AP Calculus BC – Unit 4, Part 1 Required Practice Name: answers

4.1 – Required Practice

#1. Evaluate $\int \left(3e^x + 7\sec^2(x)\right)\,dx$

$3e^x + 7\tan(x) + C$

#2. Find $f(x)$ given $f''(x)$: $f''(x) = 1 + x^{4/5}$

$f(x) = \frac{1}{2}x^2 + \left(\frac{5}{9}\right)\left(\frac{5}{14}\right)x^{14/5} + Cx + D$

#3. Find $f(x)$ given $f'(x)$:

$f'(x) = 3\sqrt{x} - \frac{1}{\sqrt{x}}, \quad f(1) = 2$

$f(x) = 2x^{3/2} - 2x^{1/2} + 2$

#4. Find $f(x)$ given $f''(x)$:

$f''(x) = 12x^2 - 6x + 2$

$f(0) = 1, \quad f(2) = 11$

$f(x) = x^4 - x^3 + x^2 - x + 1$

#5. A particle moves along the *x*-axis such that the velocity in cm²/sec at time *t* in seconds given by $v(t) = 3t^2 - 2t + 4$. If the particle is at *x* = 3 cm when *t* = 2 seconds, what is the function for the position of the particle, $x(t)$?

$x(t) = t^3 - t^2 + 4t - 9$

#6. A stone is thrown downward with a speed of 5 m/sec from the top of a 450 m tall tower.
a) Find the distance of the stone above the ground level at time *t*.
b) How long does it take the stone to reach the ground?
c) With what velocity does the stone strike the ground?

a) $s(t) = -4.9t^2 - 5t + 450$

b) $t = 9.087$ sec

c) -94.048 m/sec

Evaluate the indefinite integral.

#7. $\int\left(-\frac{6}{x^4}\right)dx$

$$\frac{-6x^{-3}}{-3}+c$$

(or)

$$\frac{2}{x^3}+c$$

#8. $\int(9t^2)\,dt$

$$9\frac{t^3}{3}+c$$

(or)

$$3t^3+c$$

#8. $\int x^{3/2}\,dx$

$$\frac{x^{5/2}}{(5/2)}+c$$

(or)

$$\frac{2}{5}x^{5/2}+c$$

#9. $\int\sqrt[3]{x}\,dx$

$$\frac{x^{4/3}}{(4/3)}+c$$

(or)

$$\frac{3}{4}x^{4/3}+c$$

#10. $\int\frac{1}{4x^2}\,dx$

$$\frac{1}{4}\frac{x^{-1}}{(-1)}+c$$

(or)

$$\frac{-1}{4x}+c$$

#11. $\int(3x^3-6x^2+2)\,dx$

$$3\frac{x^4}{4}-\frac{6x^3}{3}+2x+c$$

(or)

$$\frac{3}{4}x^4-2x^3+2x+c$$

Evaluate the indefinite integral.

#12. $\int \frac{x+6}{\sqrt{x}}\, dx$

$\frac{x^{3/2}}{(3/2)} + \frac{6x^{1/2}}{(1/2)} + c$

(or)

$\frac{2}{3}x^{3/2} + 12x^{1/2} + c$

(or)

$\frac{2}{3}x\sqrt{x} + 12\sqrt{x} + c$

#13. $\int (x+1)(3x-2)\, dx$

$3\frac{x^3}{3} + \frac{x^2}{2} - 2x + c$

(or)

$x^3 + \frac{1}{2}x^2 - 2x + c$

#14. $\int \left(2\sin(x) - 5e^x\right) dx$

$-2\sin x - 5e^x + c$

#15. $\int \left(\tan^2(\theta) + 1\right) d\theta$

$\tan(\theta) + c$

#16. $\int \left(x - \frac{5}{x}\right) dx$

$\frac{1}{2}x^2 - 5\ln|x| + c$

#17. Find $f(x)$ given $f'(x)$:

$f'(x) = 6x, \qquad f(0) = 8$

(when given an (x,y) pair, always find C)

$f(x) = 2x^2 + 8$

#18. A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second.

a) Find $v(t)$ for the ball.

b) Find $h(t)$ where h is the height of the ball above the ground.

c) What is the maximum height of the ball?

