 4}$, where I is the light intensity measured in thousands of foot-candles and P is the photosynthesis rate. For what light intensity is P a maximum?

$I = 2$ (thousand) foot-candles

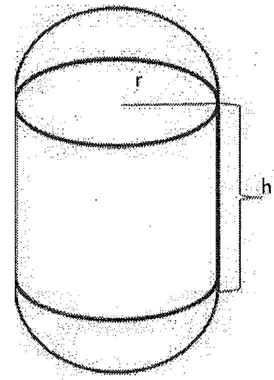
#6. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

maximum $V = 4000\ cm^3$

#7. A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible volume of such a cylinder.

maximum $V = \frac{4\pi}{27} r^2 h$

#8. A solid is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid must be 14 cm^3. Find the radius of the cylinder that requires minimum surface area.



Formulas you may need...

$$A_{\text{lateral surface, cylinder}} = 2\pi r h$$

$$A_{\text{surface, sphere}} = 4\pi r^2$$

$$V_{\text{cylinder}} = \pi r^2 h$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$r = \sqrt[3]{\frac{21}{2\pi}} \approx 1.495$ cm

($h = 0$ optimum shape is a sphere)

3.6 – Required Practice

#1. A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$ is given by $v(t) = -6t^2 + 30t - 36$. At time $t = 1$, the position of the particle is $x(1) = 4$.

a) Find the acceleration of the particle at any time, t.

b) Find the minimum acceleration of the particle.

c) When is the particle at rest?

d) When is the particle moving to the left?

e) Is the particle speeding up or slowing down at $t = 1$?

a) $a(t) = -12t + 30$

b) minimum acceleration is -30

c) particle at rest at $t = 2$ and $t = 3$

d) particle moving left for $0 \le t < 2$ and $3 < t \le 5$

e) the particle is slowing down at $t = 1$ because the signs of $v(1)$ and $a(1)$ are different.

#2. A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$ is given by $v(t) = 3(t-1)(t-3)$. At time $t = 2$, the position of the particle is $x(2) = 0$.

a) Find the acceleration of the particle at any time, t.

b) Find the minimum acceleration of the particle.

c) When is the particle at rest?

d) When is the particle moving to the left?

e) Is the particle speeding up or slowing down at $t = 2$?

a) $a(t) = 6t - 12$

b) The minimum acceleration is -12

c) particle is at rest at $t=1$ and $t=3$

d) particle is moving left for $1 < t \le 3$

e) The particle is neither speeding up nor slowing down at $t=2$.

#3. The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and the time t is measured in seconds. Selected values of $v(t)$ are given in the table.

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

Estimate the acceleration of the particle at $t = 36$ seconds.

$a(36) = \frac{3}{2}$ meters per second per second

#4. If a ball is thrown vertically upward with a velocity of 80 ft/s, then its height after t seconds is given by $s(t) = 80t - 15t^2$.

a) Find the velocity of the ball, $v(t)$.

b) What is the maximum height reached by the ball?

c) What is the velocity of the ball when it is 96 ft. above the ground, on its way up? ...on its way down?

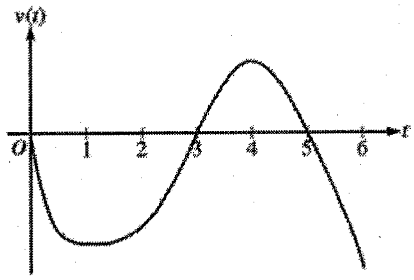
a) $v(t) = 80 - 30t$

b) max height is 106.667 ft

c) 25.298 ft/sec on the way up

−25.298 ft/sec on the way down

#5. A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 6$, is given by a differentiable function v whose graph is shown.



