AP Calculus BC – Unit 4, Part 1 Extra Practice

4.1 – Extra Practice

Evaluate the indefinite integral.

#7b. $\int\left(8x^3+\frac{1}{2x^2}\right)dx$

$8\int x^3dx+\frac{1}{2}\int x^{-2}dx$

$\boxed{\frac{8x^4}{4}+\frac{1}{2}\frac{x^{-1}}{-1}+c = 2x^4-\frac{1}{2x}+C}$

#8b. $\int 5\,dt$

$\boxed{5t+c}$

#8b. $\int 2x^{-3}\,dx$

$2\int x^{-3}dx$

$\boxed{\frac{2x^{-2}}{-2}+c = -\frac{1}{x^2}+c}$

#9b. $\int\frac{1}{x\sqrt{x}}\,dx$

$\int x^{-3/2}dx$

$\boxed{\frac{x^{-1/2}}{(-1/2)}+c = \frac{-2}{\sqrt{x}}+c}$

#10b. $\int\frac{1}{(3x)^2}\,dx = \int\frac{1}{9x^2}dx$

$=\frac{1}{9}\int x^{-2}dx$

$\boxed{=\frac{1}{9}\frac{x^{-1}}{-1}+c = \frac{-1}{9x}+c}$

#11b. $\int\left(x^2+7\right)dx$

$=\int x^2dx+\int 7dx$

$=\boxed{\frac{x^3}{3}+7x+C}$

Evaluate the indefinite integral.

#12b. $\int \frac{x^4-3x^2+5}{x^4}\,dx$

$\int \frac{x^4}{x^4}dx - 3\int \frac{x^2}{x^4}dx + 5\int \frac{1}{x^4}dx$

$\int x^3 dx - 3\int x^{-2}dx + 5\int x^{-4}dx$

$$\boxed{\begin{array}{l} \frac{x^4}{4} - 3\frac{x^{-1}}{-1} + 5\frac{x^{-3}}{-3} + C \\ \frac{1}{4}x^4 + \frac{3}{x} - \frac{5}{3x^3} + C \end{array}}$$

#13b. $\int (4t^2+3)^2\,dx = \int (4t^2+3)(4t^2+3)\,dt$

$\int (16t^4 + 24t^2 + 9)\,dt$

$16\int t^4 dt + 24\int t^2 dt + \int 9\,dt$

$$\boxed{\begin{array}{l} 16\frac{t^5}{5} + 24\frac{t^3}{3} + 9t + C \\ \frac{16}{5}t^5 + 8t^3 + 9t + C \end{array}}$$

#14b. $\int \sec(y)(\tan(y) - \sec(y))\,dy$

$\int \sec(y)\tan(y)\,dy - \int \sec^2(y)\,dy$

$$\boxed{\sec(y) - \tan(y) + C}$$

#15b. $\int (4\theta - \csc^2(\theta))\,d\theta$

$4\int \theta\,d\theta - \int \csc^2(\theta)\,d\theta$

$4\frac{\theta^2}{2} - (-\cot(\theta)) + C$

$$\boxed{2\theta^2 + \cot(\theta) + C}$$

#16b. $\int \left(\frac{4}{x} + \sec^2(x)\right)dx$

$4\int \frac{1}{x}dx + \int \sec^2(x)\,dx$

$$\boxed{4\ln|x| + \tan(x) + C}$$

#17b. Find $f(x)$ given $f'(x)$:

$f'(x) = 10x - 12x^3, \quad f(3) = 2$

(when given an (x,y) pair, always find C)

$f(x) = \int (10x - 12x^3)\,dx$

$f(x) = 10\frac{x^2}{2} - 12\frac{x^4}{4} + C$

$f(x) = 5x^2 - 3x^4 + C$

$2 = 5(3)^2 - 3(3)^4 + C$

$2 = 45 - 243 + C \Rightarrow C = 200$

$$\boxed{f(x) = 5x^2 - 3x^4 + 200}$$

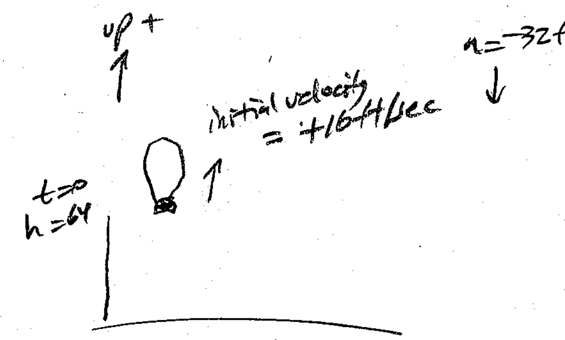
#18b. A hot-air balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant when it is 64 feet above the ground.