a) $v(t) = -32t + 60$

b) $h(t) = -16t^2 + 60t + 6$

c) $h = 62{,}25 \text{ ft}$

#19. The rate of growth of a population P is given by $\frac{dP}{dt} = 20t^3 - 35t^{4/3}$, where t is in years. The initial population is 8000 people.

a) Is the population always increasing? Justify your answer.

b) When is the population at its lowest point?

c) Find the population function and use it to estimate the population after 10 years.

a) No, because $\frac{dP}{dt} < 0$ for $0 < t < 1.399$ years.

b) $t = 1.399$ years

c) $P(t) = 5t^4 - 15t^{7/3} + 8000$

$P(10) = 54768.348$ people

always round to 3 decimal places unless the problem states otherwise

4.2 day 1 – Required Practice

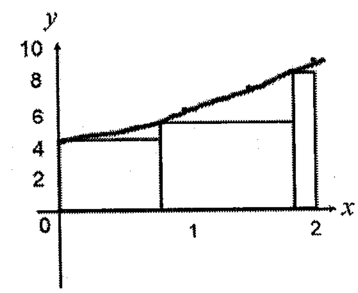
#1. Use the right side of the rectangles with n = 4 to approximate the area of the region bounded by the x-axis and the graph of the function over the given interval.

$$f(x) = x^2 + 4x, \quad [0,4]$$

area ≈ $[(5)(1) + (12)(1) + (21)(1) + (32)(1)]$ $(= 70)$

#2. The values of a function are shown in a table for specific x-values. Evaluate a Riemann Sum using rectangular partitions, left endpoints, and the specific subintervals given:

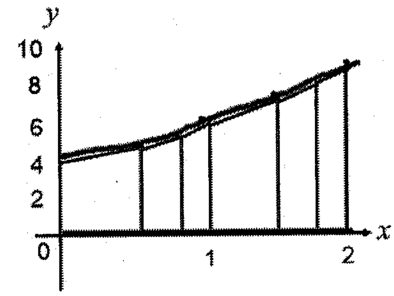
x	0.00	0.50	0.75	1.00	1.50	1.75	2.00
y	4.32	4.58	5.79	6.14	7.64	8.08	8.14



subintervals: [0.00, 0.75] [0.75, 1.75] [1.75, 2.00]

area = $[(4.32)(0.75) + (5.79)(1) + (8.08)(0.25)]$ (≈ 11.05)

#3. The values of a function are shown in a table for specific x-values. Evaluate a Riemann Sum using the Trapezoidal Rule for all the partitions included in the table.



x	0.00	0.50	0.75	1.00	1.50	1.75	2.00
y	4.32	4.58	5.79	6.14	7.64	8.08	8.14

area = 12,45

#4. Approximate the value of $\int_1^7 x^2\,dx$ using a right Riemann Sum with 3 equal-size intervals.

$\int_1^7 x^2 dx \approx [(9)(2)+(25)(2)+(49)(2)]\ (=166)$

#5. Selected values of the function $f(x)$ are given in the table:

x	0	1	2	3	4	5	6
f(x)	-6	0	8	18	30	50	80

Using three equal subintervals, approximate $\int_0^6 f(x)\,dx$ using...

a) ...left endpoints
b) ...midpoints
c) ...right endpoints

d) How does each compare to the actual value of $\int_0^6 f(x)\,dx$?

a) $\int_0^6 f(x)dx \approx [(-6)(2)+(8)(2)+(30)(2)]$ $(=64)$

b) $\int_0^6 f(x)dx \approx [(0)(2)+(18)(2)+(50)(2)]$ $(=136)$

c) $\int_0^6 f(x)dx \approx [(8)(2)+(30)(2)+(80)(2)]$ $(=236)$

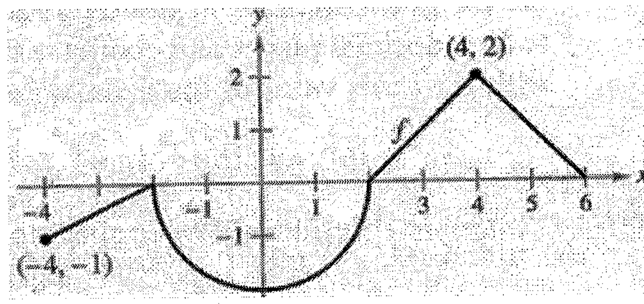
d) - left endpoints are underestimating
- right endpoints are overestimating
- can't really tell for midpoints, but likely closer to actual value than either left or right.