Graph of v

During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

acceleration is negative for $0<t<3$ and $4<t<6$

#6. If a ball is given a push so that is has an initial velocity of 5 meters per second down a certain inclined plane, then the distance it has rolled after t seconds is $s(t) = 5t + 3t^2$.

a) Find the velocity of the ball after 2 seconds.
b) How long does it take for the velocity to reach 35 meters per second?
c) What is the acceleration of the ball at 3 seconds?

a) $v(2) = 17$ m/s

b) $t = 5$ sec

c) $a(3) = 6$ m/s^2

#7. If a ball is given a push so that is has an initial velocity of 5 meters per second down a certain inclined plane, then the distance it has rolled after t seconds is $s(t) = 5t + 3t^2$.

a) Find the velocity of the ball after 2 seconds.
b) What is the average rate of change of the position of the ball over the interval $0 \le t \le 3$ seconds?
b) How long does it take for the velocity to reach 35 meters per second?
c) What is the acceleration of the ball at 3 seconds?

a) $v(2) = 17$ m/s

b) avg rate of change of position = 21 m/s

c) $t = 5$ sec

d) $a(3) = 6$ m/s^2

#8. A particle moves according to a law of motion where t is measured in seconds and s in feet:

$s(t) = 4t^3 - 8t^2 + 3t$ for $t \geq 0$

(Justify all responses)

a) Find the velocity at time t.

b) What is the velocity after 1 second?

c) When is the particle at rest?

d) When is the particle moving in the negative direction?

e) Find the acceleration of the particle at t = 1.5 seconds.

f) Find all the values of t in the interval $0 \leq t \leq 4$ for which the speed of the particle is $1\ \frac{ft}{s}$.

g) At what times in the interval $0 \leq t \leq 4$ is the particle changing direction?

h) Is the particle moving the right or the left at time t = 0.4 seconds? ...at time t = 1.3 seconds?

i) Is the particle speeding up or slowing down at time t = 0.4 seconds? ...at time t = 1.3 seconds?

a) $v(t) = 12t^2 - 16t + 3$

b) $v(1) = -1$ ft/sec

c) particle at rest at $t = 0.226$ sec and $t = 1.107$ sec

d) particle moving in negative direction for $0.226 < t < 1.107$ sec

e) $a(1.5) = 20$ ft/s^2

f) speed is 1 ft/s at $t = 0.140$ sec and $t = 1.194$ sec.

g) particle changes direction at $t = 0.226$ sec and $t = 1.108$ sec

h) particle moving left at $t = 0.4$ sec
particle moving right at $t = 1.3$ sec

i) particle is speeding up at both $t = 0.4$ sec and $t = 1.3$ sec because the signs of $v(t)$ and $a(t)$ at these times are the same.

3.7 – Required Practice

#1. Given $f(x) = x^2 + 2$, write the equation of a tangent line to the curve at $x = 1$, and then use this tangent line to approximate the value of the function at $x = 2.2$

$(y-3) = 2(x-1)$

$f(2.2) \approx 5.4$

#2. Does the approximation in #1 over- or under-estimate the true value of $f(2.2)$?

5.4 is an underestimation of f(2.2)

#3. Given $f(x) = 3x - x^2 + 4$

a) Write the equation of a tangent line to the curve at $x = 2$.

b) Use this tangent line to approximate the value of the function at $x = 3$.

c) Does this approximation over- or under-estimate the true value of $f(3)$?

a) $(y-6) = -(x-2)$

b) $f(3) \approx 5$

c) 5 is an overestimation of f(3)

#4. a) Use the tangent line at $x = 0$ to approximate the function $f(x) = \sin(x)$

b) Use this tangent line to approximate $\sin(0)$, $\sin(0.1)$, and $\sin(0.5)$

c) For each of these approximations, what is the 'error' (difference between the actual function value and the approximated value?

a) $(y-0) = 1(x-0)$

b) $f(0) \approx 0$

$f(0.1) \approx 0.1$

$f(0.5) \approx 0.5$

c)

actual	approx	error
$\sin(0) = 0$	0	0
$\sin(0.1) = 0.0998334166$	0.1	$1.666 \cdot 10^{-4} = 0.0001666$
$\sin(0.5) = 0.4794255386$	0.5	0.02055

$\left.\begin{matrix} \\ \\ \end{matrix}\right]$ overestimates

#5. Use a linear approximation to estimate $\sqrt{36.1}$

$\sqrt{36.1} \approx 6.00833$

For #6 and #7...

a) Find the linearization of the function (find a tangent line) at the given 'easy to compute' x-value.

b) Use the tangent line linearization to approximate the value of the 2nd x-value.

c) Is this approximation an over- or under-estimation of the actual function value?