a) Find $v(t)$ for the sandbag.

b) Find $h(t)$ where h is the height of the sandbag above the ground.

c) How many seconds after its release will the sandbag strike the ground?

d) At what velocity will the sandbag strike the ground?



a) $a(t) = -32 \text{ ft/s}^2$

$v(t) = \int a(t)\,dt = \int(-32)\,dt = -32t + C$

$v(0) = 16$ ft/sec (sandbag starts carried by balloon upward until release at $t=0$)

$16 = -32(0) + C \Rightarrow C = 16$

$\boxed{v(t) = -32t + 16}$

b) $h(t) = \int v(t)\,dt = \int(-32t+16)\,dt = -16t^2 + 16t + D$

$h(0) = 64$ ft

$64 = -16(0)^2 + 16(0) + D \Rightarrow D = 64$

$\boxed{h(t) = -16t^2 + 16t + 64}$

c) on ground when $h(t) = 0$
$-16t^2 + 16t + 64 = 0$ which occurs at $t = 2.5615528$ (by calculator graph)

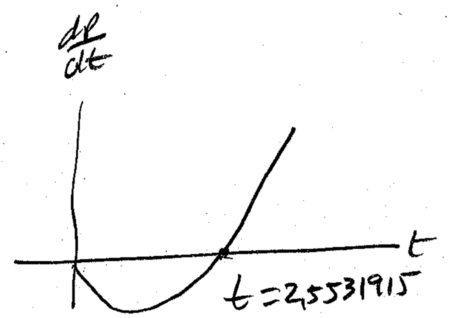
$\boxed{t = 2.562 \text{ sec}}$

d) $v(2.5615528) = \boxed{-65.970 \text{ ft/sec}}$

#19b. The rate of growth of a population P of bacteria is given by $\frac{dP}{dt} = 6t^2 - 30t^{1/4}$, where t is in years. The initial population is 2000 bacteria.

a) Is the bacteria population always increasing? Justify your answer.
b) When is the bacteria population at its lowest point?
c) Find the bacteria population function and use it to estimate the number of bacteria after 10 years.

a) population increases when $\frac{dP}{dt} > 0$



$\boxed{\text{No, because } \frac{dP}{dt} < 0 \text{ for } 0 < t < 2.553 \text{ years}}$

b) The population min occurs when $\frac{dP}{dt} = 0$ $\boxed{\text{at } t = 2.553 \text{ years}}$

(verify min: sign of $\frac{dP}{dt}$ changes from negative to positive at $t = 2.553$)

c) $P(t) = \int (6t^2 - 30t^{1/4})\,dt = 6\frac{t^3}{3} - 30\left(\frac{4}{5}\right)t^{5/4} + C = 2t^3 - 24t^{5/4} + C$

$P(0) = 2000$

$2000 = 2(0)^3 - 24(0)^{5/4} + C \Rightarrow C = 2000$

$\boxed{P(t) = 2t^3 - 24t^{5/4} + 2000}$

$\boxed{P(10) = 3573.213 \text{ bacteria}}$

recommend always rounding to 3 decimal places unless problem states otherwise

4.2 day 1 – Extra Practice

#5b. Selected values of the function $f(x)$ are given in the table:

x	2	3	4	5	6	7	8
f(x)	24	18	12	4	-4	-10	-20

Using three equal subintervals, approximate $\int_2^8 f(x)\,dx$ using...

a) ...left endpoints

b) ...midpoints

c) ...right endpoints

$\text{width} = \frac{8-2}{3} = \frac{6}{3} = 2$

d) How does each compare to the actual value of $\int_2^8 f(x)\,dx$?