#6. Approximate $\int_0^6 \left(x^2 - 3x\right) dx$ using the Trapezoidal Rule with 3 equal subintervals.

$$\int_0^6 (x^2 - 3x)dx \approx [-2 + 2 + 22] = 22$$

4.2 day 2 – Required Practice

Evaluate the definite integrals using the graph of $f(x)$



#1. $\int_{-4}^{2} f(x)\,dx$

$= -1 - 2\pi$

#2. $\int_{0}^{6} f(x)\,dx$

$= -\pi + 4$

#3. $\int_{6}^{0} f(x)\,dx$

$= \pi - 4$

#4. $\sum_{n=1}^{3} 2\left[(1+2n)^2\right]$ approximates what definite integral?

$\int_1^7 x^2 dx$

#5. $\lim_{n\to\infty} \frac{2}{n}\left[\left(1+\frac{2}{n}\right)^3+\left(1+\frac{4}{n}\right)^3+\left(1+\frac{6}{n}\right)^3+\ldots+\left(1+\frac{2n}{n}\right)^3\right]$ is equivalent to what definite integral?

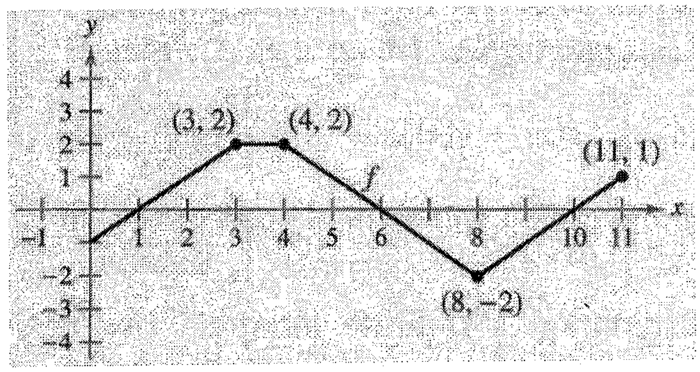
$\int_1^3 x^3 dx$

Find the definite integral which the following Riemann Sum approximates:

#6. $\lim_{n\to\infty} \frac{1}{n}\left[\left(3+\frac{1}{n}\right)^2+\left(3+\frac{2}{n}\right)^2+\left(3+\frac{3}{n}\right)^2+\ldots+\left(3+\frac{n}{n}\right)^2\right]$

$\int_3^4 x^2 dx$

#7. The graph of f consists of line segments as shown in the figure. Evaluate each definite integral using geometry.



a) $\int_{0}^{1} (-f(x))\,dx$ $= \frac{1}{2}$

b) $\int_{3}^{4} 3f(x)\,dx$ $= 6$

c) $\int_{0}^{7} f(x)\,dx$ $= 5$

d) $\int_{5}^{11} f(x)\,dx$ $= -3$

e) $\int_{0}^{11} f(x)\,dx$ $= 2$

f) $\int_{4}^{10} f(x)\,dx$ $= -2$

g) $\int_{10}^{4} f(x)\,dx$ $= 2$

#8. $\lim_{n\to\infty}\frac{4}{n}\left[\cos\left(3+\frac{4}{n}\right)+\cos\left(3+\frac{8}{n}\right)+\cos\left(3+\frac{12}{n}\right)+\ldots+\cos\left(3+\frac{4n}{n}\right)\right]$ is equivalent to what definite integral?

$\int_2^6 \cos(x)dx$

#9. $\lim_{n\to\infty}\frac{6}{n}\left[\left(\frac{6}{n}\right)^3+\left(\frac{12}{n}\right)^3+\left(\frac{18}{n}\right)^3+\ldots+\left(\frac{6n}{n}\right)^3\right]$ is equivalent to what definite integral?