#6. $f(x) = x^4 + 3x^2$ *linearization at* $x = -1$

Approximate $f(-1.5)$

a) $(y-4) = -10(x+1)$

b) $f(-1.5) \approx 9$

c) 9 is an underestimate of $f(-1.5)$

#7. $f(x) = \cos(x)$ *linearization at* $x = \frac{\pi}{2}$

Approximate $f(2)$

a) $(y-0) = -(x-\frac{\pi}{2})$

b) $f(2) \approx \frac{\pi}{2} - 2$

c) $\frac{\pi}{2} - 2$ is an overestimate of $f(2)$

#8. The function f is twice differentiable with $f(2)=1,\ f'(2)=4,\ and\ f''(2)=3.$

a) What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

b) Is this approximation an over- or under-estimation of $f(1.9)$? Justify your result.

a) $f(1.9) \approx 0.6$

b) 0.6 is an underestimation of $f(1.9)$

#9. The function g is defined for $x > 0$, with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, *and* $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

a) Find all values of x in the interval $0.12 \leq x \leq 1$ at which the graph of g has a horizontal tangent line.

b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

c) Write an equation for the line tangent to the graph of g at $x = 1$.

d) Does the line tangent to the graph of g at $x = 1$ lie above or below the graph of g for $0.3 < x < 1$? Why?

a) horiz. tangent at $x = 0.163$ and $x = 0.359$

b) g concave down for $(0.129, 0.223)$

c) $(y - 2) = 0.90292274(x - 1)$

d) line tangent lies below the graph of g at $x = 1$

3.8 – Required Practice

#1. The altitude of a triangle is increasing at a rate of 1 cm/minute while the area of the triangle is increasing at a rate of 2 cm^2/minute. How fast is the length of the base of the triangle changing when the altitude is 10 cm and the area is 100 cm^2?

$$\frac{db}{dt} = -\frac{8}{5} \text{ cm/min}$$

#2. A plane flying horizontally at an altitude of 1 mile and a speed of 500 miles/hr passes directly over a radar station. Find the rate at which the distance from the plane to the station is changing when it is 2 miles away from the station.

$$\frac{ds}{dt} = \frac{500\sqrt{3}}{2} \ \frac{\text{miles}}{\text{hr}}$$

#3. Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/minute at the same time that water is being pumped into the tank at a constant rate. The tank has a height of 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/minute, find the rate at which water is being pumped into the tank when the height of the water is 2 m.

$C = 289{,}252{,}680$ cm^3/min

#4. A child throws a stone into a still pond, causing a circular ripple to spread. If the radius of the circle increases at a constant rate of 0.5 feet per second, how fast is the area of the ripple increasing when the radius of the ripple is 30 feet?

$\frac{dA}{dt} = 2\pi(30)(0.5) = 94.248$ ft^2/sec

#5. A 20-foot long ladder is leaning against a wall. If the foot of the ladder is slipping away from the wall at a rate of 7 ft/s, how fast is the top of the ladder (the point where the ladder touches the wall) moving downward when the foot of the ladder is 4 ft from the wall?

$$\frac{dy}{dt} = \frac{-(4)(7)}{\sqrt{384}} \text{ ft/sec}$$

#6. The radius of a funnel depends upon height according to the function $r = \frac{1}{20}(3+h^2)$ where $0 \le h \le 10$ and both r and h are in inches. The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is h = 3 inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, how fast is the height of the liquid changing?

$$\frac{dh}{dt} = -\frac{2}{3} \text{ in/sec}$$

#7. The radius *r* of a sphere is increasing at a rate of 3 inches per minute. The volume of a sphere is $V_{sphere} = \frac{4}{3}\pi r^3$. Find the rate of change of the volume when the radius of the sphere is 9 inches.