a)

interval	x_i (Left)	$f(x_i) \cdot \Delta x$	= area
$[2,4)$	2	$24 \cdot 2$	$= 48$
$[4,6]$	4	$12 \cdot 2$	$= 24$
$[6,8]$	6	$-4 \cdot 2$	$= -8$

$\int_2^8 f(x)dx \approx [(24)(2)+(12)(2)+(-4)(2)]$

≈ 64

b)

interval	x_i (midpoint)	$f(x_i) \cdot \Delta x$	= area
$[2,4)$	3	$18 \cdot 2 =$	36
$[4,6]$	5	$4 \cdot 2 =$	8
$[6,8]$	7	$-10 \cdot 2 =$	-20

$\int_2^8 f(x)dx \approx [(18)(2)+(4)(2)+(-10)(2)]$

≈ 24

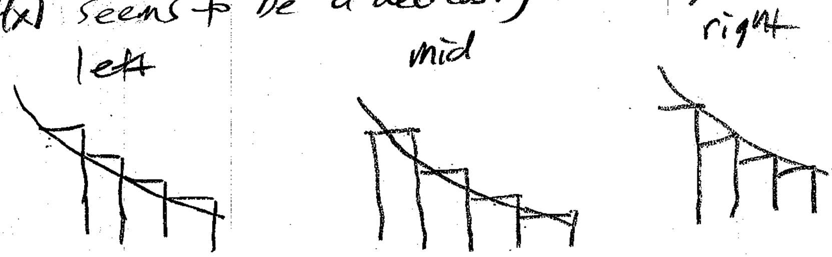
c)

interval	x_i (right)	$f(x_i) \cdot \Delta x$	= area
$[2,4]$	4	$12 \cdot 2$	$= 24$
$(4,6]$	6	$-4 \cdot 2$	$= -8$
$(6,8]$	8	$-20 \cdot 2$	$= -40$

$\int_2^8 f(x)dx \approx [(12)(2)+(-4)(2)+(-20)(2)]$

≈ -24

d) f(x) seems to be a decreasing function, therefor



- left endpoints are overestimating area
- right endpoints are underestimating area
- can't really tell for midpoints, but likely closer to the actual value than either left or right.

#6b. Approximate $\int_0^9 \left(2x^2 - 3x\right) dx$ using the Trapezoidal Rule with 3 equal subintervals.

$$\text{width} = \frac{9-0}{3} = 3$$

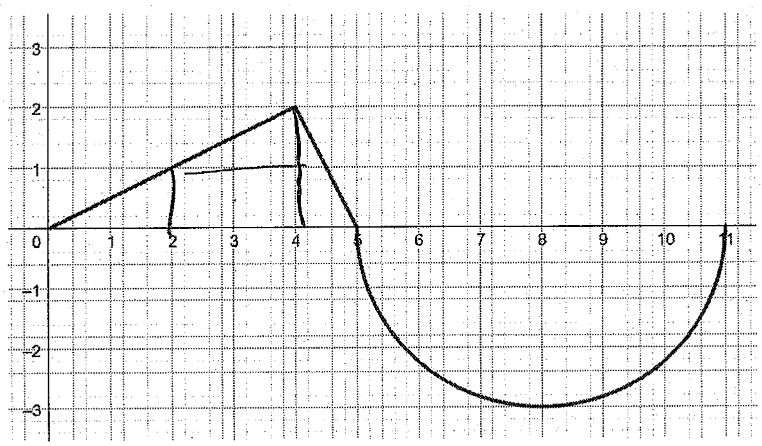
interval	f_L	f_R	Δx	area $= \frac{1}{2}(f_L+f_R)\Delta x$
$[0,3]$	$2(0)^2-3(0)=0$	$2(3)^2-3(3)=9$	3	$\frac{1}{2}(0+9)3 = \frac{27}{2}$
$[3,6]$	$2(3)^2-3(3)=9$	$2(6)^2-3(6)=54$	3	$\frac{1}{2}(9+54)3 = \frac{189}{2}$
$[6,9]$	$2(6)^2-3(6)=54$	$2(9)^2-3(9)=135$	3	$\frac{1}{2}(54+135)3 = \frac{567}{2}$

$$\boxed{\int_0^9 (2x^2-3x)dx \approx \left[\frac{27}{2} + \frac{189}{2} + \frac{567}{2}\right] = \frac{783}{2} = 391.5}$$

4.2 day 2 – Extra Practice

#7b. The graph of f consists of line segments and a semicircle as shown in the figure.