$\int_0^6 x^3dx$

#10. If $\int_0^8 f(x)\,dx=10$, $\int_0^5 f(x)\,dx=-3$, *and* $\int_5^8 g(x)\,dx=12$, find:

a) $\int_8^5 g(x)\,dx$ $= -12$

b) $\int_5^8 f(x)\,dx$ $= 13$

c) $\int_8^5 (3f(x)-g(x))\,dx$ $= 51$

d) $\int_5^5 f(x)\,dx$ $= 0$

(no width)

4.3 – Required Practice

#1. Evaluate $\int_2^4 \left(x^2+2x\right) dx$

$\frac{(4)^3}{2}+(4)^2-\frac{(2)^3}{2}-(2)^2$

(≈ 30.667)

#2. Evaluate $\int_{-1}^3 x^5\, dx$

$\frac{(3)^6}{6}-\frac{(-1)^6}{6}$

(≈ 121.333)

#3. Evaluate $g(x)=\int_2^x t^2\, dt$

$\frac{1}{3}(x)^3-\frac{1}{2}(2)^3$

#4. Evaluate $g(x)=\int_3^{x^2} 3t^3\, dt$

$\frac{3}{4}(x^2)^4-\frac{3}{4}(3)^4$

#5. Find $g'(x)$ *if* $g(x)=\int_2^x t^2\, dt$

$(x)^2$

#6. Find $g'(x)$ *if* $g(x)=\int_3^{x^2} 3t^3\, dt$

$3(x^2)^3(2x)$

Evaluate:

#7. $\frac{d}{dx}\left[\int_{x^2}^{3x^4} f(t)\,dt\right]$

$f(3x^4)\cdot 12x^3 - f(x^2)\cdot 2x$

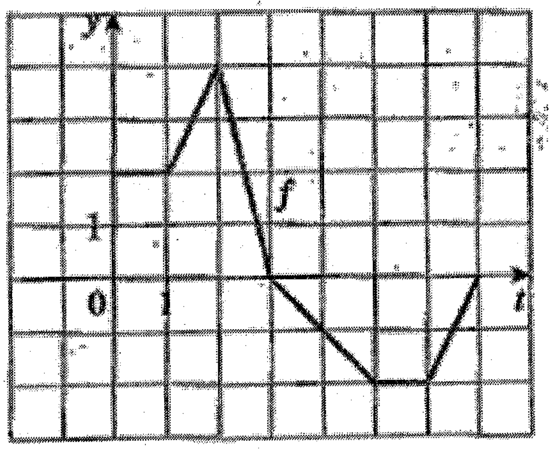
#8. $\frac{d}{dx}\left[\int_{x^3}^{2} f(t)\,dt\right]$

$-f(x^3)\cdot 3x^2$

#9. Find the interval on which the curve is concave upward: $y=\int_0^x \frac{1}{1+t+t^2}\,dt$

for $x < -\frac{1}{2}$

#10. Let $g(x)=\int_0^x f(t)\,dt$ where the graph of f is shown:



a) Evaluate $g(5)$

b) On what interval is $g(x)$ increasing?

c) Where does $g(x)$ have a maximum value?

a) $g(5) = 5$

b) $0 < t < 3$

c) $t = 3$

Evaluate the definite integral by hand. Then use your calculator (math-9) to verify your answer.

#11. $\int_{-1}^{1}(t^2-2)\,dt$

$\left[\frac{1}{3}(1)^3-2(1)\right]-\left[\frac{1}{3}(-1)^3-2(-1)\right]$

(≈ -3.333)

#12. $\int_{1}^{2}\left(\frac{3}{x^2}-1\right)dx$

$\left[-\frac{3}{2}-2\right]-\left[-\frac{3}{1}-1\right]$

$(=0.5)$

#13. $\int_{1}^{4}\frac{u-2}{\sqrt{u}}\,du$

$\left[\frac{2}{3}(4)^{3/2}-4(4)^{1/2}\right]-\left[\frac{2}{3}(1)^{3/2}-4(1)^{1/2}\right]$

(≈ 0.667)

#14. $\int_{0}^{2}(2^x+6)\,dx$

$\left[\frac{2^2}{\ln(2)}+6(2)\right)-\left(\frac{2^0}{\ln(2)}+6(0)\right]$

(≈ 16.328)

#15. $\int_{-\pi/6}^{\pi/6}\sec^2(x)\,dx$

$\tan(\frac{\pi}{6})-\tan(-\pi/6)$

(≈ 1.155)

#16. $\int_{-\pi/3}^{\pi/3}4\sec(\theta)\tan(\theta)\,d\theta$

$4\sec(\pi/3)-4\sec(-\pi/3)$

$(=0)$

Evaluate:

#17. $\frac{d}{dx}\left[\int_{-1}^{x} \sqrt{t^4+1}\, dt\right]$

$\sqrt{(x)^4+1}$

#18. $\frac{d}{dx}\left[\int_{2}^{\sin(x)} \sqrt{t}\, dt\right]$

$\sqrt{\sin(x)} \cdot \cos(x)$

4.4 – Required Practice

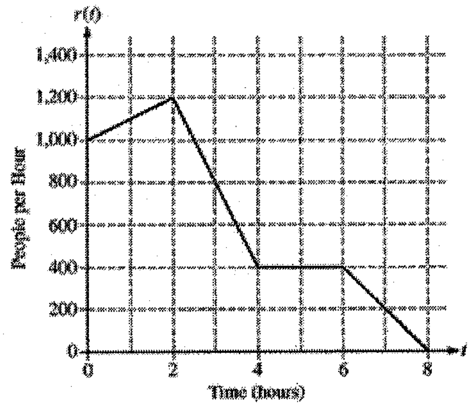
#1. A liquid flows into a storage tank at a rate of $(180+3t)$ liters per minute.
If there is 40 liters of liquid in the tank at time $t = 2$ minutes, how much liquid is in the tank at $t = 10$ minutes?

1624 liters

#2. A hot air balloon's height above the ground is changing at a rate given by $h'(t) = -110t + 550$ where h is in feet and t is in hours.
If the hot air balloon is on the ground at time $t = 0$, what is the height of the balloon at $t = 2$ hours?

880 ft

#3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.



How many people are in line for the ride at $t = 3$ hours?

1900 people

#4. The velocity function, in feet per second, is $v(t) = t^2 - t - 12 \quad \textit{for} \ 1 \le t \le 5$ for a particle moving along a straight line.

a) Find the displacement over the interval.

b) Find the total distance that the particle travels over the given interval.

a) −18.667

b) 26.333

#5. Water flows empties out of a storage tank at a rate of $(500-5t)$ gallons per hour. If there are 5000 gallons in the tank at time $t = 2$ hours, how much water is left in the tank at $t = 3$ hours?

$5000 + \left[-500(3) + \frac{5}{2}(3)^2\right] - \left[-500(2) + \frac{5}{2}(2)^2\right]$ gallons

(do not simplify on test) – especially the AP exam)

(≈ 4512.5 gallons)

#6. For $0 \le t \le 6$ the acceleration of a particle moving along a straight line is given by $a(t) = 2t - 6$. The velocity of the particle is given by $v(t)$ and its position is given by $s(t)$. When $t = 1$, $v(1) = 3$, and $s(1) = \frac{4}{3}$.

a) Find the velocity function, $v(t)$.

b) Find the position function, $s(t)$.

c) When is the particle moving to the left? Explain.

d) Find the total distance travelled by the particle from time $t = 0$ to $t = 6$.

e) Find the time t at which the particle is farthest to the left. Explain.

a) $v(t) = t^2 - 6t + 8$

b) $s(t) = \frac{1}{3}t^3 - 3t^2 + 8t - 4$

c) $2 < t < 4$

d) 14.667

e) $t = 0$

#7. People enter a line for an escalator at a rate modeled by the function *r* given by

$$r(t)=\begin{cases}44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 & for\ 0\le t\le 300\\ 0 & for\ t>300\end{cases}$$

Where *r(t)* is measured in people per second and *t* is measured in seconds. As people get on the escalaro, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time *t* = 0.

a) How many people enter the line for the escalator during the time interval $0\le t\le 300$?

b) During the time interval $0\le t\le 300$ there are always people in line for the escalator. How many people are in line at time *t* = 300?

c) For $t>300$, what is the first time *t* that there are no people in line for the escalator?

d) For $0\le t\le 300$ at what time *t* is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time.

a) 270 people

b) 80 people

c) 414.286 Seconds

d) At $t = 33.013$ sec, there are 4 people in line.

#8. A particle is moving along a straight line. Its velocity function, in feet per second, is $v(t) = t^3 - 10t^2 + 27t - 18$

a) Find the displacement over $1 \le t \le 7$.

b) Find the total distance travelled over $1 \le t \le 7$.

a) 0 ft

b) 31.5ft

Unit 4 Part 1 Test Review

Evaluate the integral.