$\frac{dv}{dt} = 4\pi(9)^2(3)$ in^3/min

#8. An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna. When the plane is 10 miles away from the antenna, the radar detects that the distance between the antenna and the airplane is increasing at a rate of 240 miles per hour. What is the speed of the airplane?

Speed of the airplane $= \frac{2400}{\sqrt{75}}$ miles/hr

#9. A 6-foot tall man walks at a rate of 5 feet per second away from a lamppost that is 15 feet tall.
a) When he is 10 feet from the base of the light, at what rate is the tip of his shadow moving?
b) When he is 10 feet from the base of the light, at what rate is the length of his shadow changing?

a) $\frac{ds}{dt} = \frac{10}{3}$ ft/sec

b) $\frac{dL}{dt} = (5 + \frac{10}{3})$ ft/sec

#10. A balloon rises at a rate of 4 meters per second from a point on the ground 50 meters from an observer. Find the rate of change of the angle of elevation of the ballon from the observer when the balloon is 50 meters above the ground.

$\frac{d\theta}{dt} = \frac{(4/50)}{(\sec(\frac{\pi}{4}))^2}$ radians/sec

Test Review for Unit 3 Part 2 Test
(recommend you also review required and extra practice for additional review)
Calculators are allowed on this test

#1. There are 320 yards of fencing available to enclose a rectangular field. How should this fencing be used so that the enclosed area is as large as possible?

#2. A rectangular area of 10,000 m^2 must be enclosed by a fence. What should the dimensions of the rectangle be if we want to enclose this area using the minimum amount of fencing?

#3. A rectangular area of 10,000 m^2 must be enclosed by a fence. One side of the fencing must be a high-security style fence which costs $10/m, and the remaining 3 sides will use lower-cost $6/m fencing. What are the dimensions of the rectangle which will minimize the cost of the fencing required?

#4. A poster is to be produced from a piece of paper with a total paper area of 180 in^2 with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What should the dimensions of the paper be to give the largest printed area?

#5. A particle moves along a line such that its position in meters at time *t* in seconds is given by $s(t) = 2t^3 - 5t^2 + 2t$
(Justify all responses)
a) Find the velocity at time *t*.
b) What is the velocity after 1 second?
c) Find the acceleration at time *t*.
d) Find the acceleration of the particle at time *t* = 4 seconds.
e) When is the acceleration of the particle zero?
f) When is the speed of the particle $1 \frac{m}{s}$?
g) At what times in the interval $0 \le t \le 8$ is the particle changing direction?
h) When is the particle at rest?
i) Is the particle moving the right or the left at time *t* = 0.4 seconds? ... at time *t* = 1.5 seconds?
j) Is the particle speeding up or slowing down at time *t* = 0.4 seconds? ... at time *t* = 1.5 seconds?

For #6, and #7
a) Find the linearization of the function (find a tangent line) at the given 'easy to compute' *x*-value.
b) Use the tangent line linearization to approximate the value of the 2nd *x*-value.
c) Is this approximation an over- or under-estimation of the actual function value?

#6. $f(x) = e^x \cos x$ *linearization at* $x = 0$. *Approximate* $f(0.3)$

#7. $f(x) = \sin(\sin(x))$ *linearization at* $x = \pi$. *Approximate* $f(3)$

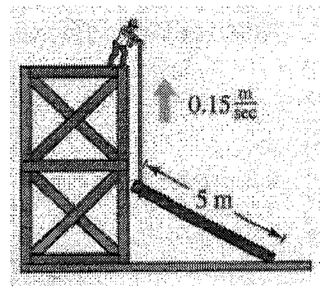
#8. Write the tangent line at $\left(\frac{-\pi}{4}, 1\right)$ for $x^2 + x\arctan(y) = y - 1$

#9. A 25-foot long ladder is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

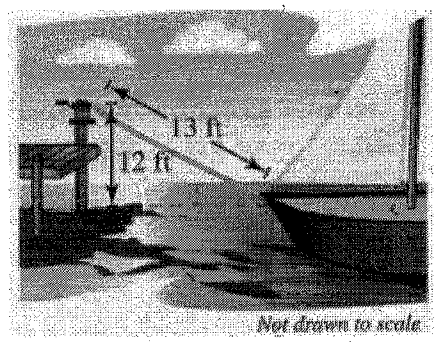
a) How fast is the top of the ladder moving down the wall when its base is 15 feet from the wall?

b) Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall.