Evaluate each definite integral using geometry.



a) $\int_0^5 f(x)\,dx = \frac{1}{2}(5)(2) = \boxed{5}$

b) $\int_4^8 2f(x)\,dx = 2\int_4^8 f(x)dx = 2\left[\frac{1}{2}(1)(2) - \frac{1}{2}\pi(3)^2\right] = 2\left(1 - \frac{9\pi}{2}\right) = \boxed{2 - 9\pi}$

c) $\int_0^{11} f(x)\,dx = \frac{1}{2}(5)(2) - \frac{1}{2}\pi(3)^2 = \boxed{5 - \frac{9\pi}{2}}$

d) $\int_0^2 (-3f(x))\,dx = -3\int_0^2 f(x)dx = -3\left[\frac{1}{2}(2)(1)\right] = -3(1) = \boxed{-3}$

e) $\int_2^4 f(x)\,dx = (2)(1) + \frac{1}{2}(2)(1) = 2 + 1 = \boxed{3}$

f) $\int_{11}^4 f(x)\,dx = -\int_4^{11} f(x)dx = -\left[\frac{1}{2}(1)(2) - \frac{1}{2}\pi(3)^2\right]$

$= -\left(1 - \frac{9\pi}{2}\right) = \boxed{\frac{9\pi}{2} - 1}$

#8b. $\lim_{n\to\infty} \frac{7}{n}\left[\tan\left(-2+\frac{7}{n}\right)+\tan\left(-2+\frac{14}{n}\right)+\tan\left(-2+\frac{21}{n}\right)+\ldots+\tan\left(-2+\frac{7n}{n}\right)\right]$

is equivalent to what definite integral?

$a=-2$, width $=\frac{b-a}{n}=\frac{7}{n}$

$b-(-2)=7$

$b+2=7$

$b=5$

$\boxed{\int_{-2}^{5}\tan(x)dx}$

#9b. $\lim_{n\to\infty}\frac{2}{n}\left[\left(\frac{2}{n}\right)^5+\left(\frac{4}{n}\right)^5+\left(\frac{6}{n}\right)^5+\ldots+\left(\frac{2n}{n}\right)^5\right]$ is equivalent to what definite integral?

$\frac{2}{n}=\left(0+\frac{2}{n}\right)$ so $a=0$

width $=\frac{b-a}{n}=\frac{2}{n}$

$\frac{b-0}{n}=\frac{2}{n}$, $b=2$

$\boxed{\int_0^2 x^5 dx}$

#10b. If $\int_0^{12} f(x)\,dx=8$, $\int_0^4 f(x)\,dx=6$, *and* $\int_4^{12} g(x)\,dx=-5$, find:

a) $\int_{12}^{4} g(x)\,dx = -\int_4^{12} g(x)dx = -[-5] = \boxed{5}$

b) $\int_4^{12} f(x)\,dx = \int_0^{12} f(x)dx - \int_0^4 f(x)dx = 8-6 = \boxed{2}$

c) $\int_{12}^{4}(3f(x)-2g(x))\,dx = 3\int_{12}^{4} f(x)dx - 2\int_{12}^{4} g(x)dx$

$= 3[5] - 2[5] = \boxed{5}$

d) $\int_4^4 f(x)\,dx = \boxed{0}$

(no width)

4.3 – Extra Practice

Evaluate the definite integral by hand. Then use your calculator (math-9) to verify your answer.