#1. $\int \left(3x^2 + \frac{1}{x^4}\right) dx = 3\int x^2 dx + \int x^{-4} dx$

$\boxed{\frac{3x^3}{3} + \frac{x^{-3}}{-3} + c}$

#2. $\int \left(x^{2/3} - \frac{1}{x^{4/5}}\right) dx = \int (x^{2/3} - x^{-4/5}) dx$

$\boxed{\frac{3}{5}x^{5/3} - 5x^{1/5} + c}$

#3. $\int_1^4 \frac{2}{x^3} dx = 2\int_1^4 x^{-3} dx$

$2\left[\frac{x^{-2}}{-2}\right]_1^4 = \left(\frac{-1}{x^2}\right]_1^4$

$\boxed{-\frac{1}{(4)^2} - \frac{1}{(1)^2}}$

#4. $\int \frac{4}{x} dx = \boxed{4\ln|x| + C}$

#5. $\int \frac{x^2 - 3x}{\sqrt{x}} dx = \int \frac{x^2}{x^{1/2}} dx - 3\int \frac{x}{x^{1/2}} dx$

$= \int x^{3/2} dx - 3\int x^{1/2} dx$

$\boxed{\frac{2}{5}x^{5/2} - 3\left(\frac{2}{3}\right)x^{3/2} + c}$

#6. $\int_0^3 t^2(3t+1)\, dt = \int_0^3 (3t^3 + t^2) dt$

$\left[\frac{3}{4}t^4 + \frac{1}{3}t^3\right]_0^3$

$\boxed{\frac{3}{4}(3)^4 + \frac{1}{3}(3)^3 - 0}$

#7. $\int (e^x - 4^x)\, dx$

$\boxed{e^x - \frac{4^x}{\ln(4)} + c}$

#8. $\int \sin(x)\, dx$

$\boxed{-\cos(x) + C}$

#9. $\int_{\pi/4}^{\pi/2} \cos(x)\, dx$

$\left[\sin(x)\right]_{\pi/4}^{\pi/2}$

$\boxed{\sin(\pi/2) - \sin(\pi/4)}$

#10. $\int 2\csc^2(x)\, dx$

$\boxed{-2\cot(x) + C}$

#11. $\int 3\csc(x)\cot(x)\, dx$

$\boxed{-3\csc(x) + C}$

#12. $\int \sec(x)(\tan(x) - \sec(x))\, dx$

$= \int \sec x \tan x\, dx - \int \sec^2 x\, dx$

$\boxed{\sec(x) - \tan(x) + C}$

#13. $\int \frac{5}{1+x^2} dx$

$\boxed{5\arctan(x) + C}$

#14. $\int \frac{7}{\sqrt{1-x^2}} dx$

$\boxed{7\arcsin(x) + C}$

#15. $\frac{d}{dx}\left[\int_{2}^{e^{3x}} \cos\left(t^2\right) dt\right]$

$\boxed{\cos\left(\left(e^{3x}\right)^2\right) e^{3x}(3)}$

#16. $\frac{d}{dx}\left[\int_{2}^{\cos(x)} \left(3\sqrt{t}+t^2\right) dt\right]$

$\boxed{\left(3\sqrt{\cos x}+(\cos x)^2\right)(-\sin x)}$

#17. Approximate $\int_{1}^{10}(2+x)\, dx$ using a left Riemann Sum with 3 equal subintervals. width $= \frac{10-1}{3} = 3$

Interval	x_i	$f(x_i)\cdot \Delta x$	$=$ area
$[1,4]$	1	$3 \cdot 3$	$= 9$
$[4,7]$	4	$6 \cdot 3$	$= 18$
$[7,10]$	7	$9 \cdot 3$	$= 27$

$\boxed{\int_{1}^{10}(2+x)dx \approx \left[(3)(3)+(6)(3)+(9)(3)\right]}$

#18. Approximate $\int_{-2}^{2}\left(x^2+3\right) dx$ using a right Riemann Sum with 2 equal subintervals. width $= \frac{2-(-2)}{2} = \frac{4}{2} = 2$

Interval	x_i	$f(x_i)\cdot \Delta x$	$=$ area
$[-2,0]$	0	$3 \cdot 2$	$= 6$
$[0,2]$	2	$7 \cdot 2$	$= 14$

$\boxed{\int_{-2}^{2}(x^2+3)dx \approx 20}$

#19. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function $g'(t)$ measured in gallons per second. Selected values of $g'(t)$ are given in the table:

t	0	60	90	120	135	150
$g'(t)$	0	0.1	0.15	0.1	0.05	0

a) Use a left Riemann sum to approximate the value of $\int_{60}^{135} g'(x)\,dx$ using intervals from the table.