#10. A construction worker pulls a 5-meter plank up the side of a building under construction by means of a rope tied to one end of the plank (see figure). Assume the opposite end of the plank follows a path perpendicular to the wall of the building and the worker pulls the rope at a rate of 0.15 meter per second. How fast is the end of the plank sliding along the ground when it is 2.5 meters from the wall of the building?



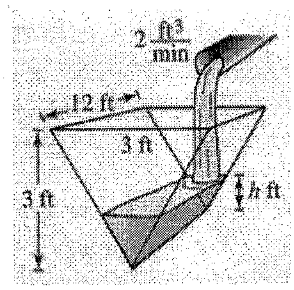
#11. A boat is pulled into a dock by means of a winch 12 feet above the deck of the boat (see figure).



a) Suppose the winch pulls in rope at a rate of 4 feet per second. Determine the speed of the boat when there is 13 feet of rope out.

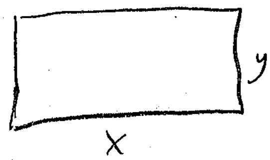
b) Instead, suppose the boat is moving at a constant rate of 4 feet per second along the surface of the water. Determine the speed at which the winch pulls in rope when there is a total of 13 feet of rope out.

#12. A trough is 12 feel long and 3 feet across the top (see figure). Its ends are isosceles triangles with altitudes of 3 feet. Suppose water is being pumped into the trough at 2 cubic feet per minute. How fast is the water level rising when the depth h is 1 foot?



Test Review for Unit 3 Part 2 Test — Solutions

#1



objective

max $A = xy$

$A = x(160 - x)$

$A = 160x - x^2$

$A' = 160 - 2x = 0$

$2x = 160$

$x = 80$ yds

$y = 160 - 80 = 80$ yds

constraint

$P = 320$

$2x + 2y = 320$

$x + y = 160$

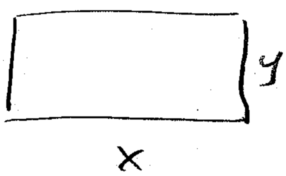
$y = 160 - x$

verify max: $A'' = -2 < 0$

$\therefore$ A is concave down

$\therefore$ this is a max area

#2



objective

min $P = 2x + 2y$

$P = 2x + 2\left(\frac{10000}{x}\right)$

$P = 2x + 20000x^{-1}$

$P' = 2 - 20000x^{-2} = 0$

$2 = \frac{20000}{x^2}$

$x^2 = 10000$

$x = 100$ m

$y = \frac{10000}{100} = 100$ m

constraint

$A = 10000$ m^2

$xy = 10000$

$y = \frac{10000}{x}$

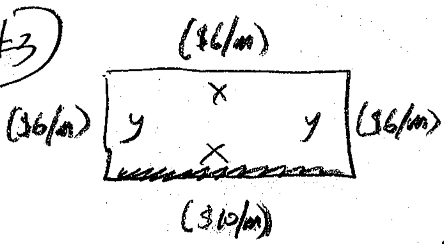
verify min:

$P'' = 40000x^{-3} = \frac{40000}{x^3} > 0$

$\therefore$ P is concave up

$\therefore$ this is a min amount of fence

#3



objective

$$\min C = 10x + 6y + 6x + 6y$$

$$C = 16x + 12y$$

$$C = 16x + 12\left(\frac{10000}{x}\right)$$

$$C = 16x + 120000x^{-1}$$

$$C' = 16 - 120000x^{-2} = 0$$

$$16 = \frac{120000}{x^2}$$

$$x^2 = \frac{120000}{16}, \quad x = \sqrt{\frac{120000}{16}} = 86.6025 \text{ m}$$