#11b. $\int_{1}^{2}\left(6x^2-3x\right)dx$

$\left[\frac{6x^3}{3}-3\frac{x^2}{2}\right]_1^2$

$\left[2x^3-\frac{3}{2}x^2\right]_1^2$

do not simplify on tests :)

$\boxed{\left[2(2)^3-\frac{3}{2}(2)^2\right]-\left[2(1)^3-\frac{3}{2}(1)^2\right]}$

$(=9.5)$

#12b. $\int_{-2}^{-1}\left(u-\frac{1}{u^2}\right)du = \int_{-2}^{-1}(u-u^{-2})du$

$\left[\frac{u^2}{2}-\frac{u^{-1}}{-1}\right]_{-2}^{-1}$

$\left[\frac{1}{2}u^2+\frac{1}{u}\right]_{-2}^{-1}$

$\boxed{\left[\frac{1}{2}(-1)^2+\frac{1}{(-1)}\right]-\left[\frac{1}{2}(-2)^2+\frac{1}{(-2)}\right]}$

$(=-2)$

#13b. $\int_{-8}^{8}x^{1/3}\,dx$

$\left[\frac{3}{4}x^{4/3}\right]_{-8}^{8}$

$\boxed{\frac{3}{4}(8)^{4/3}-\frac{3}{4}(-8)^{4/3}}$

$(=0)$

#14b. $\int_{0}^{3}\left(t-5^t\right)dt$

$\left[\frac{1}{2}t^2-\frac{5^t}{\ln(5)}\right]_0^3$

$\boxed{\left(\frac{1}{2}(3)^2-\frac{5^3}{\ln(5)}\right)-\left(\frac{1}{2}(0)^2-\frac{5^0}{\ln(5)}\right)}$

(≈ -72.546)

#15b. $\int_{\pi/4}^{\pi/2}\left(2-\csc^2(x)\right)dx$

$\left[2x+\cot(x)\right]_{\pi/4}^{\pi/2}$

$\boxed{\left[2\left(\frac{\pi}{2}\right)+\cot\left(\frac{\pi}{2}\right)\right]-\left[2\left(\frac{\pi}{4}\right)+\cot\left(\frac{\pi}{4}\right)\right]}$

(≈ 0.571)

#16b. $\int_{-\pi/2}^{\pi/2}\left(2t+\cos(t)\right)dt$

$\left[t^2+\sin(t)\right]_{-\pi/2}^{\pi/2}$

$\boxed{\left[\left(\frac{\pi}{2}\right)^2+\sin\left(\frac{\pi}{2}\right)\right]-\left[\left(-\frac{\pi}{2}\right)^2+\sin\left(-\frac{\pi}{2}\right)\right]}$

$(=2)$

Evaluate:

#17b. $\frac{d}{dx}\left[\int_1^x \sqrt[4]{t}\, dt\right]$

$\boxed{\sqrt[4]{x}}$

#18b. $\frac{d}{dx}\left[\int_2^{x^2} \frac{1}{t^3}\, dt\right]$

$\boxed{\frac{1}{(x^2)^3}(2x)}$

4.4 – Extra Practice

#5b. When the lid is opened on a container of honey and the container is turned upside down, honey drips out of the container at a rate of $(t+1)$ ounces per minute. If there are 36 ounces in the container time t = 1 minute, how much honey is left in the container at t = 4 minutes?

$$h(4) = h(1) + \int_1^4 (-(t+1))\,dt$$

"drips out"

$$h(4) = 36 - \int_1^4 (t+1)\,dt$$

$$= 36 - \left[\tfrac{1}{2}t^2 + t\right]_1^4$$

$$= \boxed{36 - \left[\tfrac{1}{2}(4)^2 + 4 - \left(\tfrac{1}{2}(1)^2 + (1)\right)\right] \text{ ounces}}$$

do not simplify on tests (especially the AP exam)

(= 25.5 ounces)

#8b. For $0 \le t \le 8$ the acceleration of a particle moving along a straight line is given by $a(t) = 3t - 2$. The velocity of the particle is given by $v(t)$ and its position is given by $s(t)$. When $t = 2$, $v(2) = 2$, and $s(2) = 6$.

a) Find the velocity function, $v(t)$.

b) Find the position function, $s(t)$.

c) When is the particle moving to the left? Explain.

d) Find the total distance travelled by the particle from time $t = 0$ to $t = 8$.

e) Find the time t at which the particle is farthest to the left. Explain.

a) $v(t) = \int a(t)dt \Rightarrow \int (3t-2)dt = \frac{3}{2}t^2 - 2t + C$, $v(2) = 2$