interval	x_i	$f(x_i)$ · Δx	
[60, 90)	60	0.1	· 30
[90, 120)	90	0.15	· 30
[120, 135]	120	0.1	· 15

(no calculator, so...)

$$\boxed{\int_{60}^{135} g'(x)\,dx \approx [(0.1)(30) + (0.15)(30) + (0.1)(15)] \text{ gallons}}$$

b) Using your approximation from part a and the fact that there are 6 gallons of gasoline in the tank at time $t = 60$ seconds, find the amount of gasoline in the tank at time $t = 135$ seconds.

$$g(135) = g(60) + \int_{60}^{135} g'(t)\,dt$$

$$\boxed{g(135) \approx 6 + [(0.1)(30) + (0.15)(30) + (0.1)(15)] \text{ gallons}}$$

#20. The rate at which a population of predators in a forest grows over time is given by $\frac{dP}{dt} = 2e^t + 3t$ where t is in months. If there are 30 predators in the forest at t = 6 months, how many predators are in the forest at t = 12 months?

$$P(12) = P(6) + \int_6^{12} (2e^t + 3t)\,dt$$

$$= 30 + \left[2e^t + \tfrac{3}{2}t^2\right]_6^{12}$$

$$\boxed{P(12) = 30 + \left(2e^{12} + \tfrac{3}{2}(12)^2\right) - \left[2e^6 + \tfrac{3}{2}(6)^2\right] \text{ predators}}$$

#21. $\lim_{n\to\infty} \frac{5}{n}\left[\left(3+\frac{5}{n}\right)^2 + \left(3+\frac{10}{n}\right)^2 + \left(3+\frac{15}{n}\right)^2 + \ldots + \left(3+\frac{5n}{n}\right)^2\right]$ is equivalent to what definite integral?

$a = 3$

$\text{width} = \frac{b-a}{n} = \frac{b-3}{n} = \frac{5}{n}$

$b - 3 = 5$

$b = 8$

$$\boxed{\int_3^8 x^2\,dx}$$

#22. $\lim_{n\to\infty} \frac{2}{n}\left[\sin\left(\frac{2}{n}\right) + \sin\left(\frac{4}{n}\right) + \sin\left(\frac{6}{n}\right) + \ldots + \sin\left(\frac{2n}{n}\right)\right]$ is equivalent to what definite integral?

$a = 0$

$\text{width} = \frac{b-a}{n} = \frac{b}{n} = \frac{2}{n}$

$b = 2$

$$\boxed{\int_0^2 \sin(x)\,dx}$$

#23. If $\int_2^{12} f(x)\,dx = 20$, $\int_2^7 f(x)\,dx = 6$, and $\int_7^{12} g(x)\,dx = 5$, find:

a) $\int_{12}^{7} f(x)\,dx = -\int_7^{12} f(x)dx = -\left[\int_2^{12} f(x)dx - \int_2^7 f(x)dx\right] = -[20 - 6] = \boxed{-14}$

b) $\int_7^{12} (2g(x) - 3f(x))\,dx = 2\int_7^{12} g(x)dx - 3\int_7^{12} f(x)dx$

$= \boxed{2(5) - 3[-(-14)]}$

#24. A hot-air balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant when it is 64 feet above the ground.

a) Find $v(t)$ for the sandbag.

b) Find $h(t)$ where h is the height of the sandbag above the ground.

a) $v(t) = \int a(t)dt = \int (-32)dt = -32t + C$ $\quad v(0) = 16$

$16 = -32(0) + C \Rightarrow C = 16$

$\boxed{v(t) = -32t + 16}$

b) $h(t) = \int v(t)dt = \int (-32t + 16)dt = -16t^2 + 16t + D$, $\quad s(0) = 64$

$64 = -16(0)^2 + 16(0) + D \Rightarrow D = 64$

$\boxed{h(t) = -16t^2 + 16t + 64}$

#25. A particle is moving in a straight line such that the x-coordinate of its position is given by $x(t) = -t^2 + 4t + 2$

a) Find $v(t)$ for the particle.

b) Find $a(t)$ for the particle.

c) At what time is the particle at rest?

a) $\boxed{v(t) = x'(t) = -2t + 4}$

b) $\boxed{a(t) = v'(t) = -2}$

c) at rest when $v(t) = 0$: $\quad -2t + 4 = 0$

$2t = 4$

$\boxed{t = 2}$