$$y = \frac{10000}{86.6025} = 115.470 \text{ m}$$

constraint

$$A = 10000 \text{ m}^2$$

$$xy = 10000$$

$$y = \frac{10000}{x}$$

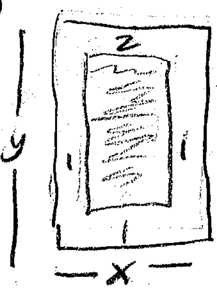
verify min:

$$C'' = 240000x^{-3} > 0$$

$\therefore$ C is concave up

$\therefore$ this is a min cost

#4



objective

$$\max A_{(print)} = (x-2)(y-3)$$

$$A = (x-2)\left(\frac{180}{x} - 3\right)$$

$$A = 180 - 360x^{-1} - 3x + 6$$

$$A' = 360x^{-2} - 3 = 0$$

$$3 = \frac{360}{x^2}$$

$$x^2 = 120$$

$$x = \sqrt{120} = 10.9545 \text{ in}$$

$$y = \frac{180}{\sqrt{120}} = 16.4317 \text{ in}$$

constraint

$$A_{paper} = 180 \text{ in}^2$$

$$xy = 180$$

$$y = \frac{180}{x}$$

verify max:

$$A'' = -720x^{-3}$$

$$= \frac{-720}{x^3} < 0$$

$\therefore$ A is concave down

$\therefore$ this is a max area

#5 $s(t) = 2t^3 - 5t^2 + 2t$

(a) $\boxed{v(t) = s'(t) = 6t^2 - 10t + 2}$

(b) $\boxed{v(1) = -2 \text{ m/s}}$

(c) $\boxed{a(t) = v'(t) = 12t - 10}$

(d) $\boxed{a(4) = 38 \text{ m/s}^2}$

(e) $a(t) = 0$ when $12t - 10 = 0$, at $12t = 10$
$\boxed{t = \frac{10}{12} \text{ sec}}$

(f) speed $= |v(t)| = 1$ m/s when $v(t) = 1$ or $v(t) = -1$
which occurs at $\boxed{t = 0{,}109 \text{ sec},\ t = 1{,}560 \text{ sec},\ t = 0.392 \text{ sec and } t = 1{,}274 \text{ sec}}$

(g) The particle changes direction when the sign of $v(t)$ changes
which occurs at $\boxed{t = 0{,}232 \text{ sec and } t = 1{,}434 \text{ sec}}$

(h) The particle is at rest when $v(t) = 0$ which occurs at $\boxed{t = 0{,}232 \text{ sec and } t = 1{,}434}$

(i) $v(0.4) = -1{,}04 < 0$ ∴ the particle is moving left at $t = 0.4$ sec
$v(1{,}5) = 0.5 > 0$ ∴ the particle is moving right at $t = 1{,}5$ sec

(j) $v(0.4) = -1{,}04 < 0$
$a(0.4) = -5{,}2 < 0$
$v(1.5) = 0{,}5 > 0$
$a(1.5) = 8 > 0$

∴ The particle is speeding up at $t = 0.4$ sec and $t = 1.5$ sec because at both these times the signs of $v(t)$ and $a(t)$ are the same.

#6 $f(x)=e^x\cos x$ at $x=0$ $\approx f(0.3)$

a) $f(0)=e^0\cos 0=1(1)=1$ $\quad (0,1)$ $m=1$

$f'(x)=e^x(-\sin x)+\cos x\, e^x$ $\quad (y-1)=1(x-0)$

$f'(0)=(1)(0)+(1)(1)=1$

b) $y-1=1(0.3-0)=0.3$ $\therefore f(0.3)\approx 1.3$

c) $f''(x)=e^x(-\cancel{\cos x})+(-\sin x)e^x+\cancel{\cos x e^x}+e^x(-\sin x)$

$=-2e^x\sin x$

$f''(0)=-2(1)(0)=0$ (inconclusive)