$2 = \frac{3}{2}(2)^2 - 2(2) + C \rightarrow C = 0$

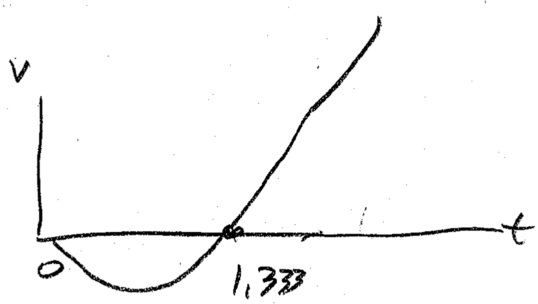
$\boxed{v(t) = \frac{3}{2}t^2 - 2t}$

b) $s(t) = \int v(t)dt = \frac{1}{2}t^3 - t^2 + D$, $s(2) = 6$

$6 = \frac{1}{2}(2)^3 - (2)^2 + D \rightarrow D = 6$

$\boxed{s(t) = \frac{1}{2}t^3 - t^2 + 6}$

c) particle is moving left when $v(t) < 0$: which occurs for $\boxed{0 < t < 1{,}333}$



d) total distance travelled $= -\int_0^{1,333} v(t)dt + \int_{1,333}^{8} v(t)dt$ (use math9)

$= -[-0{,}59259...] + [192{,}59259...] = \boxed{193{,}185}$

e) particle is farthest left at minimum of $s(t)$ which may occur where $s'(t) = v(t) = 0$ or at interval ends:

t	$s(t) = \frac{1}{2}t^3 - t^2 + 6$
0	6
1,333	5.407 ←
8	198

particle is farthest left $\boxed{\text{at } t = 1{,}333}$

#7b. From 5 AM to 10 AM, the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$ where *t* is the number of hours after 5 AM and *A(t)* is measured in vehicles per hour. Traffic is flowing smoothly at 5 AM with no vehicles waiting in line.

a) What is the total number of vehicles that arrive at the toll plaza from 6 AM (*t* = 1) to 10 AM (*t* = 5)?

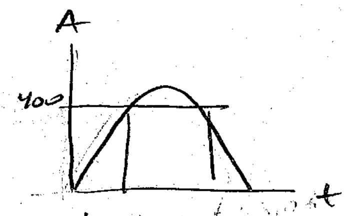
b) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time *t*, for $a \leq t \leq 4$ is given by $N(t) = \int_a^t (A(x) - 400)\,dx$ where *a* is the time when a line first begins to form. To the nearest whole number, fin the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$.

a) $\int_1^5 A(t)dt = \boxed{1502.148 \text{ vehicles}}$ (use math 9 :))

b) The greatest number of vehicles is where N(t) is a maximum which may occur when N'(t) = 0 or at interval ends.

$N'(t) = \frac{d}{dt}\left[\int_a^t (A(x) - 400)\,dx\right] = A(t) - 400 = 0$

when $A(t) = 400$



At t = 1.14693716 A(t) goes above 400 so this is when a line starts forming (a = 1.14693716)

t	$N(t) = \int_{1.14693716}^{t} (A(x) - 400)\,dx$
a = 1.4693716	0
3.5977133	71.25412877 ←
4	62.338345

$\boxed{\text{the greatest number of vehicles in line is approximately 71 vehicles}}$

#8b. A particle is moving along a straight line. Its velocity function, in feet per second, is $v(t)=t^3-8t^2+15t$

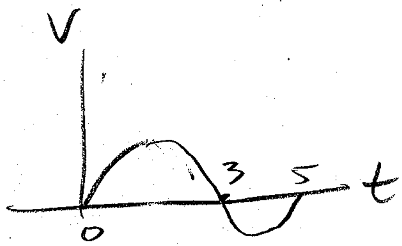
a) Find the displacement over $0 \le t \le 5$.

b) Find the total distance travelled over $0 \le t \le 5$.

(use calculator math 9 for #8 & #8b)

a) displacement $= \int_0^5 (t^3-8t^2+15t)dt = \boxed{10,417 \text{ ft}}$

b)



total distance travelled $= \int_0^3 v(t)dt - \int_3^5 v(t)dt$

$= 15,75 - [-5,333]$

$= \boxed{21,083 \text{ ft}}$