try $f''(0.3)=-0.798<0$ $\therefore f$ is concave down in this region

$\therefore 1.3$ is an over-estimate of $f(0.3)$

#7 $f(x)=\sin(\sin(x))$ at $x=\pi$ $\approx f(3)$

a) $f(\pi)=\sin(\sin\pi)=\sin(0)=0$ $\quad (\pi,0)$ $m=-1$

$(y-0)=-(x-\pi)$

$f'(x)=\cos(\sin x)\cos x$

$f'(\pi)=\cos(\sin\pi)\cos\pi=\cos(0)\cos(\pi)=(1)(-1)=-1$

b) $y=-(3-\pi)=\pi-3$ $\therefore f(3)\approx\pi-3$

c) $f''(x)=\cos(\sin x)(-\sin x)+\cos x(-\sin(\sin x)\cos x)$

$f''(\pi)=\cos(\sin\pi)(-\sin\pi)+\cos(\pi)(-1)(\sin(\sin\pi)\cos(\pi))$

$=\cos(0)(0)+(-1)(-1)\sin(0)(-1)$

$=0$ (inconclusive)

try $f''(3)=\cos(\sin(3))(-1)\sin(3)+\cos(3)(-1)\sin(\sin 3)\cos(3)=-0.278<0$

$\therefore f$ is concave down in this region

$\therefore \pi-3$ is an over-estimate of $f(3)$

#8 $x^2 + x\arctan(y) = y - 1$

(implicit differentiation)

$$\frac{d}{dx}[x^2] + x\frac{d}{dx}[\arctan y] + \arctan y\,\frac{d}{dx}[x] = \frac{d}{dx}[y] - \frac{d}{dx}[1]$$

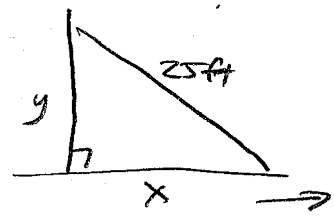
$$2x + x\frac{1}{1+y^2}\frac{dy}{dx} + \arctan y\,(1) = 1\frac{dy}{dx} - 0$$

$$\left(\frac{x}{1+y^2} - 1\right)\frac{dy}{dx} = -2x - \arctan y$$

$$m = \left.\frac{dy}{dx}\right|_{(-\frac{\pi}{4},1)} = \left.\frac{-2x - \arctan y}{\frac{x}{1+y^2} - 1}\right|_{(-\frac{\pi}{4},1)} = \frac{-2(-\frac{\pi}{4}) - \arctan(1)}{\frac{(-\pi/4)}{1+(1)^2} - 1} = \frac{\frac{\pi}{2} - \frac{\sqrt{2}}{2}}{-\frac{\pi}{8} - 1}$$

tangent line $(y-1) = \left(\frac{\frac{\pi}{2} - \frac{\sqrt{2}}{2}}{-\frac{\pi}{8} - 1}\right)\left(x + \frac{\pi}{4}\right)$

#9 a) Find $\frac{dy}{dt}$ when $x = 15ft$

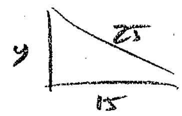


$\frac{dx}{dt} = +2$ ft/sec

$x^2 + y^2 = 25^2$

$\frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}[25^2]$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$



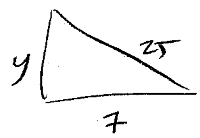
$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$y = \sqrt{25^2 - 15^2} = 20$

$(15)(2) + (20)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-30}{20} = -\frac{3}{2}$ ft/sec

b) Find $\frac{dA}{dt}$ when $x = 7ft$



$y = \sqrt{25^2 - 7^2} = 24$

$A = \frac{1}{2}xy$

$\frac{d}{dt}[A] = \frac{1}{2}x\frac{d}{dt}[y] + y\frac{d}{dt}[\frac{1}{2}x]$

$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}y\frac{dx}{dt}$

$\frac{dA}{dt} = \frac{1}{2}(7)(-\frac{7}{12}) + \frac{1}{2}(24)(2)$ ft^2/sec

$(\approx 21.958$ $ft^2/sec)$

repeat part a but with $x = 7$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

$(7)(2) + (24)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-14}{24} = \frac{-7}{12}$ ft/sec

#10

$$x^2+y^2=5^2$$

$$\frac{d}{dt}[x^2]+\frac{d}{dt}[y^2]=\frac{d}{dt}[5^2]$$

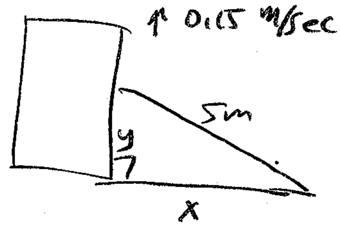
$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

$$(2{,}5)\frac{dx}{dt}+(4{,}330127019)(0{,}15)=0$$

$$\frac{dx}{dt}=-\frac{(4{,}330127019)(0{,}15)}{2{,}5}$$

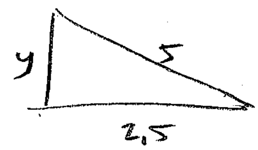
$$\frac{dx}{dt}=-0{,}260 \text{ m/sec}$$



Find $\frac{dx}{dt}$

$\frac{dy}{dt}=+0{,}15$ m/sec

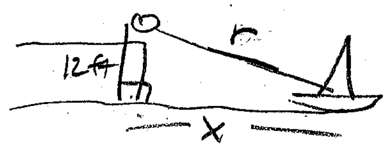
(instant)



$y=\sqrt{5^2-2{,}5^2}\approx 4{,}330127019$

#11

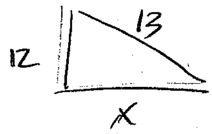
a) Find $\frac{dx}{dt}$, $\frac{dr}{dt} = -4$ ft/sec (when $r = 13$ ft)



$$x^2 + 12^2 = r^2$$

$$\frac{d}{dt}[x^2] + \frac{d}{dt}[12^2] = \frac{d}{dt}[r^2]$$

$$2x\frac{dx}{dt} + 0 = 2r\frac{dr}{dt}$$



$x = \sqrt{13^2 - 12^2} = 5$

$$x\frac{dx}{dt} = r\frac{dr}{dt}$$

$(5)\frac{dx}{dt} = (13)(-4)$, $\frac{dx}{dt} = \frac{13(-4)}{5}$ ft/sec

b) Find $\frac{dr}{dt}$, $\frac{dx}{dt} = -4$ ft/sec (when $r = 13$ ft)

$$x^2 + 12^2 = r^2$$

$\vdots$

$$x\frac{dx}{dt} = r\frac{dr}{dt}$$

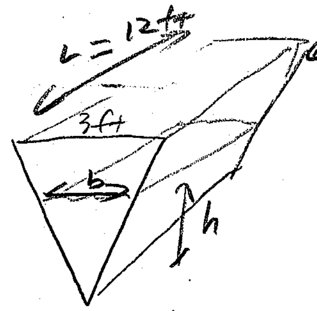
$(5)(-4) = (13)\frac{dr}{dt}$, $\frac{dr}{dt} = \frac{(-4)(5)}{13}$ ft/sec

#12

$V = \frac{1}{2}bhL$

$V = \frac{1}{2}bh(25)$

don't have a derivative for $\frac{db}{dt}$ so we need to substitute out b!



$L = 12$ ft

water in 2 ft³/min

3 ft

3 ft

b

h

Find $\frac{dh}{dt}$

$\frac{dV}{dt} = +2$ ft³/min

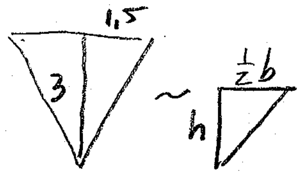
$V = \frac{25}{2}h^2$

$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{25}{2}h^2\right]$

$\frac{dV}{dt} = 25h\frac{dh}{dt}$

$(2) = 25(1)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{2}{25}$ ft/min



1.5

3

$\sim$

$\frac{1}{2}b$

h

$\frac{h}{\frac{1}{2}b} = \frac{3}{3/2}$

$\frac{3}{2}h = \frac{1}{2}b(3)$

$3h = 3b$

$b = h$